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A CLASSICAL N-BODY SIMULATION OF GROUPS OF GALAXIES

by

G. PECH\* and K.C. CHUNG

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq  
Rua Dr. Xavier Sigaud, 150  
22290 - Rio de Janeiro, RJ - Brasil

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## SUMMARY

Groups of galaxies are simulated by Monte Carlo technique. The mass distribution of the groups is assumed to follow a power-law. Furthermore, a linear relationship between mass and luminosity is considered.

The calculated velocity dispersion is compared with the observational data and provides an estimation of the range in which the galaxy masses are distributed. It is shown, in this case, that the mass discrepancy can cover up two orders of magnitude, such as pointed out in the literature.

Key-words: Groups of galaxies; Monte Carlo simulation.

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## I. INTRODUCTION

Groups of galaxies (3-30 galaxies) are believed to be a very important source of information about the mass of galaxies and the total mass of each group itself (1,2). With respect to the observational data, a number of catalogues of groups of galaxies have been compiled in these last two decades (3,4,5,6), in spite of the difficulty to identify a given observed cluster of galaxies as a group. In the same time, theoretical approaches have been worked out, giving a dynamical description of groups of galaxies (7,8,9). All of these theoretical studies basically use the classical n-body simulation technique. Recently, interesting questions as the distribution of the dark mass (10) and the formation of elliptical galaxies (11) have been also discussed in the scope of the dynamical study of the groups of galaxies.

Nowadays, it is believed that there exists a reasonable amount of observational data concerning to the groups of galaxies, so that it is possible to try to extract from them information about the galaxy mass distribution, the mass-to-luminosity ratio, the velocity dispersion, etc. As a matter of fact, Huchra and Geller have identified more than hundred of groups in the northern hemisphere, by using the so-called redshift space method. This method is considered superior than the one used in previous works by other authors, what seems to make the Huchra and Geller's catalogue the most reliable one.

On the other hand, it is well known that there is

a difference between the mass-to-light ratio such as found in a galaxy field, evaluated through intrinsic quantities, and the one encountered in groups of galaxies, which is calculated by using global quantities. This difference is shown to lead to a discrepancy of up about two orders of magnitude between the group mass given by the virial theorem and the one calculated from its luminosity (12,13). This missing mass has provoked much speculation and boosted great interest in a dynamical description of groups of galaxies.

Basically, in order to understand this discrepancy, we may assume either the observed groups are gravitationally bound systems, with the masses given by the virial theorem, or the groups are unbound and all of their masses can be evaluated from the luminosity. In the first hypothesis, some fraction of the group mass is in the form of dark matter, and in this case, it can be located in the galaxies themselves (14) or distributed somehow in the intergalactic space (10).

In order to clarify these and other correlated questions, some computations have been performed by using n-body numerical simulation of groups of galaxies. To do that, one of the main problems is to establish the group mass distribution that we must use. In fact, previous works use either a uniform mass distribution, or masses randomly chosen from a distribution which has no connection with the observed luminosity distribution. For example, Aarseth and Saslaw (7) have used an inverse square distribution, i.e.  $n(m) \propto m^{-2}$ .

Both of these approximations seem quite unrealistic, so we look for other mass distributions. The simplest and more

natural way is to extend one step further the inverse square approximation with free parameters to be fitted to the observational data. As a matter of fact, we propose a power-law distribution, i.e ,  $Y(M) \propto M^{-\tau}$ , where  $\tau$  is to be found such that the observational luminosity data is reproduced.

On the other hand, instead of a mass-to-light ratio equal a constant, we can generalize a little bit this approximation and assume the mass to vary linearly with the luminosity. Of course, this linear relationship is claimed to be hold only in the luminosity range which we are interested in, i.e,

$$0.05 \leq L \leq 50.0$$

in  $10^{10} L_{\odot}$  units.

In this paper, we perform a n-body simulation of group of galaxies, by using the Monte Carlo technique. This permit us to obtain some relevant results, specially the distribution of the velocity dispersion, to compare with the observational data, and in this way we can obtain usefull information about the mass of the galaxies which build up the groups.

The scheme of this work is the following:

In Section II, we address the question of the initial conditions of the dynamical problem. In Section III, we simulate the groups by using the Monte Carlo method and in Section IV we present our results and conclusions.

