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PATH INTEGRAL BOSONIZATION FOR MASSIVE FERMION MODELS IN TWO DIMENSIONS

by

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ABSTRACT

We study the bosonization of the massive Thirring model in the framework of the path integrals.

Key-words: Path integral; Bosonization; Two dimensional models.

Analysis of quantum field models in two space-time dimensions have proved to be useful theoretical laboratory to understand phenomena like dynamical mass generation, confinement, topological excitations, all features expected to be present in the more realistics four dimensional quantum theories.

Recently a powerful non-perturbative technique has been used to analyse several two dimensional non massive fermion models in the (euclidean) path integral approach. This technique is based on a suitable chiral change of variables.[1,2,3,4]

It is the purpose of this brief report to show how to deal with the case of massive fermion models in the framework of the above technique by studying the massive abelian Thirring model.[5]

Let us start our analysis by considering the euclidean lagrangian of the model

$$\mathcal{L}_1(\psi,\bar{\psi})(x) = (-i\bar{\psi}\gamma_\mu\partial_\mu\psi + m\bar{\psi}\psi + \frac{g^2}{2}\,(\psi\gamma_\mu\psi)^2(x) \qquad (1)$$

where $\psi=(\psi_1,\psi_2)$ denotes a two dimensional massive fermion field of (bare) mass m and g the coupling constant.

The hermitean γ-matrices we are using satisfy the (euclidean) relations

$$\{\gamma_\mu,\gamma_\nu\} = 2\gamma_{\mu\nu} \quad ; \quad \gamma_\nu\gamma_5 = i\varepsilon_{\mu\nu}\gamma_\nu \quad ; \quad \gamma_5 = i\gamma_0\gamma_1$$

$$\varepsilon_{01} = -\varepsilon_{10} = 1 \qquad (2)$$

The lagrangian (1) is invariant under the global abelian group $\psi \to e^{i\alpha}\psi$ ($\alpha \in \mathbb{R}$) with the noetherian conserved current

$$\partial_\mu(\bar{\psi}\gamma^\mu\psi)(x) = 0$$

In order to construct an equivalent bosonic theory for the model (1), we consider the quantum partition functional

$$Z = \int D[\psi(x)]D[\bar{\psi}(x)]e^{-\int d^2x\, \mathcal{L}_1(\psi,\bar{\psi})(x)} \tag{3}$$

It will be useful for our purposes to write the interaction lagrangian in (3) in a form closely parallel to the usual fermion--vector coupling in gauge theories by making use of the following identity:

$$e^{-\frac{g^2}{2}\int d^2x(\bar{\psi}\gamma^\mu\psi)^2(x)} = \int D[A_\mu(x)]e^{-\int d^2x\,\frac{1}{2}A_\mu^2(x)}$$
$$e^{+\int d^2x\, ig(\bar{\psi}\gamma^\mu\psi)(x)A_\mu(x)} \tag{4}$$

where $A_\mu(x)$ is an auxiliary abelian vector field.

After the use of (4), Z becomes

$$Z = \int D[A_\mu(x)]D[\psi(x)]D[\bar{\psi}(x)]e^{-\frac{1}{2}\int d^2x\;A_\mu^2(x)}$$
$$e^{-\int d^2x(-i\bar{\psi}\gamma_\mu(\partial_\mu - gA_\mu)\psi + m\bar{\psi}\psi)(x)} \tag{5}$$

Now, we proceed as in the massless case by making the change of variables[3]

$$\psi(x) = e^{ig\gamma_5\beta(x) + i\eta(x)}\chi(x) \tag{6}$$

$$\bar{\psi}(x) = \bar{\chi}(x).e^{ig\gamma_5\beta(x) - i\eta(x)}\chi(x) \tag{7}$$

$$A_\mu(x) = (+\epsilon_{\mu\nu}\partial_\mu\beta - \frac{i}{g}\partial_\mu\eta)(x) \tag{8}$$

At this point, it becomes important to remark that the fermionic measure $D[\psi(x)]D[\bar{\psi}(x)]$ in (5) is defined in terms of the normalized eigenvectors of the hermitean Dirac operator $-i\gamma_\mu(\partial_\mu - gA_\mu)$, since we are dealing with the massive Thirring model as a mass perturbation model around the massless case closely to the idea of the conventional bosonization scheme implemented by Coleman.[4,5]

As has been shown by Fujikawa[1] the transformations (6) and (7) are not free of cost due to the non-invariance of the functional fermionic measure under chiral change of variables. The resulting jacobian is given by[3]

$$D[\psi(x)]D[\bar{\psi}(x)] = e^{-\{\int d^2x \frac{1}{2}(1+\frac{g^2}{\pi})(\partial_\mu\beta)^2 + -\frac{1}{2}\frac{(1+\frac{g^2}{\pi})}{g^2}(\partial_\mu\eta)^2\}} D[\chi(x)]D[\bar{\chi}(x)] \tag{9}$$

