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SPONTANEOUS MAGNETISATION OF THE EXTENDED DISCRETE N-VECTOR FERROMAGNET

by

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Abstract. Within a simple real space renormalization group framework, we calculate the thermal evolution of the spontaneous magnetisation associated with the (extended) discrete N—vector ferromagnet. The corresponding critical exponents β are calculated as well.

Key-words: Critical phenomena; N-vector model magnetisation; Renormalisation Group.

The (extended) discrete N-vector model (or cubic model) unifies in a single framework a large amount of theoretically and experimentally important statistical models (see [1–6] and references therein). Its Hamiltonian is given by

$$\beta \mathcal{H} = - \sum_{<i,j>} \left[N K (\vec{S}_i \cdot \vec{S}_j) + N^2 L (\vec{S}_i \cdot \vec{S}_j)^2 \right] \tag{1}$$

where $\beta \equiv 1/k_B T$ and the spin $\vec{S}_i$ at any given site is a N-component unitary vector which can point only along the $2N$ positive or negative orthogonal coordinate directions, i.e., $\vec{S}_i = (\pm 1, 0, 0, ..., 0)$ or $(0, \pm 1, 0, ..., 0)$ or ... or $(0, 0, 0, ..., \pm 1)$. This interaction is a discrete version of the classical N-vector model. Let us list a few important particular cases:

(*i*) $N \rightarrow 0$ and arbitrary L yields [5,7] the self-avoiding walk (SAW) problem;

(*ii*) $N = 1$ and arbitrary L yields the spin 1/2 Ising model;

(*iii*) $N = 2$ reproduces the $Z(4)$ model (see [8] and references therein);

(*iv*) $R \equiv NL/K = 1$ yields the $2N$-state Potts model with dimensionless coupling constant $2NK$;

(*v*) $K = 0$ yields the N-state Potts model with dimensionless coupling constant $N^2 L$;

(*vi*) Finite K and $NL/|K| \rightarrow \infty$ yields the spin 1/2 Ising model for arbitrary N.

The criticality (phase diagram and thermal critical exponents) associated with Hamiltonian (1) is easily tractable within a correlation function-preserving real space renormalization group (RG) because no coupling constant proliferation occurs for arbitrary (real) value of N [5,6]. To discuss the square lattice we shall follow along the lines of [5] and use the self-dual Wheatstone-bridge cluster (see Fig. 1); the corresponding recursive RG equations are given by

$$K' = \frac{1}{2N} \ln (G_1/G_2) \tag{2}$$

$$L' = \frac{1}{2N^2} \ln (G_1 G_2/G_3^2) \tag{3}$$

where

$$G_1 \equiv e^{5N^2L}\left[e^{5NK} + e^{-3NK} + 2e^{-NK}\right] + 2\,(N-1)\,\left[2e^{2N^2L}\left[e^{2NK} + e^{-2NK}\right] + e^{N^2L}\left[e^{NK} + e^{-NK}\right] + 2N-4\right] \tag{4}$$

$$G_2 \equiv 2\,e^{5N^2L}\left[e^{NK} + e^{-NK}\right] + 2\,(N-1)\,\left[4\,e^{2N^2L} + e^{N^2L}\left[e^{NK} + e^{-NK}\right] + 2N-4\right] \tag{5}$$

$$G_3 \equiv 2\left\{e^{3N^2L}\left[e^{3NK} + 3\,e^{-NK}\right] + e^{2N^2L}\left[e^{2NK} + 2 + e^{-2NK}\right] + (N-2)\left[5e^{N^2L}\left[e^{NK} + e^{-NK}\right] + 2N-6\right]\right\} \tag{6}$$

We are here interested in the calculation of the spontaneous magnetisation. To perform its calculation we shall follow along the lines of [9], and add, to eqs. (2) and (3), the following one

$$\mu' = h(K,L)\,\mu \tag{7}$$

where μ is proportional to the spontaneous magnetisation and $h(K,L)$ is obtained through Table 1.

