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A SCATTERING AMPLITUDE IN QUANTUM GEOMETRY OF FERMIONIC STRINGS

by

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Abstract

In this paper, a scattering amplitude for the Polyakov quantum fermionic string is proposed. Its main feature is that it leads to a spectrum without the usual tachionic excitation, obtained in the "semi-classical" limit $D \to -\infty$, where D is the space-time dimension.

Key-words: Scattering amplitude; Fermionic dual models; Quantum geometry.

1. Introduction

Recently, A.M. Polyakov (1,2) developed a formalism for closed strings quantization; later on further generalized by including the case of open strings (3,4,5).

An important problem in the formalism concerns the definition of a scattering amplitude for these strings, whose knowledge affords (in principle) the determination of the associated spectrum. A natural definition for these scattering amplitudes remains, however, the main problem. Probably its complete solution will require the determination of the exact Q.C.D. string (6,7).

In the lack of a Q.C.D. scattering definition, a suggestion for the closed bosonic string was put forward by A.M. Polyakov (1) and generalized for the bosonic open string case in (3). A remarkable feature of these scattering amplitudes is that the standard dual (Veneziano) model can be obtained in a saddle point approximation (3).

Our aim in this letter is to propose a scattering amplitude for the open fermionic string (2,5) with the property that the spectrum does not possesses the usual tachionic excitation in the saddle point approximation $D \to -\infty$.

2. The scattering amplitude

Let us start our analysis by considering the fermionic string action in a D-dimensional Euclidean space-time (2,5,8,9); namely

$$S[\phi^{(A)}(\xi),\ \psi^{(A)}(\xi),\ e^a_\mu(\xi),\ \chi_\mu(\xi)]$$

$$= \int_{\mathbf{D}} d^2\xi\ e(\xi)\{\frac{1}{2}\ \partial_\mu\phi^{(A)}\ \partial_\nu\phi^{(A)}\ g^{\mu\nu} + \frac{1}{2} i\ \psi^{(A)}\ \gamma_\mu D_\mu \psi^{(A)}$$

$$- \frac{1}{2}\ F^2 - \frac{1}{2}\ i(\chi_\mu \gamma^\nu \gamma^\mu \psi^{(A)})(\partial_\nu \phi^{(A)} - \frac{1}{4} i\ \chi_\nu \psi^{(A)})\}(\xi) \qquad (1)$$

Here the fermionic string is characterized by two fields: firstly, the vector-position $\phi^{(A)}(\xi)$ (A=1,...,D) and secondly by $\psi^{(A)}(\xi) = (\psi_1^{(A)}(\xi),\ \psi_2^{(A)}(\xi))$, a two-dimensional majorana spinor describing the string fermionic degrees of freedom. **D** denotes a two--dimensional parameter domain (embedded in the Euclidean space) with the boundary denoted by $\partial\mathbf{D}$. The presence of the Vierbein $e^a_\mu(\xi)$ and of the two-dimensional vector-majorana spinor $\chi_\mu(\xi)$ together with the auxiliary scalar field $F(\xi)$ insures respectively that the action (1) is invariant under general Lorentz and coordinate transformations and local supersymmetry transformations ([5,8,9]).

The average of a functional $W(\phi^{(A)}(\xi),\ \psi^{(A)}(\xi)$ defined on the fermionic string random surface is given by the following prescription:

$$<W[\phi^{(A)}(\xi),\psi^{(A)}(\xi)]>_F = \frac{1}{Z}\ \{\int D[\phi^{(A)}(\xi)]D[\psi^{(A)}(\xi)]$$

$$D[e^a_\mu(\xi)].D[\chi_\mu(\xi)]e^{-S[\phi^{(A)}(\xi),\psi^{(A)}(\xi),e^a_\mu(\xi),\chi_\mu(\xi)]}W[\phi^{(A)}(\xi),\psi^{(A)}(\xi)]\} \qquad (2)$$

where Z denotes the usual measure normalization factor.

