CNPq

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-003/92

NEW REMARKS ON CHIRAL BOSONIZATION

by

Alvaro de Souza DUTRA

ABSTRACT

We discuss a certain duality between the constraints appearing in ordinary Lagrangian density and its first order counterpart for the gauged Siegel chiral boson. It is demonstrated the equivalence, at the classical level, of the two versions of the gauged Siegel chiral boson to its corresponding gauged Floreanini-Jackiw chiral bosons. It is also argued that the most general constrained Lagrangian density, that leads to a bosonic field obeying a first order differential equation of motion and preserve simultaneously Lorentz invariance, is just the Floreanini-Jackiw one.

PACS Numbers: 11.10.Ef, 11.30.Cp, 11.30.Rd.

Key-words: Two-dimensional models; Chiral bosons.

Recently Bazeia [1] showed, using a method developed by Faddeev and Jackiw [2], that a gauged version of Siegel's chiral boson proposed by Belucci, Golterman and Petcher [3,4] is equivalent, at the classical level, to the gauged Floreanini-Jackiw (FJ) chiral boson found by Harada [5,6]. More recently [7] a simple procedure to verify the Lorentz invariance was proposed for models where it is not explicitly manifest, like the FJ chiral boson [5]. Based on this approach, it was proposed a new way to gauge the FJ chiral boson, which was equivalent to another gauge invariant Siegel chiral boson [7]. The latter corresponds to a sort of "chiral gauging", where chirality is preserved under gauge transformations, as we will see later.

Here we intend to show that if one starts from the model proposed by Belucci, Golterman and Petcher and substitutes the gauged constraint by the non-gauged one ends with that "chiral gauge-invariant" version of Siegel's chiral boson mentioned above. We prove then the equivalence between it and the corresponding new version for the gauged FJ model.

First of all, let us see a simple way to test the Lorentz invariance of Lagrangian densities when it is not manifest. This was recently used to introduce the second gauge invariant version of the FJ chiral boson [7] quoted above.

We exemplify this approach by applying it to the case of FJ chiral boson [5], characterized in its local version by the Lagrangian density

$$\mathcal{L}_{FJ} = \dot{\phi}\,\phi' - \phi'^2, \qquad (1)$$

where $\dot{\phi}$ and ϕ' denotes $\partial_0\phi$ and $\partial_1\phi$ respectively. Performing a Lorentz transformation,

$$\begin{pmatrix} \phi' \\ \dot{\phi} \end{pmatrix} \to \begin{pmatrix} \cosh(\varphi) & \sinh(\varphi) \\ \sinh(\varphi) & \cosh(\varphi) \end{pmatrix} \begin{pmatrix} \phi' \\ \dot{\phi} \end{pmatrix}, \tag{2}$$

where φ is associated to the relative velocity between the two reference frames. Substituting (2) in (1) we get,

$$\mathcal{L}_{\varphi} = a(\varphi)\dot{\phi}^2 - b(\varphi)\phi'^2 + c(\varphi)\dot{\phi}\,\phi', \tag{3}$$

with

$$a(\varphi) \equiv \sinh(\varphi)(\cosh(\varphi) - \sinh(\varphi)), \tag{4a}$$

$$b(\varphi) \equiv \cosh(\varphi)(\cosh(\varphi) - \sinh(\varphi)), \tag{4b}$$

$$c(\varphi) \equiv (\cosh(\varphi) - \sinh(\varphi))^2. \tag{4c}$$

As this "rotated" Lagrangian density is not constrained, we can easily construct its corresponding first order Lagrangian density [2],

$$\mathcal{L}_{\varphi 1} = \pi_{\phi}\dot{\phi} - b(\varphi)\phi'^2 - (\pi_{\phi} - c(\varphi)\phi')^2/4a(\varphi). \tag{5}$$

Since, as required by the equivalence principle, the two systems must be indistinguishables, we impose that the new Lagrangian density obeys the same constraint ($\pi_{\phi} = \phi'$) as the Lagrangian density (1), so (5) becomes

$$\mathcal{L}_{\varphi 1}\Big|_{\pi_{\phi} = \phi'} = \phi'\dot{\phi} - \Big(4a(\varphi)b(\varphi) - (1 - c(\varphi))^2\Big)\phi'^2/4a(\varphi). \tag{6}$$