Typical phase diagrams are indicated in Fig. 2 [5], and typical results for the spontaneous magnetisation are indicated in Figs. 3–5. In particular Fig. 6 shows, for the nontrivial cubic fixed point, the critical exponent β calculated from

$$\beta = \frac{\ln\,[1/h\,(K^*,L^*)]}{\ln\,\lambda} \tag{8}$$

where (K^*,L^*) refer to the cubic fixed point, and λ denotes the largest eigenvalue of the 2×2 Jacobian matrix associated with eqs. (2) and (3). The asterisk indicates the Ising value for β obtained by Caride and Tsalis [9], which is a perfect agreement between this result and ours (N=1). It is worthy to mention that this is not the first time that non-monotonous behaviour like that of Fig. 6 is observed in hierarchical lattices (see, for instance, refs. [5] and [10]). To the best of our knowledge, this is the first time that the N-evolution of β is calculated. The present results could be numerically improved by considering larger self-dual Wheatstone-bridge-like graphs.

Figure Captions

Figure 1. The self–dual Wheatstone–bridge cluster used to obtain the RG recursive equations. • and ∘ denote, respectively, internal and terminal (root) sites.

Figure 2. N=2 typical phase diagrams (reproducing that of ref. 5).

Figure 3. Magnetisation versus reduced temperature for $N = 2$, for several values of R ($\equiv NL/K$). Curve (a) $R = 0$, (b) $R = 0.5$, (c) $R = 1$, (d) $R = 2$, (e) $R = 5$ (long dash) and $R = 10$ (short dash).

Figure 4. Magnetisation versus reduced temperature for $R \equiv NL/K = 2$, for several values of N. Curve (a) $N = 0.5$, (b) $N = 1$, (c) $N = 2$, (d) $N = 3$, (e) $N = 5$, (f) $N = 10$.

Figure 5. Magnetisation versus reduced temperature for several values of N and the corresponding value of R at the cubic fixed point. Curve (a) $N = 1$ (long dash) and $N = 2$ (short dash), (b) $N = 3$, (c) $N = 4$, (d) $N = 10$.

Figure 6. Magnetic critical exponent β versus N at the cubic fixed point. The asterisk indicates the Ising value of β obtained in ref. 9, which corresponds $N = 1$.