## II. INITIAL CONDITIONS

As mentioned in the Introduction, we assume the galaxy masses obey a power-law, i.e.,

$$\phi(M) \propto M^{-\tau} \quad , \quad (1)$$

where  $\tau$  is a constant and  $M$  the mass of the galaxy. Furthermore, we assume that  $M$  is a linear function of the luminosity  $L$ , i.e.,

$$M = \alpha L + \beta \quad , \quad (2)$$

where,  $\alpha$  and  $\beta$  are parameters to be obtained by fitting to observational data.

In Eq. (2), beside the luminosity dependent term, there is another one that remains unaltered when the luminosity is changed. This last term can take care fo the dark matter, which is assumed here to be located in the galaxies, instead of spreading in the intergalactic space.

Combining the Eqs. (1) and (2), we obtain the following luminosity distribution,

$$\phi(L) \propto (\alpha L + \beta)^{-\tau} \quad . \quad (3)$$

The parameters  $\alpha$ ,  $\beta$  and  $\tau$  are obtained by fitting the luminosity data of Turner and Gott (15) (Fig. 1). This gives  $\tau = 2.8$  and  $\beta/\alpha = 5.8$ . It should be noted that  $\beta/\alpha = 5.8$  can yield a difference of up about two orders of magnitude between the mass given by the Eq. (2) and the mass given by the luminosity-dependent term  $\alpha L$  only, within the luminosity range that

we have used in the fitting.

Unfortunately, the fitting is unable to determine the values of  $\alpha$  and  $\beta$  separately, because they depend on the normalization constant. Therefore we must resort to a dynamical study of the group, in order to determine  $\alpha$  and  $\beta$  uniquely.

To solve the equations of motion, we need initial values for the position and momentum of each galaxy, as well as the size of the group, which is assumed to be a sphere. The value for the sphere size is chosen to be the same than the most probable value of the mean pairwise separation of galaxies. According with the Huchra and Geller's catalogue, this value is equal to 0.5 Mpc. It should be noted that the results will not be changed significantly for other values slightly different from 0.5 Mpc.

So, the galaxy positions are selected randomly within a sphere of radius equal to 0.5 Mpc. In the case of the initial velocities, we follow the Bahcall's (9) prescription, and each of the three cartesian components of the velocity is randomly chosen according with a Gaussian distribution whose peak is located at the origin.

The multiplicity of each group is randomly chosen in such a way that our assembly reproduces the same multiplicity distribution observed in Geller and Huchra's catalogue (6).

### III. MONTE CARLO SIMULATION

Monte Carlo method is used to simulate the dynamics of

groups of galaxies. Specifically, in each sampling we choose randomly the multiplicity  $N$  of the group, according with an observed distribution. Also, we select by chance, for each galaxy of the group, the initial position and velocity, such as explained in Sect. II, and the mass, according with the power-law. The luminosity is straightforward obtained by mean of the linear relationship with the mass. The system, then, is put to evolve, obeying the equations of motion until the virialization equilibrium is attained. From this time on, the system can be regarded as a bound system of galaxies and the relevant quantities, such as the velocity dispersion, calculated at this stage, are registered and stored. The procedure is repeated again and again, until the statistics is considered satisfactory. In this case, an average of the calculated quantities is performed in order to compare with observational data.

To obtain the equations of motion, we consider, for a group of multiplicity  $N$ , the following Hamiltonian:

$$H = \frac{1}{2} \sum_i^N M_i V_i^2 - G \sum_{i < j}^N \frac{M_i M_j}{(r_{ij}^2 + \epsilon^2)}, \quad (4)$$

where  $G$  is the gravitational constant,  $M_i$  the mass of the  $i$ -th galaxy,  $V_i$  the velocity of the  $i$ -th galaxy with respect to the center-of-distribution of the group,  $\vec{r}_{ij}$  the inter-distance and  $\epsilon$  is the softened parameter.

This Hamiltonian describes a classical  $n$ -body system. The interparticle potential is modified by the introduction of the parameter  $\epsilon$ , in order to take into account the finite size



of the galaxies (16). In our calculation, we have used  $\epsilon = 0.01$  Mpc, but for other values, we have obtained essentially the same results.

The virial theorem states that

$$2 \langle T \rangle_t + \langle W \rangle_t = 0 \quad (5)$$

where  $T$  is the total kinetic energy,  $W$  the potential energy of the system and  $\langle \rangle_t$  denotes time average. When Eq. (5) is satisfied, the virial equilibrium is said to be attained and in this case the system is bound. For convenience, we define  $\gamma$  at the instant  $t$ , as

$$\gamma = \frac{T}{|W|} \quad (6)$$

so that,  $\gamma = \frac{1}{2}$  corresponds to an instantaneous virialization (virial equilibrium) of the system.