Substituting the variables $a(\varphi)$, $b(\varphi)$ and $c(\varphi)$, and using trigonometric relations in (6), we obtain that

$$\mathcal{L}_{\varphi 1}\Big|_{\pi_\phi = \phi'} = \mathcal{L}_{FJ}. \tag{7}$$

So, this Lagrangian density is Lorentz invariant because under the chiral constraint requirement it is unchanged by such transformation. This method can be easily applied to the cases of the gauged FJ models [6,7].

However one could study a generalized constrained Lagrangian density like

$$\mathcal{L}_{\varphi} = g_1 \dot{\phi}\, \phi' + g_2 \phi'^2, \tag{8}$$

that leads to the equation of motion,

$$\partial_1 (g_1 \partial_0 + g_2 \partial_1)\phi = 0 \tag{9}$$

which reduces to

$$(g_1 \partial_0 + g_2 \partial_1)\phi = 0, \tag{10}$$

after using appropriate boundary conditions. From Eq.(10) one can see that it has the FJ chiral boson as a particular case. However, there still remains an important question: is this model Lorentz invariant for arbitrary values of g_1 and g_2 ? Before we address an answer to this question we observe that, in fact, up to a finite renormalization, the expression (8) can be rewritten as

$$\mathcal{L}_{\varphi} = \alpha\, \dot{\phi}\, \phi' + \phi'^2, \tag{11}$$

with $\alpha \equiv g_1/g_2$. This model can be obtained from the covariant free boson

$$\mathcal{L} = (1/2)\partial_\mu\phi\ \partial^\mu\phi, \tag{12}$$

writing its first order Lagrangian density [2] and then imposing the reduction of the phase space defined by $\pi_\phi = \alpha\phi'$.

Now, we apply the method above to verify the Lorentz invariance of the Lagrangian density (11). Making the Lorentz rotation, the rotated Lagrangian density becomes non-constrained, so that we can easily obtain the corresponding Hamiltonian

$$\mathcal{H}^k = \left[\left(\alpha(k^2+1) + k^2 - 1\right)(k^2-1)\right]^{-1}\left[k^2\pi_\phi^2 - \pi_\phi\phi'\left(\alpha(k^4+1) + k^4 - 1\right) + \alpha\ k^2\phi'^2\right], \tag{13}$$

where $\varphi \equiv \log(k)$, with φ being related to the relative velocity between the reference frames in equation (2). From the above Hamiltonian density one can write down the first order Lagrangian density in the rotated frame. Then, keeping the constraint of the original frame in the new one, as requested by the equivalence principle, we get for the difference between the Lagrangian densities,

$$\mathcal{L}_\varphi - \mathcal{L}^k\Big|_{\pi=\alpha\phi'} = \left(\alpha(k^2+1) + k^2 - 1\right)^{-1}\left[(\phi')^2(1-k^2)(\alpha^2-1)\right] \tag{14}$$

that vanishes only (implying Lorentz invariance) for $\alpha = \pm 1$, precisely the two chiralities of the FJ chiral boson.

From now on we are able to apply this simple method in order to verify the Lorentz invariance of any non-manifestly covariant model.

Let us now exhibit a kind of "duality" between the constraints of the two gauge invariant FJ models. For this we write down the Lagrangian density of Belucci et al,

$$\mathcal{L} = (1/2)\partial_\mu\phi\ \partial^\mu\phi + e\ \partial_+\phi\ A_- + (M^2/2)A_\mu A^\mu + \lambda(\partial_-\phi + eA_-)^2, \tag{15}$$

where $\partial_- \equiv \partial_0 - \partial_1$ and $A_- \equiv A_0 - A_1$. The corresponding first order Lagrangian density is, as observed by Bazeia [1], given by