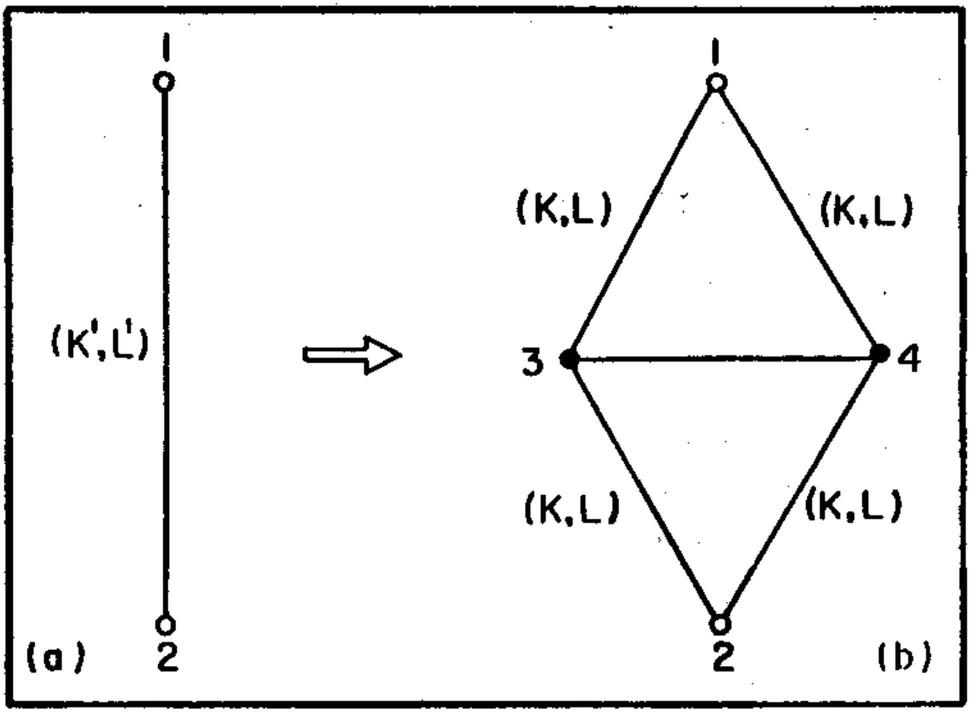


FIG. 1

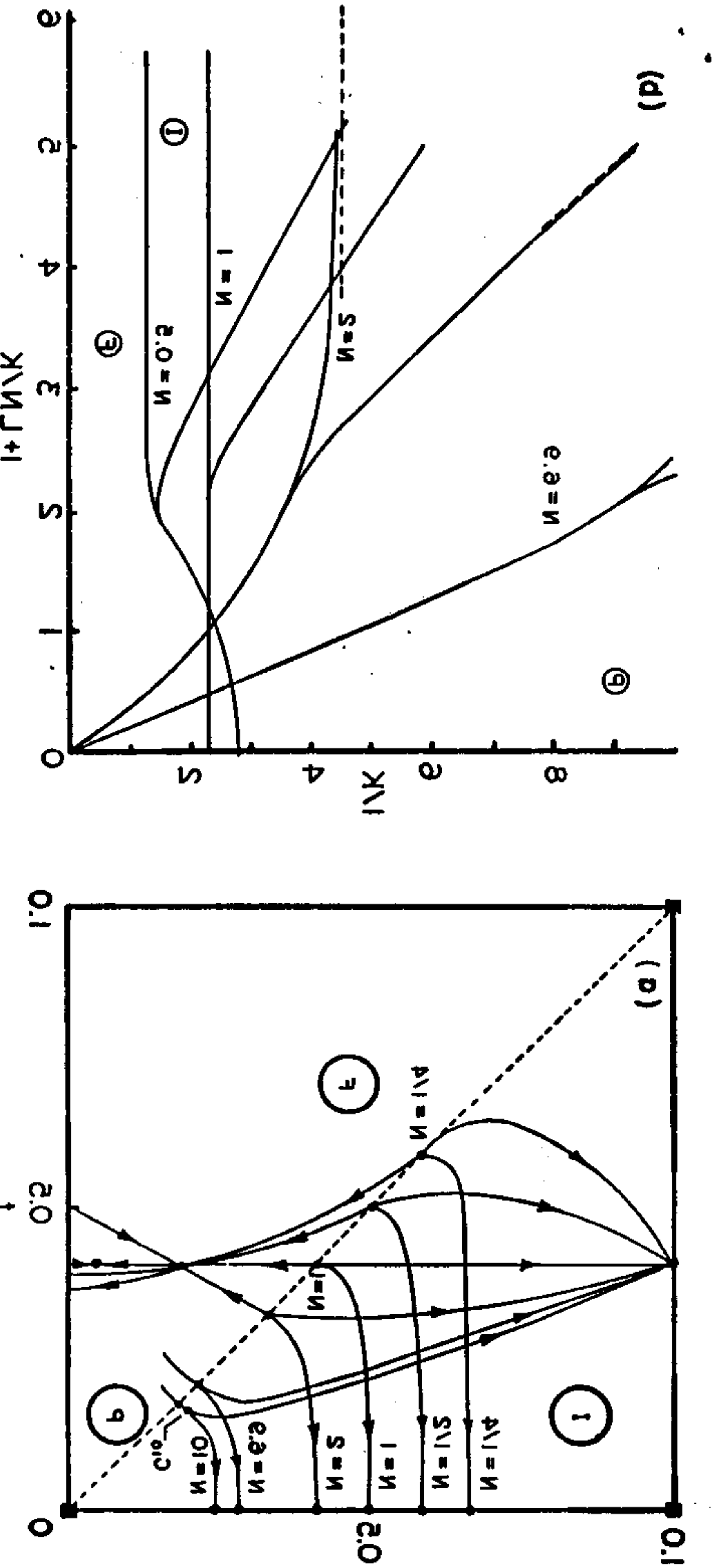


FIG. 2