For each randomly chosen group, we calculate  $\gamma$  at every instant of the dynamical evolution of the system. Fig. 2 displays  $\gamma$  for different groups with multiplicities  $N = 10, 15, 20$  and  $30$ . It is noted that for  $t > 3/2 T_c \pi$ , where  $T_c = (3/5)^{1.5} R_h / \langle v^2 \rangle^{1/2}$  is the crossing time such as defined by Gott et al. (17) and  $R_h$  the harmonic radius,  $\gamma$  oscillates around  $0.5$  indicating that, at this moment, the system is close to the virial equilibrium. However, this does not occur for all simulated groups. Indeed, in some of them,  $\gamma$  still oscillates, but the average value deviates from  $0.5$ . In this work, we have only considered groups in which the deviation is less than  $10\%$ .

To compare some dynamical quantities obtained in this simulation with the observational data, we have used informations taken from a sample formed by 48 groups from Geller and Huchra's catalogue (6), with multiplicity  $N \geq 7$ . It should be noted that, in this case, all the groups have the crossing time less than 0.2, indicating that they have a self-dynamics. Assuming that the groups are bound systems, the virial theorem provides a way to estimate the instant of system equilibration. All the dynamical quantities of the system, then, can be calculated at the equilibrium instant.

#### IV. RESULTS AND DISCUSSION

It was pointed out, in Sect. II; that the coefficient  $\alpha$  and  $\beta$  in Eq. (2), can not be determined separately by fitting the luminosity data of Turner and Gott, but only their ratio  $\beta/\alpha$ , which was shown to be 5.8.

In order to determine  $\alpha$  and  $\beta$  uniquely, we have calculated the velocity dispersion, defined (5) as

$$\sigma = \left[ \frac{1}{N-1} \sum_{i=1}^N v_{zi}^2 \right]^{1/2}, \quad (7)$$

where  $v_{zi}$  is the projection along the z-axis of the velocity of the i-th galaxy with respect to the center-of-mass of the group. To be sure about the virial equilibrium, the velocities of each group in Eq. (7) was calculated at the instant equal or greater than  $t = 2\pi T_c$ , being  $T_c$  the crossing time of group.

In Fig. 3, we display the histogram of  $\sigma$  for three different values of  $\alpha$  ( $\beta = 5.8\alpha$ ), i.e,  $\alpha = 8, 16$  and  $30$  (full lines). The number of simulated groups were 1000. We can compare the calculated histograms with the observational data, such as given by Geller and Huchra (6) (dot lines). It is apparent that for  $\alpha = 16$  ( $\beta = 92$ ), the calculated result fits better the data.

With these values of  $\alpha$  and  $\beta$ , the mass range for galaxies belonging to groups can be evaluated. In fact, with the luminosity range considered in this work, i.e,  $0.05 \leq L \leq 50.0$  in units of  $10^{10} L_{\odot}$ , the mass range is  $0.9 < M < 9.0$  in units of  $10^{12} M_{\odot}$ . This range is in good agreement with results obtained by other authors (5,9). It should be noted that the three decades in the luminosity range have shrunk to one decade only in the mass range. This is a direct consequence of the linear relationship between mass and luminosity we have assumed.

The mean value of the mass-to-light ratio for galaxies is given by

$$\langle \frac{M}{L} \rangle = \frac{\int_{L_i}^{L_f} L^{-2} (\alpha + \beta L^{-1}) \phi(L) dL}{\int_{L_i}^{L_f} L^{-2} \phi(L) dL}, \quad (8)$$

where  $\phi(L)$  is the luminosity distribution function.

Replacing  $\alpha$ ,  $\beta$  and  $\tau$  by their fitted values in Eq. (8), we find  $\langle M/L \rangle \approx 115 M_{\odot}/L_{\odot}$ . This result is close to the value found by Geller and Huchra (6), which is around  $170 M_{\odot}/L_{\odot}$ .

We can also estimate the mass-to-light ratio for groups as a whole. To do that, we assume the luminosity of the group  $L_g$  as given by the sum over all galaxies in the group, i.e.,  $L_g = \sum_i^N L_i$ . Thus using the Eq. (2) we obtain

$$\frac{M_g}{L_g} = 16 + 92 N L_g^{-1} , \quad (9)$$

where  $M_g = \sum_i^N M_i$  is the mass of the group with the binding energy neglected. In Fig. 4, we plot  $M_g/L_g$  against  $L_g$ . The open circles denote the results obtained from Eq. (9) with values of  $N$  and  $L_g$  taken from Gott and Turner's catalogue (18). The crosses denote the values given by Gott and Turner's catalogue itself. In spite of the different behaviour of these two sets of results, Fig. 4 shows that the mass-to-light estimated from this model reproduces nicely the mean value of the observational data.

