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A MODULI-DEPENDENT LAGRANGIAN FOR (2,2) THEORIES ON CALABI-YAU N-FOLDS*

by

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Abstract

We provide an explicitly moduli-dependent realization of the (2,2), $c = 3n$ superconformal algebra, utilizing an interacting collection of b, c, β, γ systems. Our construction follows from a (2,2) superconformal Lagrangian describing the critical surface of a $N = 2$ Landau Ginzburg Lagrangian or of a σ model on a deformation class of Calabi-Yau n-folds.

Key-words: Superstring theory; Calabi-Yau compactifications; Moduli spaces.

1 INTRODUCTION

Superconformal field-theories [1,2a] of type (2.2) [3] have been analyzed in depth and under many view-points.

Their applications are in two different fields:

i) superstring compactification [4-8];

ii) 2-dimensional critical phenomena [9,10].

The first application relies on the fact that for $c = 9$,(2,2)-theories represent the internal degrees of freedom of the heterotic superstring vacua with $N = 1$ target supersymmetry and E_6 chiral gauge group.

Geometrically the (2,2)-vacua correspond to a compactification where the internal 6-space is Calabi-Yau manifold, namely a complex three-fold F_3 of vanishing first Chern class: $c_1(F_3) = 0$ [11].

The second application of (2,2)-theories is in the field of critical phenomena and it is not related to a specific value of the central charge. However, there is a conjecture by Gepner [5], supported by overwhelming evidences [10], that any $c = 3n$ ($n \in N$) (2,2)-theory is the exact quantum solution of an $N = 2$ σ-model on a complex n-fold F_n of vanishing first Chern class: $c_1(F_n) = 0$. Such spaces have always SU(n) holonomy and are named Calabi-Yau n-folds.

This correspondence leads to a geometric interpretation of the abstract (2,2)-theory deformations in terms of the moduli of the underlying Calabi-Yau manifold. In general one finds an isomorphism between the Dolbeault cohomology ring of the F_n n-fold and the chiral ring of the corresponding abstract (2,2)-theory [12].

Furthermore the Zamolodchikov metric [13] on the moduli-space of the (2,2)-theory is to be indentified with the Weyl-Petersson metric on the corresponding F_n moduli-space. In particular, in the $c = 9$ case the complete moduli-space of the (2,2)-theory is the direct product of two special Kählerian manifolds [14]:

$$M_{moduli} = SK_{(1,1)} \otimes SK_{(2,1)} \tag{1.1}$$

where the complex dimensions of the two submanifolds (dim $SK_{(1,1)} = h_{(1,1)}$; dim $SK_{(2,1)} = h_{(2,1)}$) are the two independent Hodge numbers of the Calabi-Yau 3-fold. From the algebraic view-point $SK_{(1,1)}$ is the parameter space of the Kähler class deformations, associated with harmonic (1,1)-forms, while $SK_{(2,1)}$ is the parameter space of the complex structure deformations associated with harmonic (2,1)-forms.

From the abstract (2,2) viewpoint $SK_{(2,1)}$ is the parameter space of the superconformal theory deformations induced by chiral-chiral primary fields characterized by conformal weights $h = \bar{h} = \frac{1}{2}$ and U(1) charges $q = \bar{q} = 1$, while $S_{(1,1)}$ is the parameter space of the deformations induced by chiral-antichiral primary fields having $h = \bar{h} = \frac{1}{2}$ and $q = -\bar{q} = 1$.

Following the notation of [7] let $\Psi_a\begin{pmatrix}\frac{1}{2} & , & \frac{1}{2}\\ 1 & , & 1\end{pmatrix}(z,\bar{z})$ be the chiral-chiral primary fields mentioned above ($a = 1,\ldots,h_{(1,1)}$); then, evaluating the operator product expansion with N = 2 supercurrent:

$$G^-(z)\tilde{G}^-(\bar{z})\Psi_a\begin{pmatrix}\frac{1}{2} & , & \frac{1}{2}\\ 1 & , & 1\end{pmatrix}(\omega,\bar{\omega}) = \frac{1}{|z-\omega|^2}\,\Phi_a\begin{pmatrix}1 & , & 1\\ 0 & , & 0\end{pmatrix}(\omega,\bar{\omega}) + \text{reg} \qquad (1.2)$$

we obtain a set of $h_{(2,1)}$ fields of conformal weights $h = \bar{h} = 1$ that can be used to deform the Lagrangian of the (2,2)-theory:

$$\mathcal{L}^{(2,2)}(z,\bar{z}) \to \mathcal{L}^{(2,2)}(z,\bar{z}) + \delta M^a\Phi_a\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}(z,\bar{z}) = \mathcal{L}^{(2,2)}(z,\bar{z}) \qquad (1.3)$$