The functional measures in (2) are invariant under local supersymmetry and general Lorentz and coordinate transformations. They are obtained as the functional element of volume associated to the following functional Riemman metrics [2,5]:

$$||\delta\phi^{(A)}||^2 = (\int_D d^2\xi\ e(\xi)(\delta\phi^{(A)}(\xi).\delta\phi^{(A)}(\xi))) + \Gamma_1(\chi_\mu(\xi),\phi^{(A)}(\xi),\psi^{(A)}(\xi),e^a_\mu(\xi)) \tag{3-A}$$

$$||\delta\psi^{(A)}||^2 = (\int_D d^2\xi\ e(\xi)(\delta\psi^{(A)}(\xi).\delta\psi^{(A)}(\xi))) + \Gamma_2(\chi_\mu(\xi),\phi^{(A)}(\xi),\psi^{(A)}(\xi),e^a_\mu(\xi)) \tag{3-B}$$

$$||\delta e^a_\mu||^2 = (\int_D d^2\xi\ e(\xi)\{e^{\mu'}_a e^{\mu}_{a'}(\delta e^a_\mu)(\delta e^{a'}_{\mu'}) + c e^{\mu}_a e^{\mu'}_{a'}(\delta e^a_\mu)(\delta e^{a'}_{\mu'}) + c' e^{\mu}_a e^{a\mu'}(\delta e^a_\mu)(\delta e_{a\mu'})\}) \\ + \Gamma_3(\chi_\mu(\xi),\phi^{(A)}(\xi),\psi^{(A)}(\xi),e^a_\mu(\xi)) \tag{3-C}$$

$$||\delta\chi_\mu(\xi)||^2 = (\int_D d^2\xi\ e(\xi)(g^{\mu\nu}\delta\chi_\mu.\delta\chi_\nu(\xi)) + \Gamma_4(\chi_\mu(\xi),\phi^{(A)}(\xi),\psi^{(A)}(\xi),e^a_\mu(\xi) \tag{3-D}$$

where C and $C'>1$ are arbitrary constants and $\Gamma_i(\chi_\mu(\xi),\ \phi^{(A)}(\xi),\ \psi^{(A)}(\xi),e^a_\mu(\xi))\ (i=1,\ldots,4)$ represents terms of these functional metrics which vanish for $\chi_\mu(\xi)\equiv 0$ and insure invariance of the associated element of volume by local supersymmetry transformations. As we will explain below, its explicit expression is not necessary.

For the evaluation of the average (2), one has to fix the gauge associated to the local symmetries of the action (1), quoted above. As proposed by A. Polyakov [2], a natural gauge is the super-conformal gauge specified by the relations

$$e^a_\mu(\xi) = e^{\delta(\xi)}\delta^a_\mu \quad ; \quad e(\xi) = e^{2\delta(\xi)} = \rho(\xi) \quad ;$$
$$\chi_\mu(\xi) = \frac{1}{2}\gamma_\mu\chi(\xi) = e^{-\frac{1}{2}\delta(\xi)}\gamma_\mu\zeta(\xi) \tag{4}$$

Thus, the integrand becomes an effective functional of the fields $\delta(\xi)$, $\zeta(\xi)$ and an auxiliary field $f(\xi)$ necessary to insure the remnants of the local analytic supersymmetry, which are not destroyed by the gauge (4). Because of this residual symmetry, we can evaluate (2) for $\chi_\mu(\xi) \equiv 0$ and use this residual super-symmetry to determine the dependence of the effective integrand in terms of the fields $\zeta(\xi)$ and $f(\xi)$. We notice that, as a consequence of this fact, we need not know the expressions

$$\Gamma_i(\chi_\mu(\xi), \phi^{(A)}(\xi), \psi^{(A)}(\xi), e^a_\mu(\xi)) \quad \text{in} \quad ((3\text{-A}), \ldots, (3\text{-D}))$$

After having described above the formalism to compute averages in the theory, we now pass to the problem of defining an off-shell scattering amplitude. For this task, we follow Polyakov's basic idea: the proposed N-point off-shell scattering amplitude is given by the sum over all fermionic random surfaces which contain a given set of fixed points $\{X_j\}$ $(j=1,\ldots,N)$; i.e.:

$$A(X_1,\ldots,X_N) = \Big\langle \int_{\mathbf{D}} \prod_{j=1}^{N} d^2\xi_j^{(H)}\, e(\xi_j)\, d\theta_1^{(j)}\, d\theta_2^{(j)}$$

$$\delta^{(D)}(\phi^{(A)}(\xi_j) + i\theta_1^{(j)}\psi_1^{(A)}(\xi_j) + i\theta_2^{(j)}\psi_2^{(A)}(\xi_j) - X_j)\Big\rangle_F \qquad (5)$$