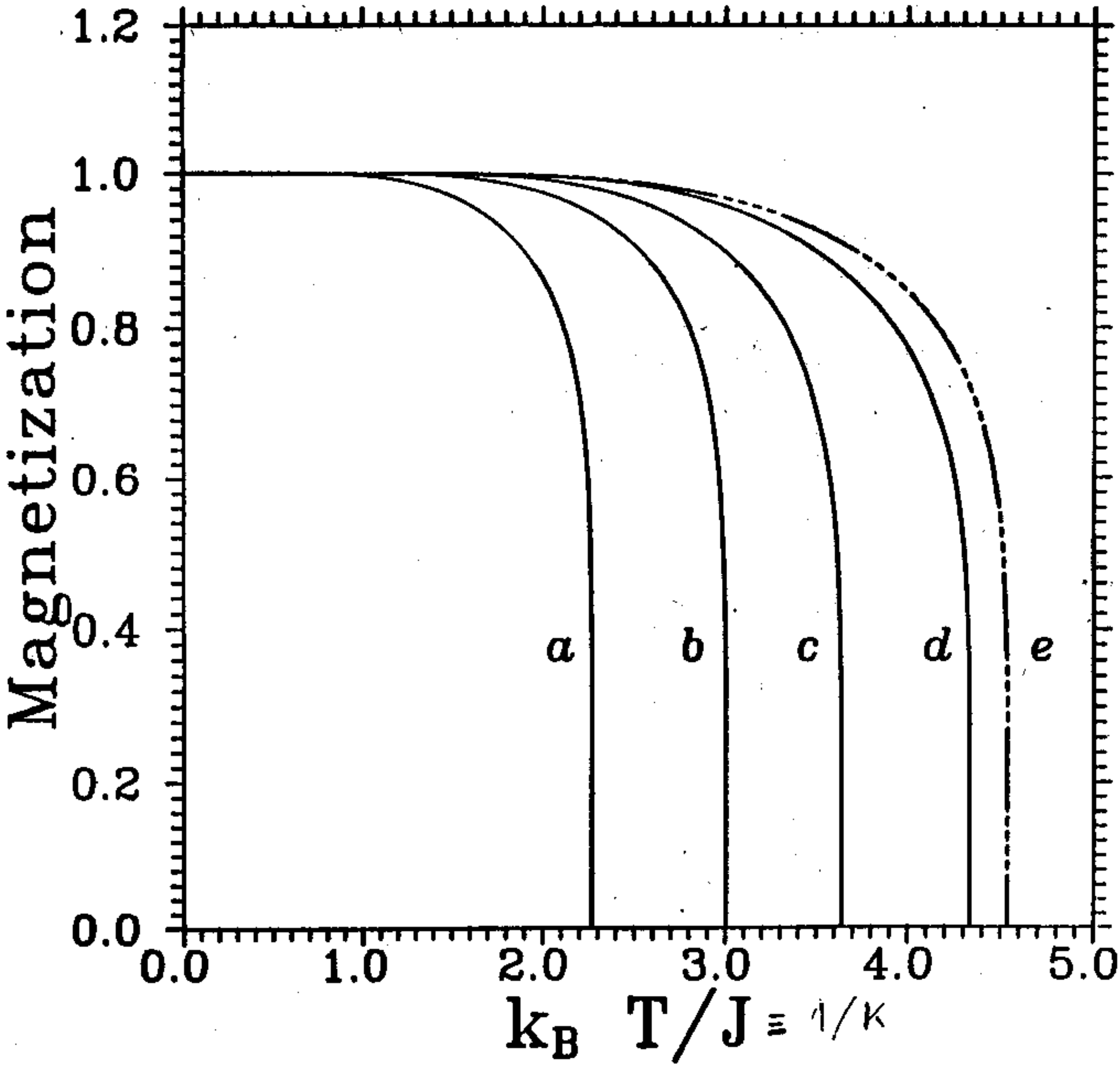

Magnetization
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0
5.0
k_B T/J ≡ 1/K
a
b
c
d
e