The parameters δM^a are the differentials of the (2,2)-theory of the (2,1)-moduli and the new Lagrangian $\mathcal{L}^{(2,2)}$ defines a new (2,2)-theory with the same $h_{(2,1)}$ and $h_{(1,1)}$ numbers and the same central charge.

Iterating the procedure we can reconstruct (at least in principle) a Lagrangian $\mathcal{L}^{(2,2)}(z,\bar{z},M^a)$ which explicitly depends on a set of $h_{(2,1)}$

parameters M^a and, for all values of these parameters, defines a (2,2)-theory with fixed Hodge numbers and fixed central charge.

Setting:

$$\Phi_a\begin{pmatrix}1 & , & 1\\ 0 & , & 0\end{pmatrix}(z,\bar{z},M) = \frac{\partial}{\partial M^a}\,\mathcal{L}^{(2,2)}(z,\bar{z},M) \qquad (1.4)$$

the Zamolodchikov metric $g_{a,\bar{b}}(M.\bar{M})$ is defined by the 2-point correlator:

$$<\bar{\Phi}_{\bar{b}}(z,\bar{z},M)\Phi_a(\omega,\bar{\omega},M)> = \frac{g_{a,\bar{b}}(M,\bar{M})}{|z-\omega|^4} \qquad (1.5)$$

Furthermore, from the Lagrangian $\mathcal{L}^{(2,2)}(z,\bar{z},M)$, by calculating the associated Noether currents, one can derive an explicitly moduli-dependent expression for the generators of the (2,2) superconformal algebra.

The problem with the programme we have outlined is that usually it cannot be carried through in explicit terms.

Writing the (2,2) Lagrangian as a σ-model Lagrangian one needs an explicit metric and torsion on the Calabi-Yau n-fold. These items cannot be randomly chosen but have to be determined as the zeros of the associated β-functions, so that the theory is really superconformal. This involves calculating and resumming the whole perturbation series of the σ-model, which is clearly beyond reach. Alternatively, in the orbifold limiting case [15], where the superconformal metric and torsion are known (the flat ones) only a subset of the primary fields $\Phi\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}(z,\bar{z})$ can be written in terms of the original fields appearing in the underformed lagrangian (untwisted moduli). The remaining deformations (twisted moduli) are expressed in terms of the spin fields that are non-local functionals of the original orbifold variables. This means that the outlined programme can be carried through only for the untwisted moduli [16].

In the present paper we want to show that there is at least one deformation class of smooth algebraic Calabi-Yau n-folds for which the Lagrangian $\mathcal{L}^{(2,2)}(z,\bar{z},M)$ can be explicitly written in terms of all the algebraic moduli: furthermore the generators of the (2,2)-algebra can also be written in an explicitly moduli-dependent way and fulfill the N = 2 OPE's at any value of the moduli parameters. The deformation class with the above properties is $CP_{n+1}[n + 2]$, namely the set of complex n-folds defined by a homogeneous polynomial constraint [5] of degree n + 2 in CP_{n+1}.

Our strategy to obtain the result will now be outlined and from this sketch it will be clear why we focus on the particular case $CP_{n+1}[n + 2]$.

The basic idea is that of utilizing a new set of variables, different from those of the σ-model, that capture the topological and analytic properties of the target-manifold bypassing all its metric properties, this is the same aim pursued by the Landau-Ginzburg formulations [9] where the analytic and topological properties are encoded in the superpotential, while the metric properties, encoded in the kinetic terms, are disregarded. The difference between the Landau-Ginzburg approach and our formulation is that the former is an ordinary field theory, becoming superconformal ony at some critical point, while the latter is a (2,2)-theory never moving off-criticality. In the Landau-Ginzburg Lagrangian the basic variables are scalar superfields representing the coordinates of the ambient space in which the Calabi-Yau n-fold is immersed as a complete intersection of polynomial constraints. Classically these fields have canonical dimension $\frac{(D-2)}{2} = 0$ but quantum mechanically they acquire anomalous dimensions related with the structure of the superpotential. In our Lagrangian the basic variables are an appropriate collection of the β-γ-b-c systems [2] whose conformal weights encode the appropriate anomalous dimensions related with the defining polynomial.

