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ON SPONTANEOUS SYMMETRY BREAKING MECHANISM IN LIGHT-FRONT QUANTIZED FIELD THEORY

by

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Abstract

Following the (*standard*) Dirac procedure, we describe the spontaneous symmetry breaking in light-front quantized scalar field theory. The zero mode operator of the field is shown to commute with the nonzero mode operators, and thus may be looked upon as a background field. In the light-front framework a nonlocal constraint must be satisfied. The values of the background field at the tree level, as a consequence, are shown to follow from the minimization of the light-front energy functional. We are thus led to a description parallel to the one made in equal-time framework where we appeal to physical considerations and minimize the energy. These values characterize the various (non-perturbative) vacua over which the corresponding physical sectors may be built by applying the non-zero mode operators.

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1. The possibility of building a Hamiltonian formulation of relativistic dynamics on light-front surface, $\tau = (t + z) = const.$, was pointed out by Dirac [1] and rediscovered by Weinberg [2] in the guise of old-fashioned perturbation theory in the infinite-momentum frame. Since the longitudinal momentum k^+ turns out to be necessarily positive, it is hoped that, the vacuum being trivial (perturbative), the non-perturbative effects may be easier to handle. The perturbative field theory is, in fact, much simplified and with the introduction of the discretized light-cone quantization (DLCQ) [3] it has developed into a useful tool to handle the non-perturbative calculations as well. Other interesting developments are the recent studies on Light-front Tamm-Dancoff Field Theory [4] to study non-perturbative effects and the begining of a systematic study of perturbative renormalization theory [5].

The problem of non-perturbative vacuum structure, Higgs mechanism, the fermionic condensates, and other related problems, in the framework of light-front quantized theory, however, has remained without a clear understanding [6] even at the tree level, say, a description parallel to the well known familiar one in the case of equal-time quantization.

We will consider here, for concreteness, the simplest case of spontaneous symmetry breaking in scalar field theory. In equal-time formulation the tree level description of the (non-perturbative) vacuum is given in terms of the constant background field (zero modes) obtained by minimizing the energy functional appealing to physical considerations. In the light-front frame work we lack a justification for minimizing the light-front energy. It is, however, suggested that the zero modes should also play a similar role here. We show here that we do find the answer if we handle carefully the constrained dynamical system at hand by the well tested and widely used *standard* procedure of Dirac [7] for constructing a canonical framework which may be later quantized. We find a non-local constraint in the theory (eq. 8) which at the tree level does give rise to a justification for minimizing the light-front energy for finding the background fields which describe the (non-perturbative) vacua even in the light-front framework. Such constraints may also arise in other interacting field theories written in terms of light-front coordinates. Their consideration seems to have been overlooked in the literature and it is clear that their implications at the quantum level need further study. A special feature of any action written in terms of light-front coordinates is that it describes a constrained dynamical system. Since most of the essential points may be seen in the case of scalar field theory in 1+1 dimensions we will adopt this simplification. The extensions to higher dimensions and continuous symmetry are straightforward. There have been some earlier attempts along this direction which, however, are rather inconclusive or incomplete [8,9,10]. We will follow here the *standard* Dirac procedure by finding out all the constraints in the theory (the constraint $p \approx 0$ obtained below was missed in [8]) and do not introduce any modifications neither in the procedure nor argue to increase the number of constraints [10]. It would be, in the least, very embarassing for the highly successful Dirac procedure as applied to gauge theories and other constrained systems if we are required to introduce modifications to it in order to handle the very simple case under discussion. We will also not discretize the modes, as is usually done, in order not to introduce in the discussion spurious zero modes, for example, coming from the sgn function.