FIG. 3

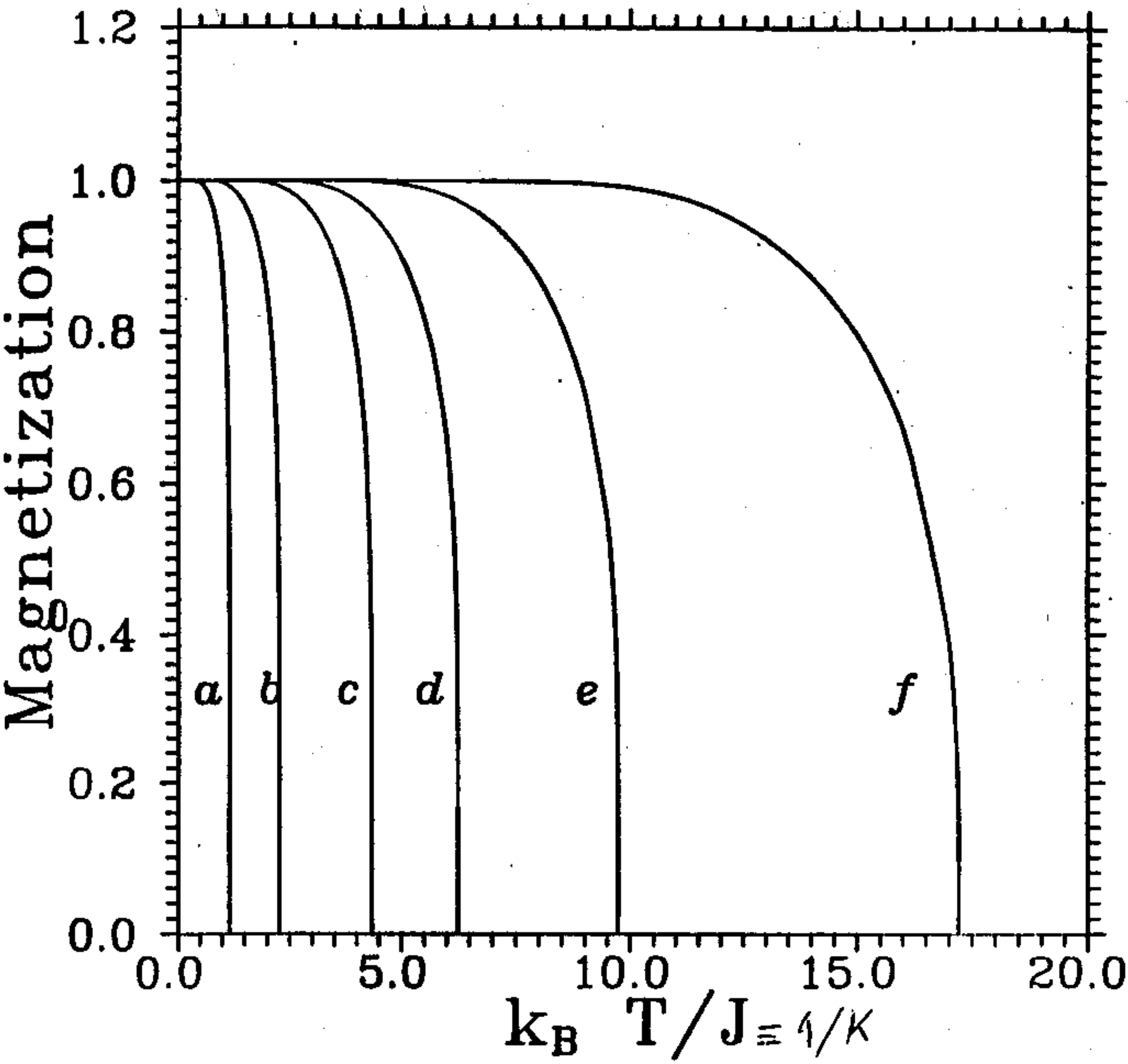

1.2
1.0
0.8
0.6
0.4
0.2
0.0
Magnetization
a
b
c
d
e
f
0.0
5.0
10.0
15.0
20.0
k_B T/J ≡ 1/K