Concerning the transformation (8), we have the result[3]

$$D[A_\mu(X)] = \mathrm{Det}[\frac{i}{g}(\partial_0^2 - \partial_1^2)]\, D[\beta(x)]\, D[\eta(x)] \tag{10}$$

Substituing eq.(9) and eq.(10) in eq.(5), we obtain the expression

$$Z = \int D[\beta(x)]D[\eta(x)]\mathrm{Det}[\frac{i}{g}(\partial_0^2 - \partial_1^2)] \\ e^{-\{\int d^2x[\frac{1}{2}(1+\frac{g^2}{\pi})(\partial_\mu\beta)^2 - \frac{1}{2}\frac{(1+\frac{g^2}{\pi})}{g^2}(\partial_\mu\eta)^2\}} \\ (\int D[\chi(x)]D[\bar{\chi}(x)]e^{-\{\int d^2x(-\bar{\chi}\, i\gamma_\mu\partial_\mu\chi + m\,\bar{\chi}\, e^{2ig\gamma_5\beta}\chi)(x)\}}) \tag{11}$$

Now we note that the (unphysical) $\eta(x)$ field is decoupled in the effective partition functional given by the equation (11); since it is related to the spurios longitudinal part of the conserved U(1) current of the model (note that at the classical level the field $A_\mu(x)$ coincides with $(\bar{\psi}\gamma^\mu\psi)(x)$). As a consequence solely the transversal part of $A_\mu(x)$ effectively contribute to the partition functional (11). Thus, we get the effective result

$$Z = \int D[\beta(x)]\, e^{-\{\int d^2x\ \frac{1}{2}(1+\frac{g^2}{\pi})(\partial_\mu\beta)^2(x)\}}$$
$$(\int D[\chi(x)]D[\bar{\chi}(x)]\, e^{-\{\int d^2x(-\bar{\chi}\, i\gamma_\mu\partial_\mu\chi + m\bar{\chi}\, e^{2ig\gamma_5\beta}\chi)(x)\}} \quad (12)$$

Now we note that, opposite to the massless case, we have not decoupled completely the massive fermions from the field $A_\mu(x)$, since it remains in eq.(12) the mass coupling term

$$m\left(\bar{\chi}\, e^{2ig\gamma_5\beta}\chi\right)(x) = m\left(\bar{\chi}\left(\frac{1+\gamma_5}{2}\right)\chi\, e^{2ig\beta} + \bar{\chi}\left(\frac{1-\gamma_5}{2}\right)\chi\ e^{-2ig\beta}\right)(x) \quad (13)$$

We, then, face the problem to evaluate the fermionic functional integral

$$I[\beta(x)] = \int D[\chi(x)]D[\bar{\chi}(x)]\, e^{-\{\int d^2x(-\bar{\chi}(i\gamma_\mu\partial_\mu)\chi)(x)}$$
$$+ m(\sigma_+ e^{2ig\beta} + \sigma_- e^{-2ig\beta})(x)\} \quad (14)$$

where we have introduced the objects

$$\sigma_{\pm}(x) = \left(\bar{\chi}\left(\frac{1\pm\gamma_5}{2}\right)\chi\right)(x) \tag{15}$$

In order to evaluate (14), we make a serie expansion of the term $\exp\{-m\int d^2x(\sigma_+ e^{2ig\beta} + \sigma_- e^{-2ig\beta})(x)\}$ in powers of the (bare) fermion mass m.

Explicitly:

$$I[\beta(x)] = \sum_{n=0}^{\infty} \frac{(-m)^n}{n!} \{\int d^2x_1 \ldots d^2x_\mu (\int D[\chi(x)]\, D[\bar{\chi}(x)] e^{-\int d^2x(\bar{\chi}(-i\gamma_\mu\partial_\mu)\chi(x)}$$

$$(\sigma_+ e^{2ig\beta} + \sigma_- e^{-2ig\beta})(x_1) \ldots (\sigma_+ e^{2ig\beta} + \sigma_- e^{-2ig\beta})(x_N) \tag{16}$$

Now it is a well-known result that the only nonzero terms in (16) are those with equal number of σ_+'s and σ_-'s, [5,6] i.e.:

$$\int D[\chi(x)]D[\bar{\chi}(x)]e^{-\int d^2x(\bar{\chi}(-i\gamma_\mu\partial_\mu)\chi)(x)} \prod_{i=1}^{k} \sigma_+(x_k) \cdot \prod_{i=1} \sigma_-(y_k)$$

$$= \left(\frac{1}{2\pi}\right)^{2k} \frac{\prod_{i>j}^{k} (x_i-y_j)^2 \cdot (y_i-y_j)^2}{\prod_{(i,j)}^{k} (x_i-y_j)^2} \tag{17}$$

with the massless fermion propagator given by

$$(i\gamma_\mu\partial_\mu)^{-1}(x,y) = +\frac{1}{2\pi}\gamma_\mu \cdot \frac{(x_\mu-y_\mu)}{(x-y)^2} \tag{18}$$