where $\phi^{(A)}(\xi_j) + i\theta_1^{(j)}\psi_1^{(A)}(\xi_j) + i\theta_2^{(j)}\psi_2^{(A)}(\xi_j)$ denotes the "fermionic--position" of the fermionic string random surface with $(\theta_1^{(j)}, \theta_2^{(j)})$ Grassmanian parameters and $\prod_{j=1}^{N} d^2\xi_j^{(H)}$ is the Möbius invariant Haar measure, which takes into account the (physical) residual symmetry of the projective group not fixed by the conformal gauge $e^a_\mu(\xi) = e^{\delta(\xi)}\delta^a_\mu$. Their explicit expression is given by

$$\prod_{j=1}^{N} d^2\xi_j^{(H)} = \prod_{\substack{j=1 \\ j\neq a,b,c}}^{N} d^2\xi_j \left|\xi_b-\xi_a\right|^2 \left|\xi_c-\xi_b\right|^2 \left|\xi_c-\xi_a\right|^2 \tag{6}$$

The indexes a,b,c are fixs but choosen arbitrarily. We observe that the effective number of integrated variables in (6) is N-3 and is related to the maximum number of mutually non-overlapping channels of the scattering process.

The physical spectrum is determined by considering the poles in the $\{X_j\}$-Fourier transformed expression for such amplitude, whose associated residues are identified with the on-shell scattering amplitudes.

In order to evaluate (5) it is convenient to write (5) in momentum-space:

$$\tilde{A}(P_1,\ldots,P_N) = <\int_{\mathbb{D}} \prod_{j=1}^{N} d^2\xi_j^{(H)} e(\xi_j) e^{i(P_j^{(A)};\phi^{(A)}(\xi_j))}$$

$$(P_j^{(A)};\psi_1^{(A)}(\xi_j))(P_j^{(A)};\psi_2^{(A)}(\xi_j))>_F \tag{7}$$

where (;) means the Euclidean scalar product over the Lorentz indexes.

In the super-conformal gauge (4), the interaction Lagrangian involving the vector-spinor $X_\mu(\xi)$ vanishes and the functional integration over the "matter" fields $(\phi^{(A)}(\xi),\psi^{(A)}(\xi))$ becomes of the Gaussian type. In order to evaluate these functional integrations we have to choose appropriate boundary conditions since we are in the presence of a quantum theory defined in a two-dimensional space-time $\mathbb{D}$ with a non-trivial boundary. At this point we fix the domain $\mathbb{D}$ as the upper-half plane R_2^+ with the real-axis being the boundary. Then, we assume as in ref.([5]) that the "matter fields" satisfy the supersymmetric boundary conditions corresponding to the Nevew-Schwarz model (see eq. (3-7)-(1);(5)) and the Faddev-Popov determinants associated to (4), the boundary conditions as discussed in ref.([4]).

By introducing the family of self-adjoint operator acting on an appropriate space of two-component real function on R_2^+ with boundary conditions indicated by N(Neumann) or D(Dirichlet) ([4])

$$\mathcal{L}_j = (-\rho^{-(j+1)}\partial_{\bar{z}}\rho^j\partial_z) \tag{8}$$

we can thus perform the Gaussian functional integration over the scalar fields $\phi^{(A)}(\xi)$ with the result

$$\mathrm{Det}^{-\frac{D}{4}}(\mathcal{L}_0^{NN})e^{-\{\sum_{(i,j)=1}^{N}(P_i^{(A)};P_j^{(A)})K^{(\varepsilon)}(z_i,z_j,2\delta(z_i,z_i^*))\}} \tag{9}$$

where $K^{(\varepsilon)}(z,z',2\delta(z,z^*))$ is the conformally regularizated Green's function for the Laplacian in the metric $g_{\mu\nu}(z,z^*)=e^{2\delta(z,z^*)}\delta_{\mu\nu}$ with the Neumann boundary conditions along the real axis ([3]). Its expression reads:

$$K^{(\varepsilon)}(z,z',2\delta(z,z^*)) = \begin{cases} -\frac{1}{2\pi}(\ell n|z-z'||z-z'^*|) & z\neq z \\ \\ \frac{\delta(z,z^*)}{2\pi}-\frac{1}{4\pi}\ell n\varepsilon-\frac{1}{2\pi}\ell n|z-z^*| & z=z' \end{cases} \tag{10}$$

The integration over the majorana fields $\psi^{(A)}(\xi)$ is carried out by using the fact that the Green function $(i\gamma_\mu D_\mu)^{-1}_{(N)}(z_i,z_j)$, with the Neumann boundary conditions along the real axis; is related to the corresponding flat propagator $(i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j)$ by (see eq. (6.11)-(10))

$$(i\gamma_\mu D_\mu)^{-1}(z_i,z_j) = e^{-\delta(z_i,z_i^*)}(i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j)e^{-\delta(z_j,z_j^*)} \tag{11}$$

where

$$(i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j) = (i\gamma_a\partial_a)[-\frac{1}{2\pi}\ln(|z_i-z_j||z_i-z_j^*|)] \tag{12}$$