In Fig. 5, we plot the kinetic energy spectra for galaxies with masses within three different mass ranges: (i)  $90 < M < 91$ ; (ii)  $100 < M < 101$  and (iii)  $110 < M < 111$  in units of  $10^{10} M_\odot$ . All these spectra display exponential behaviour (dot lines). This exponential decay is independent of the initial velocities and reflect a Maxwell-Boltzmann behaviour. In order to obtain the temperature parameter, we have fitted the energy spectra to the Maxwell-Boltzmann distribution. The result shows that this parameter is roughly equal to  $10 T_{52}$  ( $T_{52} = 10^{52} \text{J}$ ).

In summary, considering a power-law mass distribution and a linear function between mass and luminosity, we have fitted

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the parameters  $\alpha$ ,  $\beta$  and  $\tau$  to the observational data of luminosity and of velocity dispersion, obtaining  $\tau = 2.8$ ,  $\alpha = 16$  and  $\beta = 92$ . The value of  $\beta/\alpha$  is 5.8 and this value can explain the difference of up about two orders of magnitude between the total mass and the mass given solely by the luminosity. This very simple model may be very useful to study the mass distribution in groups of galaxies, in particular, the problem of missing mass, such as mentioned in the Introduction. Specifically, we have found the mass interval for the galaxies belonging to groups (0.9 to 9.0 in units of  $10^{12} M_{\odot}$ ). Our results depend strongly upon the available data given by catalogues. At the present, the number of observed groups of galaxies is small and gives a very poor statistics. With coming data, including those of southern hemisphere, our simulation may yield more precise results.

## FIGURE CAPTIONS

- Fig. 1: Plot of the luminosity distribution for groups of galaxies. The observational data from Turner and Gott (15) are given by dots and the fitting with  $\tau = 2.8$  and  $\beta/\alpha = 5.8$  by the full curve.
- Fig. 2: Evolution of  $\gamma$  with the time  $t$  for groups with multiplicity  $N = 10, 15, 20$  and  $30$ .
- Fig. 3: Histograms of the velocity dispersion  $\sigma$ , obtained from the Geller and Huchra's (6) data (dashed lines) and from our simulation (full lines).
- Fig. 4: The mass-to-light ratio for groups  $M_g/L_g$  is plotted as function of  $L_g$ . The results obtained from Eq.(9) are denoted by open circles and the observational data from Gott and Turner (18), by crosses.
- Fig. 5: Energy spectra of galaxies belonging to groups for three different galaxy mass ranges.