FIG. 4

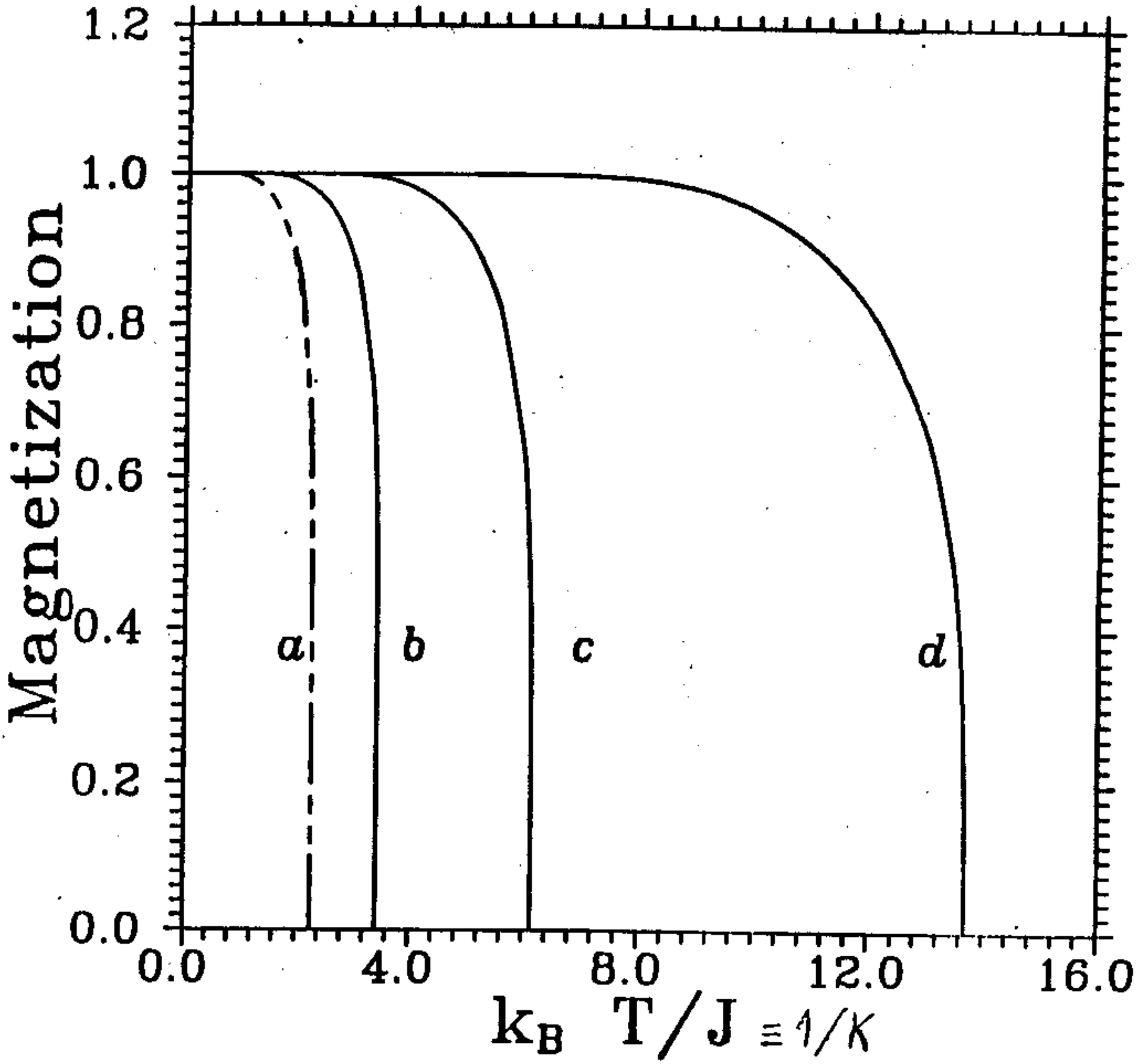


FIG. 5

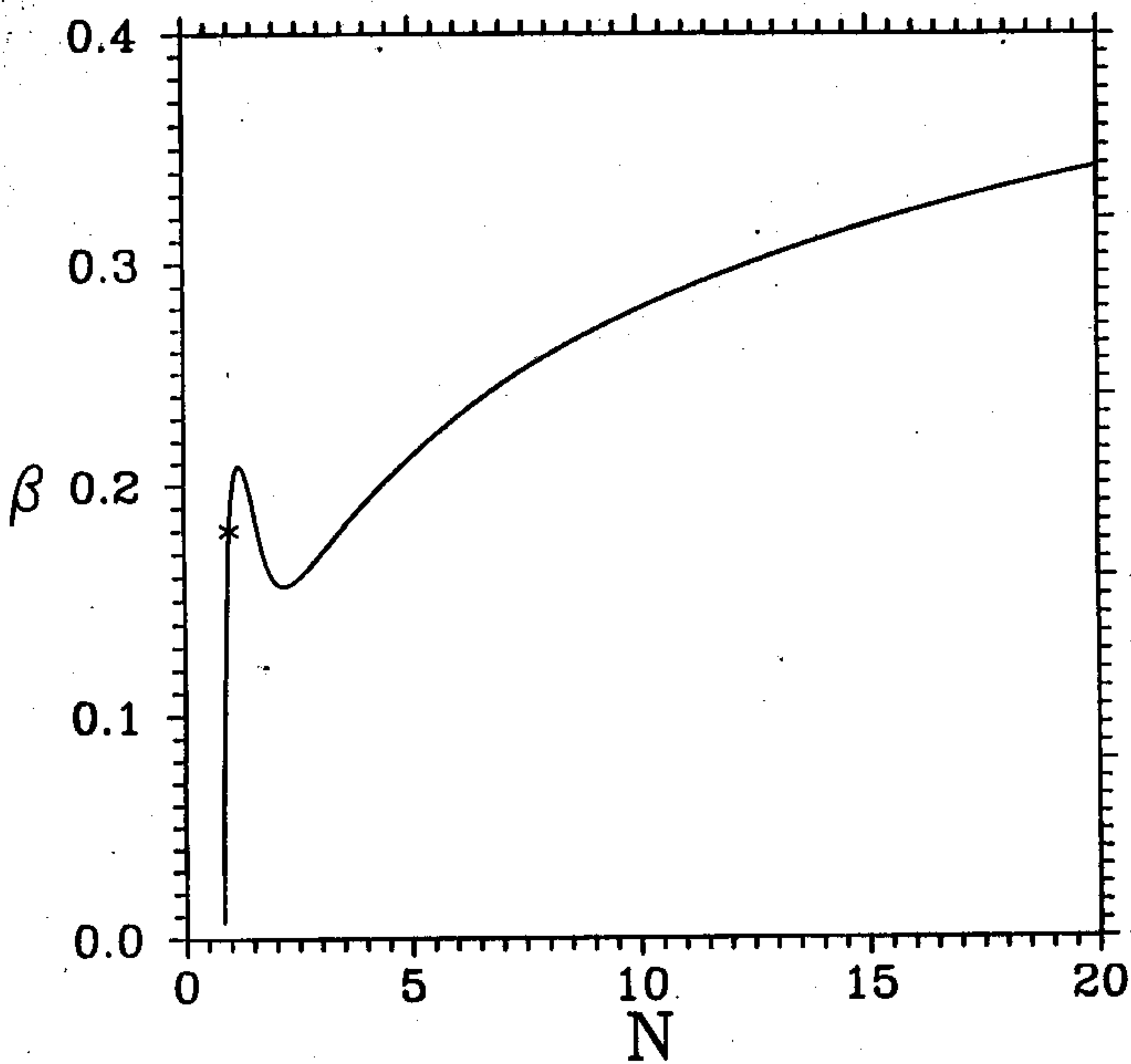
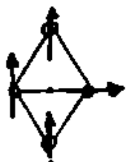
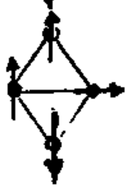