The argument leading to a formulation in terms of β-γ-b-c fields is the following. On one hand we know from Gepner's work that an interesting class of c = 3n (2,2)-theories can be built as tensor product of N = 2 minimal models. On the other hand, it is a result shown in the present paper, that N = 2 minimal model of central charge

$$c = \frac{3k}{k + 2} \tag{1.6}$$

can be realized in terms of the supersymmetric β-γ-b-c system [2] where the conformal weights of the four fields are as follows:

$$h[\beta] = \lambda = \frac{1}{2k + 4} \tag{1.7a}$$

$$h[\gamma] = 1 - \lambda = \frac{2k + 3}{2k + 4} \tag{1.7b}$$

$$h[b] = \lambda + \frac{1}{2} = \frac{k + 3}{2k + 4} \tag{1.7c}$$

$$h[c] = \frac{1}{2} - \lambda = \frac{k + 1}{2k + 4} \tag{1.7d}$$

Hence, for the (2,2)-theories obtained from the Gepner's tensor product construction, a Lagrangian written in terms of β-γ-b-c fields does always exist. The next question is whether $h = \bar{h} = \frac{1}{2}$ chiral-chiral primary fields can be written solely in terms of β-γ-b-c fields. If this condition is verified, then the deformed Lagrangian is still constructed out of the same set of fields and the deformation procedure can be successfully iterated. In this way one obtains the Lagrangian $\mathcal{L}^{(2,2)}(z,\bar{z},M)$ as a power series in the moduli parameters M^a(Noether coupling method). Fixing $c = 3n$, the critical condition we have just mentioned is certainly met by the following tensor product:

$$\mathcal{F}_n \stackrel{def}{=} (n)^{n+2} \tag{1.8}$$

which corresponds to the Fermat curve of degree n + 2 in CP_{n+1}[5]:

$$FC_n \; : \; Z_1^{n+2} + Z_2^{n+2} + \ldots + Z_{n+2}^{n+2} = 0 \tag{1.9}$$

The Fermat's curve FC_n is obviously a special point in the moduli-space of the deformation class $CP_{n+1}[n+2]$ so that the moduli-dependent Lagrangian $L^{(2,2)}(z,\bar{z},M)$ we obtain with our Noether procedure describes this deformation class.

In order to explain why $F_n = (n)^{n+2}$ has the property we have required and why it is specifically chosen, let us recall the results of ref. [5] concerning the algebraic interpretation of a general tensor product model.

Consider the tensor product of N discrete series of level $k_i (i = 1,\ldots,N)$ and to each of this minimal models associate a complex coordinate Z_i. Define the least integers $r_i (i = 1,\ldots,N)$ and d such that

$$r_i = \frac{d}{k_i + 2} \tag{1.10}$$

The (2,2)-theory obtained from the above tensor product corresponds to the (N-2)-fold defined by the following degree d equation

$$WC_{N-2} \; : \; Z_1^{k_1+2} + Z_2^{k_2+2} + \ldots + Z_N^{k_N+3} = 0 \tag{1.11}$$

in the weighted projective space $WCP_{N-1}(r_1,\ldots,r_N)$.

Note that a minimal model with k = 0 contributes zero to the central charge and corresponds to the identity representation of the superconformal algebra. Hence $k = 0$ factors are irrelevant in the superconformal language. However they cannot be disregarded while writing the corresponding algebraic equation (1.11). If $c = 3n$ the total number of coordinates Z_i has to be $n + 2$ in order to obtain an n-fold by means of a polynomial constraint in an $n + 1$-weighted projective space. In a tensor product with $N < n + 2$ the missing coordinated are associates with $k = 0$ models and have weight $\frac{d}{2}$, d being defined by equation (1.10). Note also that $k = 0$ factors correspond to a β-γ-b-c system with conformal weight $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{1}{4}$ (in the given order) (see eq. (1.7)). Hence, also in the Lagrangian we are proposing $k = 0$ factors cannot be disregarded.

Clearly the weighted Farmat's curve (1.11) is a particular case of the deformation class $WCP_{N-1}(d;r_1,\ldots,r_N)$ (Here we follow the notation of ref. [17]).