2. The light-front Lagrangian for the scalar field ϕ is

$$\int_{-\infty}^{\infty} dx\, [\dot{\phi}\phi' - V(\phi)], \tag{1}$$

where $V(\phi) \geq 0$, for example, $V(\phi) = (\lambda/4)(\phi^2 - m^2/\lambda)^2$, the potential with the wrong sign for the mass term and $\lambda \geq 0$. Here an overdot and a prime indicate the partial derivations with respect to the light-front coordinates $\tau \equiv x^+ = (x^0 + x^1)/\sqrt{2}$ and $x \equiv x^- = (x^0 - x^1)/\sqrt{2}$ respectively and $x^+ = x_-, x_+ = x^-$ while $d^2x = d\tau dx$. The Euler equation of motion, $\dot{\phi}' = (-1/2)V'(\phi)$, where a prime on V indicates the variational derivative, shows that classical solutions, for instance, $\phi = const.$, are possible to obtain. We start out by seperating the zero mode, $\omega = \omega(\tau)$, and write $\phi(x,\tau) = \omega(\tau) + \varphi(x,\tau)$ with the corresponding Fourier transform $\tilde{\phi}(k,\tau) = \sqrt{2\pi}\omega(\tau)\delta(k) + \tilde{\varphi}(k,\tau)$ so that φ has no zero mode and on integrating it over space variable it gives vanishing result. It is then easily seen that the Lagrangian density may be written as

$$\mathcal{L} = \dot{\varphi}\varphi' - V(\phi). \tag{2}$$

which is of first order in $\dot{\varphi}$ and contains no kinetic term for the zero mode and consequently describes a constrained dynamical system. Indicating by p and π the momenta conjugate to ω and φ, respectively, the primary constraints are $p(\tau) \approx 0$ and $\Phi \equiv \pi - \varphi' \approx 0$ while the canonical Hamiltonian density is shown to be $\mathcal{H}_c = V(\phi)$. Here $\approx$ stands for the weak equality [6]. We postulate now the standard Poisson brackets at equal τ, $\{p,\omega\} = -1$, $\{\pi(x),\varphi(x)\} = -\delta(x-y)$ etc., and define an extended Hamiltonian

$$H'(\tau) = H_c(\tau) + \mu(\tau)p(\tau) + \int dy\; u(\tau,y)\Phi(\tau,y) \tag{3}$$

where μ and u are Lagrange multipliers. Using (3) we derive

$$\dot{p} \;=\; \{p, H'\} \approx -\int dx\; V'(\phi) \;\equiv\; -\beta(\tau), \tag{4}$$

$$\dot{\Phi} \;=\; \{\Phi, H'\} \approx \;-\; V'(\phi) \;-\; 2u'. \tag{5}$$

The requirement of the persistency, $\dot{p} \approx 0$, leads to a secondary constraint $\beta \approx 0$, while $\dot{\Phi} \approx 0$ results in a consistency condition and does not generate any any new constraint. Defining the next extended Hamiltonian by adding a term $\nu(\tau)\beta$ to H' and repeating the procedure we find that no more secondary constraints are generated if we set $\nu \approx 0$.

We easily verify that the three constraints $p \approx 0$, $\beta \approx 0$, $\Phi \approx 0$ in our system are second class. They may be implemented in the theory by defining Dirac brackets and this may be performed iteratively. The Dirac bracket with respect to the pair $p \approx 0$, $\beta \approx 0$, $\{\beta(\tau),p(\tau)\} \equiv \alpha(\tau) = \int dx\; V''(\phi)$, is easily shown to be

$$\{f(x),g(y)\}^* = \{f(x),g(y)\} - \frac{1}{\alpha}[\{f(x),p\}\{\beta,g(y)\} - (\beta \leftrightarrow p)]. \tag{6}$$

We may now set $p = 0$ and $\beta = 0$ as strong equalities since $\{f,p\}^* \;=\; \{f,\beta\}^* \;= 0$ for any arbitrary functional f. The nonvanishing brackets, at this stage, are $\{\omega,\pi\}^* = \{\omega,\Phi\}^* = -\alpha^{-1}V''(\phi)$, $\{\pi(x),\varphi(y)\}^* = -\delta(x-y)$. It may, however, be shown that, for the cases relevant to our discussion, α is , in fact, infinite and as such ω has vanishing bracket with all the surviving canonical variables. We have

$$\alpha(\tau) = L\,V''(\omega) + \frac{1}{2}V^{(IV)}\int dx\,\varphi^2, \tag{7}$$

where we set $L = 2\pi\,\delta(0)$ and assume terms up to order ϕ^4 in the potential. We have to combine this with the constraint

$$\beta(\tau) = L\,V'(\omega) + \frac{1}{2}V'''(\omega)\int dx\,\varphi^2 + \frac{1}{6}V^{(IV)}\int dx\,\varphi^3 = 0. \tag{8}$$