FIG. 6

Table 1. $h(K,L) = \Big\{\Big[1\cdot e^{5(NK+N^2L)}\cdot 10 + 1\cdot e^{(NK+5N^2L)}\cdot 6 + (2N-2)\cdot e^{3(NK+N^2L)}\cdot 8 + 2\cdot e^{(-NK+5N^2L)}\cdot 4 + 2\cdot e^{(-NK+5N^2L)}\cdot 0 + 2(2N-2)\cdot e^{(-NK+3N^2L)}\cdot 2 + 2(2N-2)\cdot e^{2(NK+N^2L)}\cdot 7 + 2(2N-2)\cdot e^{2N^2L}\cdot 3 + \ldots\Big] \,/\, \Big[1\cdot e^{5(NK+N^2L)} + 1\cdot e^{(NK+5N^2L)} + (2N-2)\cdot e^{3(NK+N^2L)} + 2\cdot e^{(-NK+5N^2L)} + 2\cdot e^{(-NK+5N^2L)} + 2(2N-2)\cdot e^{(-NK+3N^2L)} + 2(2N-2)\cdot e^{2(NK+N^2L)} + 2(2N-2)\cdot e^{2N^2L} + \ldots\Big]\Big\} \Big/ \Big\{5\Big[1\cdot e^{NK'+N^2L'}\cdot 2 + 1\cdot e^{-NK'+N^2L'}\cdot 0 + (2N-2)\cdot 1\cdot 1\Big] \,/\, \Big[1\cdot e^{NK'+N^2L'} + 1\cdot e^{-NK'+N^2L'} + (2N-2)\cdot 1\Big]\Big\}$. The term inside the first pair of curly brackets comes from the first part of this table, while the term inside the second pair of curly brackets comes from the second part of this table. The factor 5 in this last term comes from the number of bonds in the cluster. In the configurations below, horizontal arrows represent spins in any of the $N-1$ axes perpendicular to the one where the spin at the top site is.

Configuration	Multiplicity	Weight	m
	1	$e^{5(NK+N^2L)}$	10
	1	$e^{(NK+5N^2L)}$	6
	$2N-2$	$e^{3(NK+N^2L)}$	8
	2	$e^{(-NK+5N^2L)}$	4

	2	$e^{(-NK+5N^2L)}$	0
	$2(2N-2)$	$e^{(-NK+3N^2L)}$	2
	$2(2N-2)$	$e^{2(NK+N^2L)}$	7
	$2(2N-2)$	e^{2N^2L}	3
	.	.	.
	.	.	.
	.	.	.

Configuration	Multiplicity	Weight	m
	1	$e^{NK'+N^2L'}$	2
	1	$e^{-NK'+N^2L'}$	0
	$2N-2$	1	1

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