It is immediate to see the correspondence between the algebraic deformations of the defining polynomial (1.11) and the chiral-chiral primary fields that are expressible in terms of the β_i-γ_i-b_i-c_i variables. It suffices to establish the following correspondence between the coordinates of the weighted projective space and the β_i-fields:

$$Z_i = \beta_i(z)\bar{\beta}_i(\bar{z}) \tag{1.12}$$

Given a polynomial $\Pi^{(k+2)}(Z_i)$ of degree $k_i + 2$ in the Z_i coordinate (homogeneous of degree d in $WCP_{N-1}(r_{1,\ldots,}r_N)$, if we perform the formal substitution (1.12), we obtain the operator $\Pi^{(k_i+2)}(\beta_i,\bar{\beta}_i)$ that is the chiral-chiral with $h = \bar{h} = \frac{1}{2}$, $q = \bar{q} = 1$. This follows from eq. (1.7a) and the U(1) charge of β_i fields is $q(\beta_i) = \frac{1}{k_i + 2}$. In general, however, the complex structure deformations of a $WCP_{N-1}(d;r_1,\ldots,r_N)$ class are many more than the polynomial deformations. This means that there are other expression as polynomials in the original β_i-variables (twisted moduli). The only case where the polynomial deformations encompass all the deformations is the case of a hypersurface in CP_{N-1}, namely the case

$$k_1 = k_2 = \ldots = N - 2 = n \tag{1.13}$$

leading back to the proper Fermat's curve and to the deformation class $CP_{n+1}[n+2]$†. This is the reason why we focus on this deformation class. We present now the explicit derivation of our results.

2 Coupling of a β-γ-δ-d system to 2D supergravity: (1,0) local supersymmetry versus (2,0) global supersymmetry

In this section we consider the coupling to 2D-supergravity of a system composed of two Bose fields β, γ with conformal weights λ, $1 - \lambda$ and of two Fermi fields b, c with conformal weights $\lambda + \frac{1}{2}$, $\frac{1}{2} - \lambda$. In this way we obtain an action endowed with (1,0) local supersymmetry from which we can work out the stress energy tensor and the "local" supercurrrent of the β-γ-b-c system. They close the OPEs of an N = 1 superalgebra with central charge:

$$c = 3 - 12\lambda \tag{2.1}$$

The gauge fixed action, however, is endowed with a larger (2,0) global supersymmetry whose associated Noether currents fulfill the OPEs of the N = 2 superconformal algebra. The central charge is obviously the same and is given in eq. (2.1). This is the starting point for identification (1.7). In order to obtain a unitary representation we must equate the value of c given in (2.1) with that given in (1.6) and we obtain eqs. (1.7).

To carry through our programme we utilize the rheonomy approach [18] and we use the notations of ref. [19].

The supergravity background is described by the sweiben 1-forms $\{e^+, e^-\}$ and by the gravitions 1-form ζ that obey the following structural

† Note that for n = 2 we have the surface K3 and, exceptionally, there is a 20th non algebraic modulus besides the 19 algebraic ones.

equations

$$T^+ \equiv de^+ + \omega\wedge e^+ = \frac{i}{2}\,\zeta\wedge\zeta \tag{2.2a}$$

$$T^- \equiv de^- - \omega\wedge e^- = 0 \tag{2.2b}$$

$$T^\bullet \equiv d\zeta + \frac{1}{2}\,\omega\wedge\zeta = \tau e^+\wedge e^- \tag{2.2c}$$

$$R \equiv d\omega = \mathcal{R} e^+\wedge e^- - i\tau\zeta\wedge e^- \tag{2.2d}$$

where the ω is the SO(1,1) spin connection. $\mathcal{R}$ is the world-sheet curvature and τ the gravitino field strength.

The special supperconformal gauge is selected by choosing

$$e^+ = dz + \frac{i}{2}\,\theta d\theta \tag{2.3a}$$

$$e^- = d\bar{z} \tag{2.3b}$$

$$\zeta = d\theta \tag{2.3c}$$

yielding $\omega = 0$, $\mathcal{R} = \tau = 0$.