$$\mathcal{L}_1 = \pi_\phi(\partial_-\phi + eA_-) + e\ \phi' A_- - (e^2/2)(A_-)^2 + (M^2/2)A_\mu A^\mu +$$

$$- \frac{1}{(1+\lambda)}(\pi_\phi - \phi')^2, \tag{16}$$

with $\phi' \equiv \partial_1\phi$. At this point we can obtain from the Euler-Lagrange equation for the Lagrange multiplier, that

$$(\pi_\phi - \phi')^2 = 0, \tag{17}$$

which, at the classical level, is equivalent to the imposition $\pi_\phi = \phi'$ (at the quantum level they will be different, as a consequence of ordering ambiguities). So, choosing the hipersurface in the phase space where the constraint $\pi_\phi = \phi'$ holds, we get the Harada version for the gauged FJ chiral boson,

$$\mathcal{L}_{ch} = \dot{\phi}\ \phi' - \phi'^2 + 2e\phi' A_- - (e^2/2)A_-^2 + (M^2/2)A_\mu A^\mu. \tag{18}$$

It is important to observe that in the usual Lagrangian density (5), the constraint imposed is $(\partial_-\phi + eA_-)^2 = 0$, and that when it is constructed its first order Lagrangian density, the non-gauged constraint $(\pi_\phi - \phi')^2 = 0$ arises quite naturally. Besides, using the method described above in order to verify the Lorentz invariance, one can see that the corresponding constraint to be changed in (15) must be that of the opposite chirality, $(\partial_+\phi)^2 = 0$. This is in agreement with the observation in Ref. [7], where it was verified that, starting from the left-handed CSM there exist two relativistically possible constraints: $\pi_\phi = \phi'$ and $\pi_\phi = -\phi' + eA_-$; and that for the right-handed one they are: $\pi_\phi = -\phi'$ and $\pi_\phi = \phi' + eA_+$. It is not difficult to verify that the model in (15), up to a finite coupling renormalization, is the left-handed CSM subjected to the constraint $(\partial_-\phi + eA_-)^2 = 0$, so that its dual constraint must be $(\partial_+\phi)^2 = 0$. With this in mind we invert the order and impose this second constraint in the ordinary Lagrangian density,

$$\mathfrak{L} = (1/2)\partial_\mu\phi\, \partial^\mu\phi + e\, \partial_+\phi\, A_- + (M^2/2)A_\mu A^\mu + \lambda(\partial_+\phi)^2, \tag{19}$$

that corresponds to gauging only one of the chiralities, so that

$$\begin{aligned} \phi &\to \phi + \epsilon(x^-) \\ A_- &\to A_- - (1/e)\partial_-\epsilon(x^-) \end{aligned} \tag{20}$$

as observed in [7]. Now the corresponding first order Lagrangian density is given by

$$\mathcal{L}_1 = \pi_\phi(\partial_-\phi) + 2e\phi' A_- + (M^2/2)A_\mu A^\mu - \frac{1}{2(1+\lambda)}(\pi_\phi - \phi' - eA_-)^2, \qquad (21)$$

From its equations of motion for the Lagrange multiplier we are lead to the choice of the constraint $(\pi_\phi - \phi' - eA_-)^2 = 0$. Then, after the imposition of such a constraint in (21) we get

$$\mathcal{L}_{ch} = \dot{\phi}\,\phi' - \phi'^2 + e\,\partial_+\phi\,A_- + (M^2/2)A_\mu A^\mu, \qquad (22)$$

that was recently considered [7], and corresponds to the above mentioned "chiral gauging". Now we can do an analogous analysis for the case of the linear constraint imposed by Srivastava [8]. Starting with the gauged version of this model, where the constraint appears linearly,

$$\mathcal{L} = (1/2)\partial_\mu\phi\,\partial^\mu\phi + e\,\partial_+\phi\,A_- + (M^2/2)A_\mu A^\mu + \lambda(\partial_-\phi + eA_-), \qquad (23)$$

we get, as stated by Bazeia, for the first order Lagrangian density,

$$\mathcal{L}_1 = \pi_\phi\dot{\phi} - (1/2)\phi'^2 + e\,\phi' A_- - (e^2/2)(A_-)^2 + (M^2/2)A_\mu A^\mu$$