By following Coleman,[5] we introduce a massless scalar field $\phi(x)$ with the (infrared regularized) Green function given by $\Delta(x) = -\frac{1}{4\pi}\,\ell g\,\frac{x^2}{\varepsilon^2}$ (ε is a infrared cut-off with mass dimension) and re-write eq.(17) in the form

$$\left(\frac{1}{2\pi}\right)^{2k} e^{-2k\Delta(0)} (\int D[\Phi(x)]e^{-\int d^2x\,\frac{1}{2}(\partial_\mu\phi)^2(x)}$$

$$e^{\sqrt{8\pi}\, i\, (\sum_{i=1}^{k} \phi(x_i) - \phi(y_i))} \tag{19}$$

By noting that the averages

$$\int D[\phi(x)]e^{-\int d^2x\ \frac{1}{2}(\partial_\mu\phi)^2(x)}\ e^{\sqrt{8\pi}.i(\sum_{i=1}^{k}\phi(x_i)-\sum_{i=1}^{\ell}\phi(y_i))}$$

are zero for $k\neq\ell$ due to the infrared divergencies of the massless scalar field $\phi(x)$ [5] we can write $I[\beta(x)]$ (see eq.(14)) in the form

$$I[\beta(x)] = \int D[\phi(x)]e^{-\{\int d^2x\ \frac{1}{2}(\partial_\mu\phi)^2(x)\}}$$
$$e^{-\int d^2x\{(m\ e^{-\Delta(0)}).\frac{1}{4\pi}\cos(2g\beta+\sqrt{8\pi}\ \phi)(x)\}} \tag{20}$$

where we see that the (bare) mass parameter gets a multiplicative (ultraviolet) renormalization $m_R = m\ e^{-\Delta(0)}$.[5]

Finally, substituing eq.(20) in eq.(12), we get the effective bosonic action for the fermionic massive Thirring model (see eq.(3):

$$Z = \int D[\beta(x)]D[\phi(x)]\ e^{-\{\int d^2x\ \frac{1}{2}[(1+\frac{\gamma^2}{\pi})(\partial_\mu\beta)^2\ +\ (\partial_\mu\sigma)^2](x)\}}$$
$$e^{-\frac{m_R}{4\pi}\int d^2x\ \cos(2g\beta+\sqrt{8\pi}\ \sigma)(x)} \tag{21}$$

In order to analyse the physical spectrum associated to the effective bosonic action eq.(21) we introduce the new fields

$$\begin{aligned} 2g\,\tilde{\beta}(x) &= (2g\,\beta+\sqrt{8\pi}\ \phi)(x) \\ \tilde{\phi}(x) &= (c_o\beta+c_1\phi)(x) \end{aligned} \tag{22}$$

where the arbitrary constants c_o, c_1 satisfy the relation

$$c_1 c_o = - \frac{\sqrt{8\pi}}{2g(1+g^2)}$$
$$\frac{c_1}{c_o} \neq \frac{\sqrt{8\pi}}{2g} \qquad (23)$$

In terms of these new fields, the effective lagrangean in eq.(21) takes, then, the more transparent form:

$$\mathcal{L}_2(\tilde{\beta},\tilde{\phi})(x) = (1+\frac{g^2}{\pi})\frac{(c_1^2+\frac{8\pi}{4g^2})}{(c_1-c_o\frac{\sqrt{8\pi}}{2g})^2}(\partial_\mu\tilde{\beta})^2(x) +$$
$$\frac{1}{(c_1-c_o\frac{\sqrt{8\pi}}{2g})^2}(1+(1+\frac{g^2}{\pi})c_o^2)(\partial_\mu\tilde{\phi})^2(x)$$
$$+\frac{m_R}{4\pi}\cos(2g\,\tilde{\beta}(x)) \qquad (24)$$

There is thus, in the spectrum of the model a massless scalar field $\tilde{\phi}(x)$ and a sine-Gordon field $\tilde{\beta}(x)$. We remark that this result agrees with these obtained in a conventional operator approach.[5]

It is instructive to point out that the massless scalar field $\tilde{\phi}(x)$ is a remnant of "almost long-range order" of the Kosterlitz--Thouless type which occurs in the infrared region of the massless Thirring model ($m_R=0$).[7]

Finally, we note that similar analysis can be straightforward implemented for the non-abelian version of the model,[3] or for the massive (abelian and non-abelian) fermion gauge theories.[8,9]

As we have shown, chiral changes in path integrals even for massive fermion models provide a quick, mathematically and conceptually simple way to analyse two dimensional fermion models.

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