We conclude from (8), since $L \to \infty$, that the allowed values for ω are given by solving $V'(\omega) = 0$ which corresponds to minimizing the light-front energy H_c. It is worth remarking that if we had followed the above discussion through the discretized (box quantization) formulation with the periodic boundry conditions the symbol L would there stand for the finite extension along the longitudinal x direction while in (8) the integrals would run from $-L/2$ to $L/2$ with similar modifications in other expressions. Returning to our discussion, if we ignore the massless case and do not allow any cubic term (which will explicitly break the reflection symmetry), we find that the coefficient of L in (7) is nonvanishing for any allowed value for ω and hence $\{\omega,\pi\}^* = \{\omega,\Phi\}^* = 0$ for the interesting cases. Corresponding to the potential mentioned earlier we find $\omega = 0,\ \pm m/\sqrt{\lambda}$. In the case of the correct sign for the mass term, e.g., $m^2 \to -m^2$, or in the free theory $\omega = 0$. The same results of course may be shown to follow if we quantize the system in a box of length L along x direction.

The remaining constraint $\Phi \approx 0$ may now be implemented; it is second class by itself. From the bracket $\{\Phi(x),\Phi(y)\}^* = -2\partial_x\delta(x-y) \equiv C(x,y) = -C(y,x)$ and $C^{-1}(x,y) = -C^{-1}(y,x) = -\epsilon(x-y)/4$ it follows that the final Dirac bracket which implements all the constraints is given by

$$\{f(x),g(y)\}_D = \{f(x),g(y)\}^* + \frac{1}{4}\int\int du dv \{f(x),\Phi(u)\}^*\epsilon(u-v)\{\Phi(v),g(y)\}^*. \tag{9}$$

We may now also set $\pi = \varphi'$ as a strong relation . Both p and π are removed from the theory and we are left with the constraint (8) while H' in (3) reduces to the canonical form H_c. From (9) we derive $\{\omega,\varphi(x)\}_D = 0$, $\{\omega,\omega\}_D = 0$ as well as the usual light-front result for the non-zero mode field, e.g., $\{\varphi(x),\varphi(y)\}_D = -(1/4)\epsilon(x-y)$. The commutation relations for the corresponding operators in the quantized field theory are obtained by the correspondence $i\{f,g\}_D \to [f,g]$. For the field φ they may be realized in momentum space through the following expansion ($\tau = 0$)

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \, \frac{\theta(k)}{\sqrt{2k}} \, [a(k)e^{-ikx} + a^\dagger(k)e^{ikx}] \tag{10}$$

The operators $a(k)$ and $a^\dagger(k)$ satisfy the usual commutation relations, viz, $[a(k), a(k')^\dagger] = \delta(k-k')$, $[a(k), a(k')] = 0$, and $[a(k)^\dagger, a(k')^\dagger] = 0$ while the zero mode commutes with them and thus is proportional to the identity operator. It may, in our case, be looked upon as constant background field which characterizes the physical sectors. The vacuum state is defined to be annihilated by the destruction operators, $a(k)|vac\rangle = 0$. The normal ordering with respect to the creation and destruction operators, which is interaction independent, may be introduced. The longitudinal momentum density is $:\varphi'^2:$ and we find $[a(k), P^+] = k\,a(k)$, $[a^\dagger(k), P^+] = -k\,a^\dagger(k)$ which gives a justification for the above normal ordering. In the case of the potential with reflection symmetry mentioned above, the allowed values $\omega = \pm m/\sqrt{\lambda}$ describe the assymetric sectors built on non-perturbative vacua which break the reflection symmetry spontaneously, $\langle|\phi|\rangle_{vac} = \pm m/\sqrt{\lambda}$ while, $\omega = 0$ describes the unbroken symmetry symmetric sector, $\langle|\phi|\rangle_{vac} = 0$. The Fock space may be built over any of these vacua. There are no operators in the theory which will take us from one sector to another. In our example, $P^- = H = \int dx : V(\phi) :$ and $P^-|vac, \omega = \pm m/\sqrt{\lambda}\rangle = 0$ for the degenerate vacua while on the $\omega = 0$ phase we obtain an infinite value. It is also easily seen that the (operator) constraint (8) applied on the vacuum state also lead to $V'(\omega) = 0$ as a consequence of the of the normal ordering, the positivity of the longitudinal momentum, and its conservation in the framework of light-front quantization.