The action of the $\beta - \gamma - b - c$ system in the supergravity background (2.2) has the following form:

$$S^{\beta\gamma bc} = \int_{M_2} \mathcal{L}^{\beta\gamma bc} \tag{2.4}$$

$$\mathcal{L}^{\beta\gamma bc} = \left[-\lambda\beta d\gamma + (1-\lambda)\gamma d\beta - (\lambda + \frac{1}{2})bdc + (\lambda - \frac{1}{2})cdb\right]\wedge e^{+}$$

$$+ e^{\frac{i\pi}{4}}\sqrt{2}\zeta\wedge\left[\frac{1}{2}(\lambda - \frac{1}{2})cd\beta + \frac{1}{2}\lambda bdc - \gamma be^{+}\right] \quad (2.5)$$

$$+ \frac{1}{2}\zeta\wedge\zeta\left[(\lambda - \frac{1}{2})bc + \lambda\beta\gamma\right]$$

From its variation we obtain the rheonomic parametrizations

$$db = \partial_{+}be^{+} + \partial_{-}be^{-} + \frac{1}{2}\zeta\partial_{+}\beta \quad (2.6a)$$

$$dc = \partial_{+}ce^{+} + \partial_{-}ce^{-} + 2\zeta\gamma \quad (2.6b)$$

$$d\beta = \partial_{+}\beta e^{+} + \partial_{-}\beta e^{-} + 2\zeta b \quad (2.6c)$$

$$d\gamma = \partial_{+}\gamma e^{+} + \partial_{-}\gamma e^{-} + \frac{1}{2}\zeta\partial_{+}c \quad (2.6d)$$

and the equations of motion:

$$\partial_{-}b = \partial_{-}c = \partial_{-}\beta = \partial_{-}\gamma = 0 \quad (2.7)$$

Eqs. (2.6) imply that the supersymmetry transformation rules with respect to which the action (2.5) is "locally" invariant are:

$$\delta b = \frac{1}{2}\,\varepsilon\partial_{+}\beta \tag{2.8a}$$

$$\delta c = 2\varepsilon\gamma \tag{2.8b}$$

$$\delta\beta = 2\varepsilon b \tag{2.8c}$$

$$\delta\gamma = \frac{1}{2}\,\varepsilon\partial_{+}c \tag{2.8d}$$

$$\delta e^{+} = i\varepsilon\zeta \tag{2.8e}$$

$$\delta e^{-} = 0 \tag{2.8f}$$

$$\delta\zeta = d\varepsilon \tag{2.8g}$$

The stress-energy tensor T(z) and the "local" supercurrent G(z) generating the transformations (2.8) are easily obtained by varying the action (2.5). Following [19] we set

$$\delta S = -\frac{1}{2\pi}\int\left(T_{++}e^{+}\wedge\delta e^{+} + T_{--}e^{-}\wedge\delta e^{-} - \frac{1}{2\sqrt{2}}\,e^{\frac{i\pi}{4}}\;Ge^{+}\wedge e^{-}\right) \tag{2.9}$$

and in the special superconformal gauge we obtain:

$$T = -\,\lambda\beta\partial\gamma + (1-\lambda)\gamma\partial\beta - \left(\lambda + \frac{1}{2}\right)b\partial c + \left(\frac{1}{2} - \lambda\right)c\partial b \tag{2.10a}$$

$$G = -\left(\lambda - \frac{1}{2}\right)c\partial\beta - \lambda\beta\partial c - 2\gamma b \qquad (2.10b)$$

where we have denoted $\frac{\partial}{\partial z} = \partial$. (We shall use $\frac{\partial}{\partial \bar{z}} = \bar{\partial}$). In order to verify the above currents satisfy the OPE's of the N = 1 superconformal algebra:

$$T(z)T(\omega) = \frac{c}{2}\frac{1}{(z-\omega)^4} + \frac{2T(\omega)}{(z-\omega)^2} + \frac{\partial T(\omega)}{(z-\omega)} + \text{reg} \qquad (2.11a)$$

$$T(z)G(\omega) = \frac{3}{2}\frac{1}{(z-\omega)^2}\,G(\omega) + \frac{\partial G(\omega)}{(z-\omega)} + \text{reg} \qquad (2.11b)$$

$$G(z)G(\omega) = \frac{2}{3}\,c\,\frac{1}{(z-\omega)^3} + \frac{2T(\omega)}{(z-\omega)} + \text{reg} \qquad (2.11c)$$

with the value (2.1) of the central charge one has to utilize the fundamental OPEs:

$$c(z)b(\omega) = \frac{1}{(z-\omega)} + \text{reg} \qquad (2.12a)$$

$$b(z)c(\omega) = \frac{1}{(z-\omega)} + \text{reg} \qquad (2.12b)$$

$$\gamma(z)\beta(\omega) = \frac{1}{(z-\omega)} + \text{reg} \qquad (2.12c)$$

$$\beta(z)\gamma(\omega) = -\frac{1}{(z-\omega)} + \text{reg} \qquad (1.12d)$$

that follows, via canonical quantization with Dirac brackets (we have second class constraints) from the gauge fixed action (2.4):

$$S^{\beta\gamma bc}_{\text{gauge fixed}} = \int_{M_2} d\bar{z}dz\mathcal{L}_o^{\beta\gamma bc}(z,\bar{z}) \tag{2.13a}$$

$$\mathcal{L}_o^{\beta\gamma bc} = -\lambda\beta\bar{\partial}\gamma + (1-\gamma)\gamma\bar{\partial}\beta - (\lambda + \frac{1}{2})b\bar{\partial}c + (\lambda - \frac{1}{2})c\bar{\partial}b \tag{2.13b}$$

(see for instance [19] for a more detailed derivation). Th crucial point is that the action (2.13) is invariant under a larger (2,0) global supersymmetry. Naming ε^+ and ε^- the two constant supersymmetry parameters, under the transformations

$$\delta\beta = 2e^- b \quad , \quad \delta b = \frac{1}{2}\,\varepsilon^+\partial\beta \tag{2.14a}$$

$$\delta\gamma = \frac{1}{2}\,\varepsilon^+\partial c \quad , \quad \delta c = 2\varepsilon^-\gamma \tag{2.14b}$$

the variation of the Lagrangian is a total divergence:

$$\delta\mathcal{L}_o^{\beta\gamma bc} = \varepsilon^-(\bar{\partial}f^-_z + \partial f^-_{\bar{z}}) + \varepsilon^+(\bar{\partial}f^+_z + \partial f^+_{\bar{z}}) \tag{2.15}$$

where

$$f^-_z = b\gamma \tag{2.16a}$$

$$f^-_{\bar{z}} = 0 \tag{2.16b}$$

$$f^{+}_{z} = \frac{1}{2}\left[\lambda\beta\partial c + \left(\lambda - \frac{1}{2}\right)\partial\beta c\right] \tag{2.16c}$$

$$f^{+}_{\bar{z}} = -\left[\lambda\beta\bar{\partial}c + \left(\lambda - \frac{1}{2}\right)\bar{\partial}\beta c\right] \tag{2.16d}$$

Using Noether theorem we can calculate the conserved currents generating these supersymmetries. Let $\mathcal{L}(\phi_i, \partial\phi_i, \bar{\partial}\phi_i)$ be a 2D-Lagrangian for a collection of fields ϕ_i and let us assume that under a variation:

$$\delta\phi_i = \varepsilon_A T^A(\phi) \tag{2.17}$$

we have

$$\delta\mathcal{L} = \varepsilon_A\left(\bar{\partial}f^{A}_{z} + \partial f^{A}_{\bar{z}}\right) \tag{2.18}$$

The corresponding currents are given by the formula:

$$j^{A}_{z} = T^{A}_{i}(\phi)\,\frac{\partial\mathcal{L}}{\partial\left(\bar{\partial}\phi_i\right)} - f^{A}_{z} \tag{2.19a}$$

$$j^{A}_{\bar{z}} = T^{A}_{i}(\phi)\,\frac{\partial\mathcal{L}}{\partial\left(\partial\phi_i\right)} - f^{A}_{\bar{z}} \tag{2.19b}$$

and are conserved

$$\bar{\partial}j^{A}_{z} + \bar{\partial}j^{A}_{\bar{z}} = 0 \tag{2.20}$$

If one of the two components of j vanishes, the other is holomorphic (respectively antiholomorphic). Inserting eq.s (2.14) and (2.16) into (2.19) we obtain.