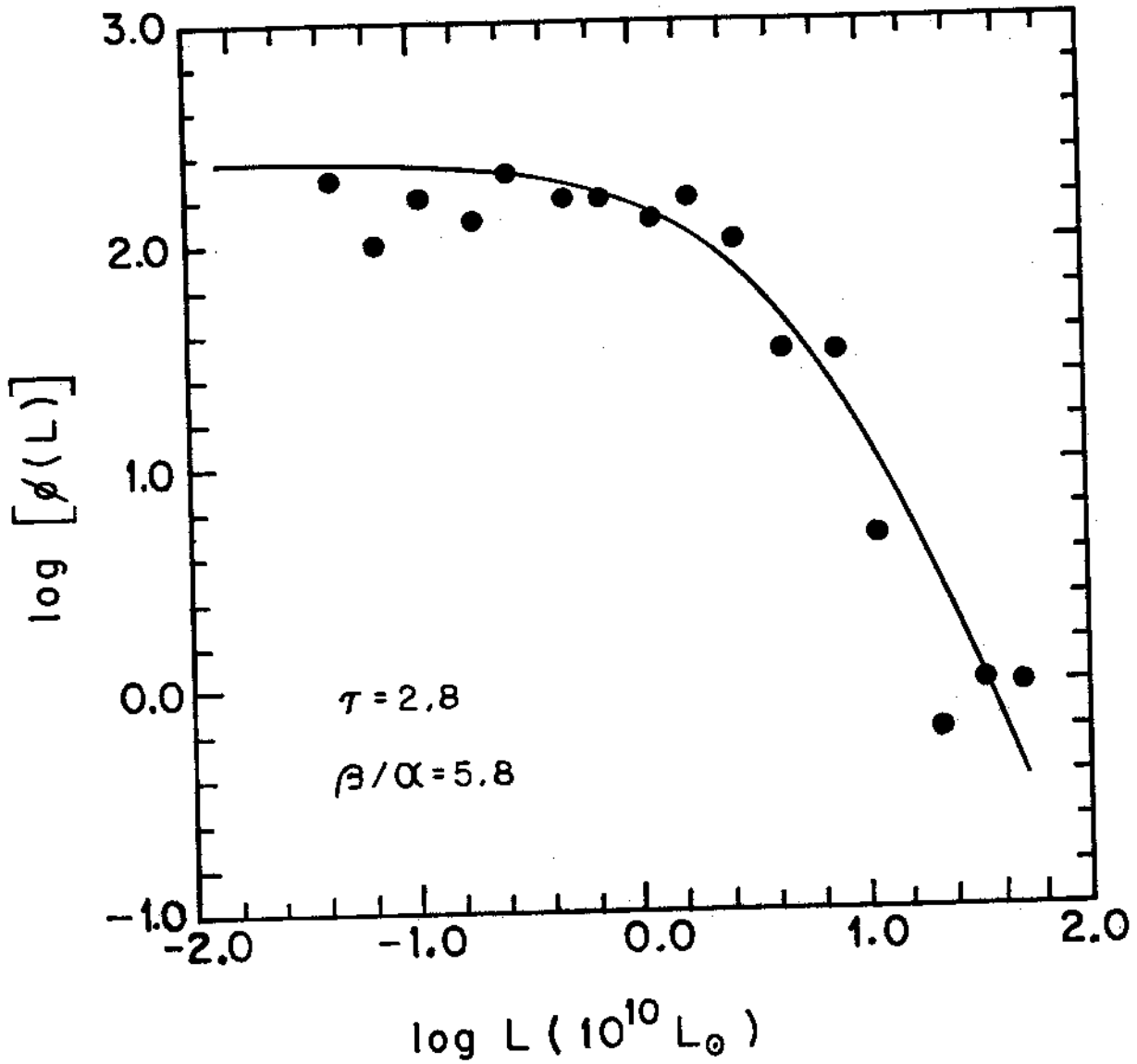


Fig. 1

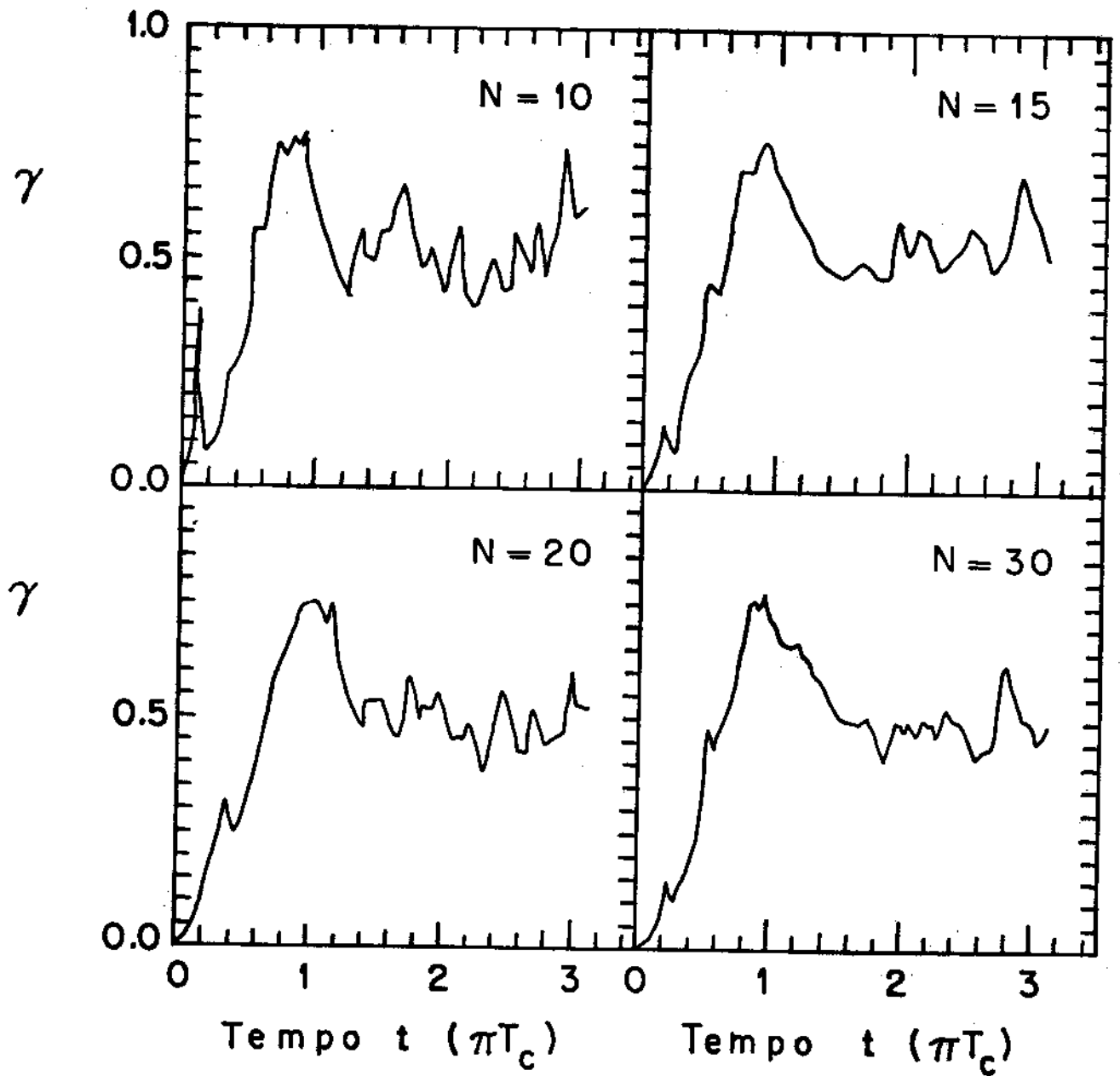


Fig. 2