3. We thus obtain, on following the Dirac procedure carefully a description, in the light-front framework, of the tree level spontaneous symmetry breaking mechanism parallel to what is known when we perform the quantization on $t = const.$ planes. The constant background field is found by minimizing the light-front energy functional now. This is not at all obvious at the start since the Lagrangian is degenerate and we lack a phsical consideration to minimize the light-front energy to obtain it. The Dirac procedure in our case gives rise to a non-local constraint which results in the above criteria. The constraint $\beta \approx 0$ is also evident even at the Lagrangian level, if we assume appropriate boundary conditions and integrate the equation of motion. However, we need to build a canonical framework for quantizing the theory. The scalar field zero mode here is found to be a background field characterizing the different (non-perturbative) vacua. This is in contrast to the case of the light-front quantization of the bosonic version of the Schwinger model where the zero mode of the scalar field must be an operator. In fact, in this case in order to ensure at the quantum level the symmetry of the Lagrangian with respect to the shift by a constant of the scalar field (chiral symmetry), a zero mode from the only other field available, viz, the gauge field, must be an operator and canonically conjugate to the zero mode operator of the scalar field. This model can also be handled by following the standard Dirac procedure without any modifications [11]. Finally, the Higgs mechanism may also be shown to be described [12] by following the same procedure.

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References

[1.] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[2.] S. Weinberg, Phys. Rev. **150**, 1313 (1966).

[3.] H.C. Pauli and S.J. Brodsky, Phys. Rev. D **32** , 1993 (1985); Phys. Rev. D **32**, 2001 (1985).

For a complete list of reference see, for example,

S.J. Brodsky and G.P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller (World Scientific, Singapore, 1989 and the recent review by S.J. Brodsky and H.C. Pauli, *Light-Cone Quantization of Quantum Chromodynamics*, SLAC preprint SLAC-PUB-5558/91.

[4.] K.G. Wilson, in *Lattice '89*, Proceedings of the International Symposium, Capri, Italy, 1989, edited by R. Petronzio *et al.* [Nucl. Phys. B (proc. Suppl.)] **17** (1990); R.J. Perry, A. Harindranath, and K.G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990).

[5.] D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson, Phys. Rev. D **43**, 3411 (1991).

[6.] The problem has been under constant attention. See, for example , ref. [3] and *Sources for light-front physics* compiled by A. Harindranath at Ohio State. Some recent papers on the topic containing earlier references are

A. Harindranath and J.P. Vary, Phys.Rev. D **36**, 1141 (1987);

S. Glazek, Phys. Rev. D **38**, 3277 (1988);

G. McCartor, Z. Phys. C **41**, 271 (1988);

Th. Heinzl, St. Krusche, and E. Werner, Z. Phys. A **334**, 443 (1989);

F. Lenz, M. Thies, S. Levit, and K. Yazaki, Ann. Phys. **208**, 1 (1991);

G. McCartor and D.G. Robertson, SMU preprint SMUHEP/91-04;

K. Hornbostel, preprint, Cornell University, CLNS 91/078 (1991).

[7.] P.A.M. Dirac, Canad. J. Math. **1**, 1 (1950); *Lectures in Quantum Mechanica*, Benjamin, New York, 1964.

[8.] T. Maskawa, K. Yamawaki, Prog. Theor. Phys., **56**, 270 (1976).

[9.] R.S. Wittman,in *Nuclear and Particel Physics on the Light Cone*, edited by M.B. Johnson and L.S. Kisslinger, World Scientific, Singapore, 1989.

[10.] Th. Heinzl, St. Krusche, and E. Werner, Regesburg preprint, TPR 91-23.

[11.] P.P. Srivastava, *in preparation.* A modified Dirac procedure was used by Th. Heinzl, St. Krusche, and E. Werner, Phys. Lett. B **256**, 55 (1991).

[12.] P.P. Srivastava, *Higgs mechanism in light-front quantized field theory*, Ohio State preprint, December 1991.