As again, the functional integration over the majorana fields are Gaussian, we get the result:

$$\mathrm{Det}^{\frac{D}{4}}(\mathcal{L}_{-\frac{1}{2}})^{ND}\{e^{-(\sum_{i=1}^{N}\delta(z_i,z_i^*))}\sum(\prod_{(i,j)}^{N}(P_i^{(A)};P_j^{(A)})\prod_{(\alpha_1,\alpha_1)}((i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j))_{\alpha_1\alpha_2})\} \tag{13}$$

where the $\sum$ in (13) means that we have to sum over all ways of pairing the fermion fields in (7) and the subscripts (α_1,α_2) denots the matrix indexes of the propagator (12).

We note that N should be a even number. This implies that the Polyakov fermionic string model possesses a quantum number which is subject to conservation and can be related to the N.S.-G parity [11]. By evaluating the Faddev-Popov determinants associated to the gauge (4), we get the effective action and hence the final expression conformally regularizated for the N-point off-shell scattering amplitude.

$$\tilde{A}^{(E)}(P_1,\ldots,P_N) = \frac{1}{2}\{\int D[\delta]D[\zeta]D[f]e^{-S_{EFF}[\delta,\zeta,f]}$$

$$(\int_{R_+^2} \prod_{j=1}^{N} d^2\xi_j^{(H)}\, e^{(\sum_{j=1}^{N} 2\delta(z_j,z_j^*))}\, e^{-(\sum_{\substack{(i,j)\\ i=j}}^{N} (P_i^{(A)};P_j^{(A)})K^{(\varepsilon)}(z_i,z_j,2\delta(z_i,z_i^*))}$$

$$e^{-(\sum_{j=1}^{N}\delta(z_i,z_j^*)}\,(\sum \prod_{(i,j)}^{N} (P_i^{(A)};P_j^{(A)}) \prod_{(\alpha_1,\alpha_2)} ((i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j)_{\alpha_1\alpha_2} \quad (14)$$

where the effective action is given by the expression [5]:

$$S_{EFF}[\delta,\zeta,f] = \frac{10-D}{8\pi}\{\int_{R_2^+} d^2\xi[\frac{1}{2}(\partial\delta)^2 - \frac{1}{2}i\,\zeta^T(\gamma.\partial.)\zeta - \frac{1}{2}f^2]$$

$$+\frac{1}{4}i(\int_{-\omega}^{+\infty} d\xi_o(\zeta\gamma_5\zeta)\Big|_{\xi_1=0}) + \frac{D}{8\pi}[\mu.\int_{R_2^+} d^2\xi\; e^{\delta(\xi)}$$

$$(f-\frac{1}{2}i\zeta\gamma_5\zeta)(\xi) - \mu\int_{-\infty}^{+\infty} d\xi_o(e^{\delta})\Big|_{\xi_1=0} - \int_{-\infty}^{+\infty} d\xi_o[f+\frac{\partial}{\partial\xi_1}\delta]\Big|_{\xi_1=0}]\} \quad (15)$$

It was pointed out in [5] that the term $f(\xi)e^{\delta(\xi)}$ in (15) produces a Liouville term after being formally integrated over f.

Since the complete solution of the supersymmetric Liouville field theory in R_2^+ was not found yet; which would provide the complete solution of (14); we implement a Saddle-point approximation to evaluate (14) as introduced in refs.[2,5]: we take the majorana field $\zeta\equiv0$ and consider the classical motion equation for the resulting action ([5]):

$$\Delta\delta = \frac{D^2}{(10-D)^2}\mu^2 e^{2\delta} - \delta'(\xi_1)\{\frac{D}{10-D} - \xi_1[\frac{D^2}{(10-D)^2}\mu e^{\delta} + \frac{D}{10-D}\mu e^{\delta} + \partial_{\xi_1}\delta]\} \qquad (16)$$

A solution of (16) having the property of vanishing automatically at the boundary conditions is the Poincaré metric in R^2_+, namely:

$$\delta(\xi_1,\xi_2) = \ln(\frac{D}{10-D}\frac{1}{\mu\xi_1}) = \ln(\frac{D}{10-D}\frac{1}{\mu|z-z^*|}) \qquad (17)$$