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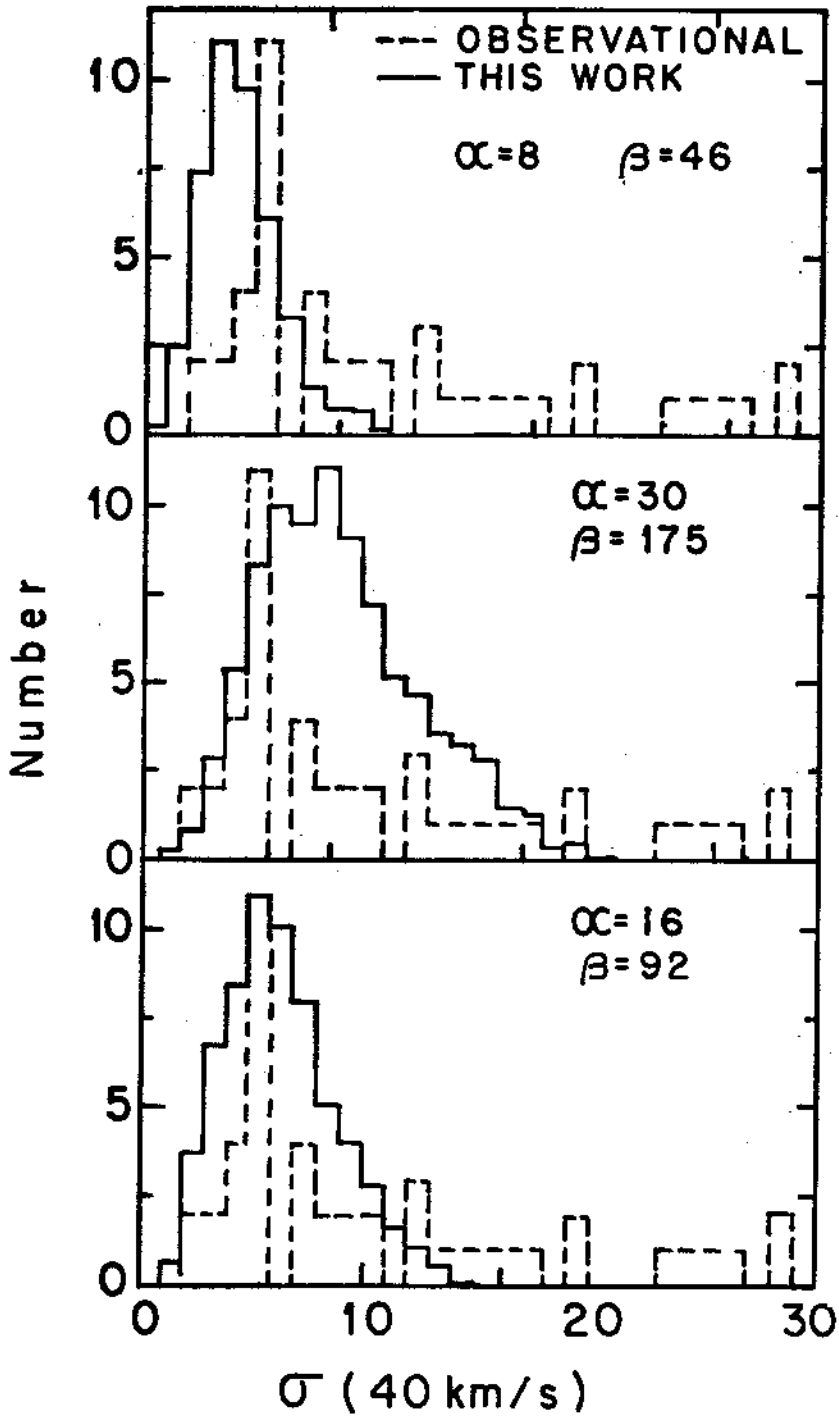