$$- \lambda(\phi' - eA_-) - (1/2)\Big[\pi - \lambda - eA_-\Big]^2. \qquad (24)$$

Using the solution for the Lagrange multiplier $\lambda = \pi_\phi - \phi'$, one can find that

$$\mathcal{L}_1 = \pi_\phi(\partial_-\phi) + e(\pi_\phi + \phi')A_- - (e^2/2)(A_-)^2 + (M^2/2)A^\mu A_\mu, \qquad (25)$$

from which one can see that, for the Harada constraint ($\pi_\phi = \phi'$), we recover his result (18). If we impose the gauged version of the chiral constraint ($\pi_\phi = \phi' + eA_-$), the Lagrangian density obtained is not Lorentz invariant as can be easily verified by using the method described at the beginning of this work. At this point it is important to observe that the criterion of Lorentz invariance is quite fundamental in such cases because, since all of these Lagrangian densities are not explicitly invariant, one must verify the invariance to obtain reliable results.

The "dual version" of expression (23) is given by

$$\mathcal{L} = (1/2)\partial_\mu\phi\,\partial^\mu\phi + e\,\partial_+\phi\,A_- + (M^2/2)A_\mu A^\mu + \lambda(\partial_-\phi), \tag{26}$$

that has as first order counterpart,

$$\mathcal{L}_1 = \pi_\phi\dot{\phi} - (1/2)\phi'^2 + e\dot{\phi}'A_- + (M^2/2)A_\mu A^\mu + \lambda(\pi_\phi - \phi') + - (1/2)\left[\pi_\phi - \lambda - eA_-\right]^2, \tag{27}$$

Substituting the solution for λ ($\lambda = \pi_\phi - \phi' - eA_-$), and imposing the "gauged chiral constraint" $\pi_\phi = \phi' + eA_-$, we get

$$\mathcal{L}_1 = \phi'(\partial_-\phi) + e\partial_+\phi\,A_- + (M^2/2)A^\mu A_\mu, \tag{28}$$

Which is the model described in [7], that is Lorentz invariant. If on the contrary we use Harada's constraint, the Lagrangian density obtained becomes non-invariant under Lorentz transformations.

It can also be observed that, as occurs in the usual gauged Siegel boson, in this new manner of gauging the chiral boson (19), the non explicitly covariant case (22) can be obtained choosing $\lambda = -1/2$ in (19). Note however that for this value of the parameter λ, the Lagrangian density becomes constrained, and so it must be carefully treated. We must be careful also to make this choice after the imposition of the "gauged chiral constraint", in analogy to what occurs in the previous case [1].

As a final remark let us mention that all of the study above can be repeated for the case of the right-handed CSM subjected to its corresponding constraints: $(\partial_+\phi + eA_+)^2 = 0$ and $(\partial_-\phi)^2 = 0$, that have as their counterparts in the FJ-type models: $\pi_\phi = -\phi'$ and $\pi_\phi = \phi' + eA_+$, respectively.

ACKNOWLEDGEMENTS: The author thanks to CNPq (Conselho Nacional de Desenvolvimento Cientifico e Tecnológico) of Brazil, by partial financial support, and to FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) for providing computer facilities.

REFERENCES:

[1] D. Bazeia, Mod. Phys Lett. A **5** (1991) 2497.

[2] L. Faddeev and R. Jackiw, Phys Rev. Lett. **60** (1988) 1692.

[3] W. Siegel, Nucl. Phys. B **238** (1984) 307.

[4] S. Belucci, M. F. L. Golterman and D. N. Petcher, Nucl. Phys. B **326** (1989) 307.

[5] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59** (1987) 1873.

[6] K. Harada, Phys. Rev. Lett. **64** (1990) 139.

[7] S. A. Dias and A. de Souza Dutra, CBPF preprint NF-037/91, submitted for publication.

[8] P. P. Srivastava, Phys. Rev. Lett. **63** (1989) 2791.