By substituing this expression in the eq. (14) and taking into account that the action evaluated in (17) cancels out with same term arising from the normalization factor, we finally get:

$$A^{(E)}(P_1,\dots,P_N) = \int_{R_2^+} \prod_{j=1}^{N} d^2 z_j^{(H)} \left(\varepsilon^{\sum_{i=1}^{N}\frac{(P_i^2)}{2\pi}}\right)$$

$$\left(\frac{D}{(10-D)\mu}\right)^{\sum_{i=1}^{N}\frac{(1-P_i^2)}{2\pi}} \left(\prod_{i<j}^{N}(|z_i-z_j'||z_i-z_j'^*|)\right)^{\frac{(P_i^{(A)};P_j^{(A)})}{\pi}} \left(\prod_{i=1}^{N}|z_i-z_i^*|^{\frac{P_i^2}{\pi}-1}\right)$$

$$\left(\sum \prod_{\substack{(i,j)\\ i\neq j}}^{N} (P_i^{(A)};P_j^{(A)}) \prod_{(\alpha_1,\alpha_2)} ((i\gamma_a\partial_a)^{-1}_{(N)}(z_i,z_j))_{\alpha_1\alpha_2}\right) \qquad (18)$$

In order to isolate the on-shell scattering amplitudes, we have first to find the poles in the external momentum variables $(P_i)^2 = (P_i^{(A)};P_i^{(A)})$. Such poles occur when z_i and z_i^* come close together, i.e. the only contribution for the associated residues come only from the region $Im(z_i) \to 0$ in the integrand in (18). This phenomenon reduces the integration over R_2^+ to the integration along the real axis. As a result, there exist (Euclidean) poles when

$$\frac{(P_i)^2}{\pi} - 1 = -1,-2,... \tag{19}$$

or

$$\frac{(P_i)^2}{\pi} = 0,-1,-2,... \tag{20}$$

This fact implies that the proposed scattering amplitude (5) leads to a spectrum without the usual lowest state being a Tachyon (compare with the bosonic case, eq. (4.21-3).

For the lowest massless excitation, we obtain a expression similar to the S-matrix elements encountered in Nevew-Scharz model [11]

$$S(P_1,...,P_N) = \left(\frac{D}{(10-D)\mu}\right)^N \left\{ \int_{-\omega}^{+\omega} \prod_{j=1}^{N} d^1 z_j^{(H)} \left(\prod_{i<j}^{N} |z_i - z_j| \frac{2(P_i^{(A)};P_j^{(A)})}{\pi} \right.\right.$$

$$\left.\left. \left(\sum \prod_{\substack{(i,j) \\ i\neq j}}^{N} 2(P_i^{(A)},P_j^{(A)}) \prod_{(\alpha_1,\alpha_2)} ((i\gamma_a \partial_a)^{-1}(z_i,z_j))_{\alpha_1\alpha_1} \right) \right\} \tag{21}$$

where now

$$((i\gamma_a\partial_a)^{-1}(z_i,z_j))_{\alpha_1\alpha_1} = ((i\gamma_a\partial_a)(-\frac{1}{2\pi}\ell n|z_i - z_j|))_{\alpha_1\alpha_2} \tag{22}$$

and the Möbius invariant Haar measure $\prod_{j=1}^{N} d^1 z_j^{(H)}$ is taken over the real axis.

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References

(1) A.M. Polyakov, Phys. Lett. 103B (1981) 207.

(2) A.M. Polyakov, Phys. Lett. 103B (1981) 211.

(3) B. Durhuus, H.B. Nielsen, P. Olesen and J.L. Petersen, Nucl. Phys. B196 (1982) 498.

(4) B. Durhuus, P. Olesen and J.L. Petersen, Nucl. Phys. B198 (1982) 157.

(5) P. di Vecchia, B. Durhuus, P. Olesen and J.L. Petersen, Nucl. Phys. B207 (1982) 77.

(6) A.A. Migdal, Nucl. Phys. B189 (1981) 253.

(7) E.S. Fradkin and A.A. Tseytlin, Annals of Physics 143 (1982) 413.

(8) S. Deser and B. Zumino, Phys. Lett. 65B (1976) 369.

(9) Lars Brink and John H. Schwarz, Nucl. Phys. B121 (1977) 285.

(10) N.D. Birrell and P.C.W. Davies, Phys. Rev. D, vol 18 , nº 12 (1978) 4408.

(11) A. Neveu and J.H. Schwarz, Nucl. Phys. B31 (1971) 86.