Fig. 3

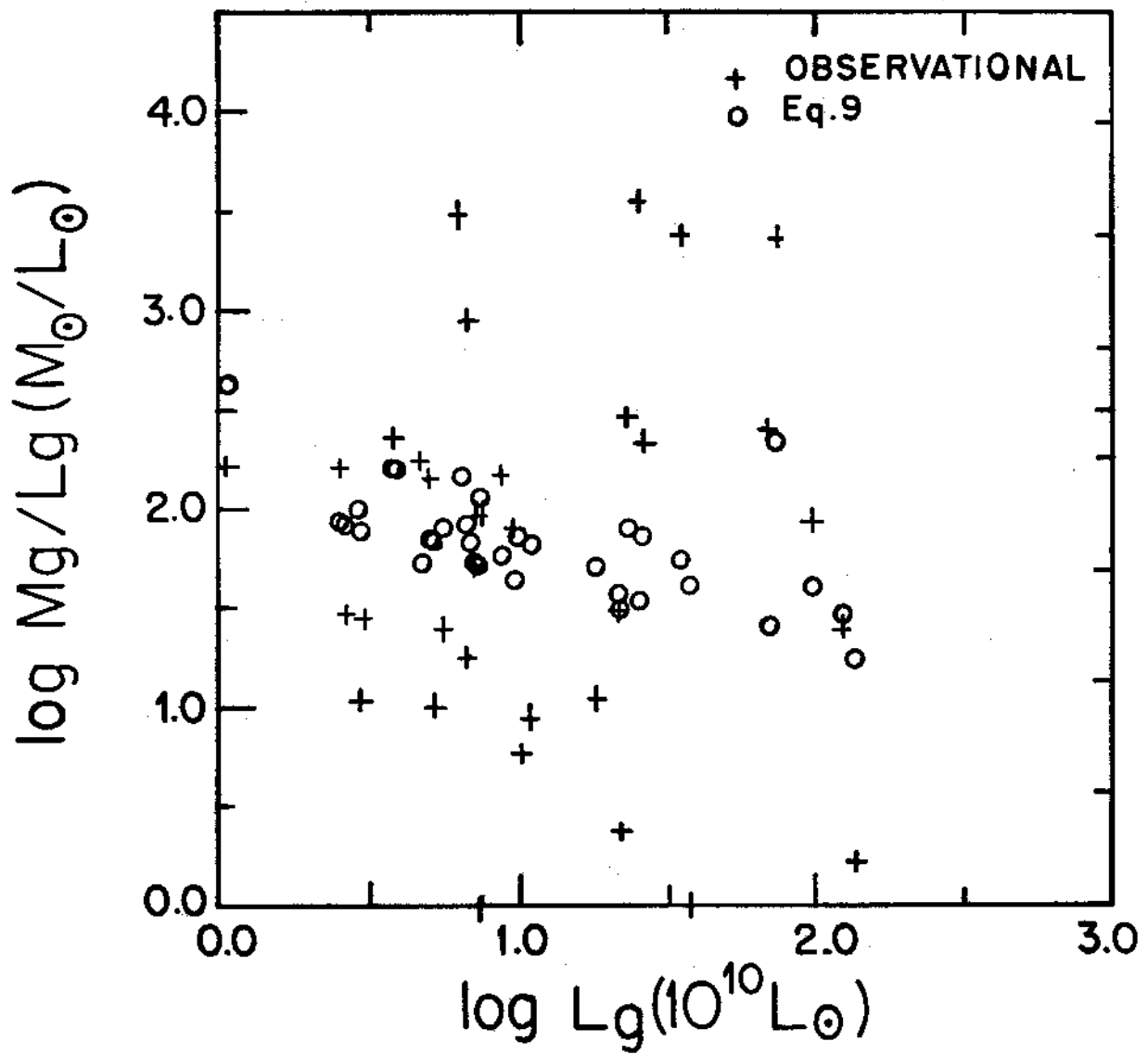


Fig. 4

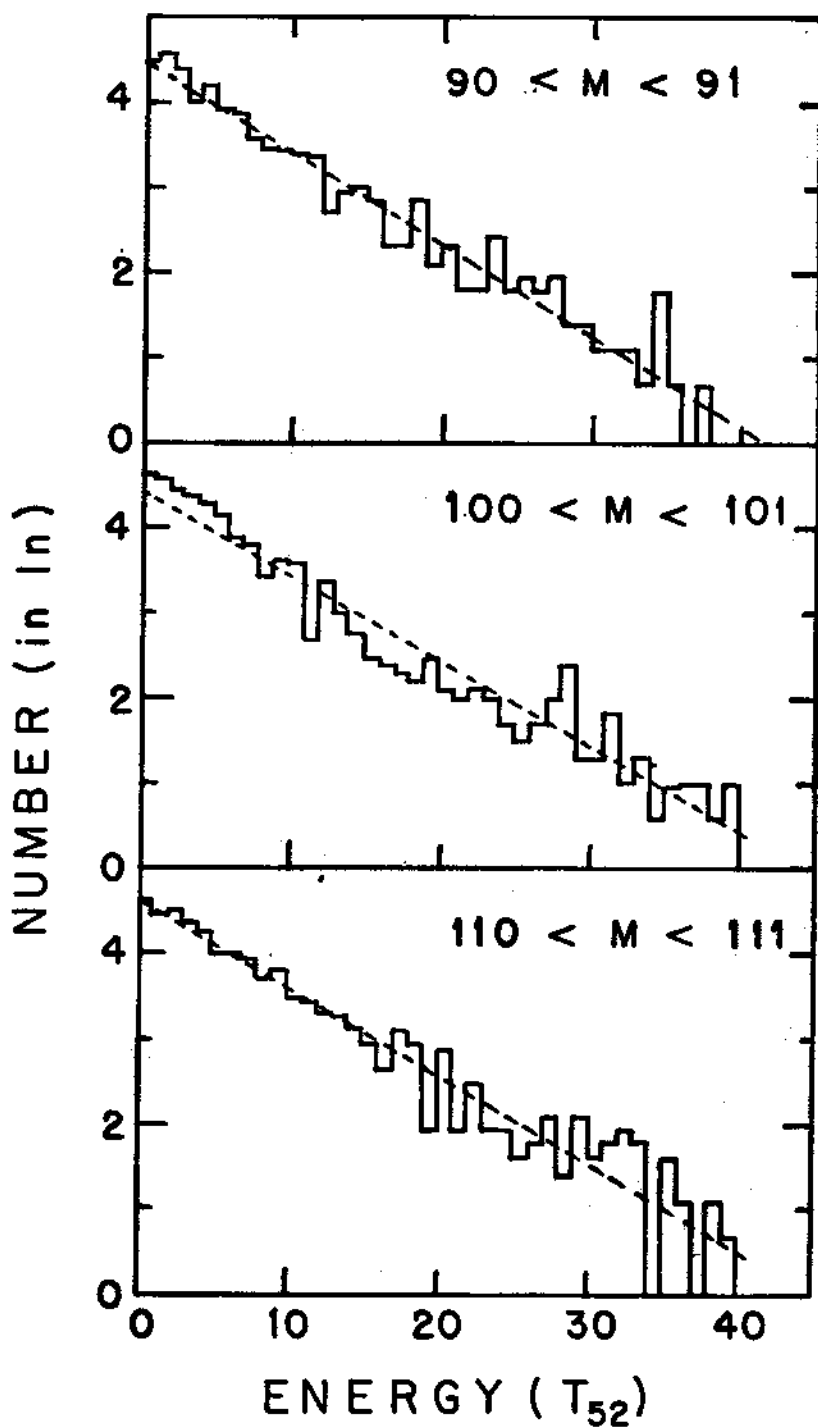


Fig. 5

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