$$G^-_z = 2\gamma b \tag{2.21a}$$

$$G^-_{\bar z} = 0 \tag{2.21b}$$

$$G^+_z = -\left(\lambda - \frac{1}{2}\right)c\partial\beta - \lambda\beta\partial c \tag{2.21c}$$

$$G^+_{\bar z} = \lambda\beta\bar\partial c + \left(\lambda - \frac{1}{2}\right)\bar\partial\beta c \approx 0 \tag{2.21d}$$

where weakly zero (≈ 0) means zero upon use of the field equations. This is a very subtle point. Equations of motion have to be utilized in while verifying current conservation (eq. (2.20)); yet weakly zero quantities cannot be disregarded in the calculation of OPEs, the same way as weakly zero objects cannot be disregarded in the calculation of Poisson brackets. The holomorphic supercurrents

$$G^-(z) = G^-_z = 2\gamma b \tag{2.22a}$$

$$G^+(z) = G^+_z = -\left(\lambda - \frac{1}{2}\right)c\partial\beta - \lambda\beta\partial c \tag{2.22b}$$

together with the U(1) current

$$J(z) = -(1 - 2\lambda)bc + 2\lambda\beta\gamma \tag{2.23}$$

and the stress-energy tensor (2.10a) fulfill the OPEs of the $N = 2$ superalgebra:

$$T(z)T(\omega) = \frac{c}{2}\frac{1}{(z-\omega)^4} + \frac{2T(\omega)}{(z-\omega)^2} + \frac{\partial T(\omega)}{(z-\omega)} + \text{reg} \qquad (2.24a)$$

$$T(z)G^{\pm}(\omega) = \frac{3}{2}\frac{1}{(z-\omega)^2} G^{\pm}(\omega) + \frac{\partial G^{\pm}(\omega)}{(z-\omega)} + \text{reg} \qquad (2.24b)$$

$$T(z)J(\omega) = \frac{1}{(z-\omega)^2} J(\omega) + \frac{1}{(z-\omega)}\partial J(\omega) + \text{reg} \qquad (2.24c)$$

$$G^{+}(z)G^{-}(\omega) = \frac{1}{3} c \frac{1}{(z-\omega)^3} + \frac{J(\omega)}{(z-\omega)^2} + \frac{T(\omega) + \frac{1}{2}\partial J(\omega)}{(z-\omega)} + \text{reg} \qquad (2.24d)$$

$$J(z)G^{\pm}(\omega) = \pm\frac{G^{\pm}(\omega)}{(z-\omega)} + \text{reg} \qquad (2.24e)$$

$$J(z)J(\omega) = \frac{c}{3}\frac{1}{(z-\omega)^2} + \text{reg} \qquad (2.42f)$$

with the value (2.1) of the central charge. This shows that we have not only a global but also a local holomorphic N = 2 supersymmetry. Indeed the reader can verify that the action (2.13) is invariant against the more general transformation

$$\delta\beta = 2\varepsilon^{-}b \qquad (2.25a)$$

$$\delta\gamma = \frac{1}{2}\varepsilon^{+}\partial c - \left(\lambda - \frac{1}{2}\right)\partial\varepsilon^{+}c \qquad (2.25b)$$

$$\delta b = \frac{1}{2}\, \varepsilon^{+}\partial\beta + \lambda\partial\varepsilon^{+}\beta \tag{2.25c}$$

$$\delta c = 2\varepsilon^{-}\gamma \tag{2.25d}$$

where the parameters are arbitrary holomorphic functions:

$$\bar{\partial}\varepsilon^{\pm} = 0 \tag{2.26}$$

These transformations are retrieved from the structure (2.21) of the supercurrents utilizing the general formula

$$\begin{aligned}\delta\phi(\omega,\bar{\omega}) &= \oint_{\omega} \frac{dz}{2i\pi} \left[\varepsilon^{+}(z)G^{+}_{z}(z,\bar{z}) + \varepsilon^{-}(z)G^{-}_{z}(z,\bar{z})\right]\phi(\omega,\bar{\omega}) \\ &+ \oint_{\omega} \frac{d\bar{z}}{2i\pi} \left[\varepsilon^{+}(z)G^{+}_{\bar{z}}(z,\bar{z}) + \varepsilon^{-}(z)G^{-}_{\bar{z}}(z,\bar{z})\right]\phi(\omega,\bar{\omega})\end{aligned} \tag{2.27}$$

which holds true for any field $\phi(\omega,\bar{\omega})$. In this way we have shown that we can realize the minimal discrete series of the N = 2 algebra in terms of β-γ-b-c fields as a consequence of the enlarged supersymmetry of the action (2.13): as already stated, it suffices to impose unitary by setting $\lambda = \frac{1}{2k+4}$ ($k \in N$).

As a final remark note that the local supercurrent (2.10b) is just the sum of the two global ones (2.22a-b)

$$G = G^{+} + G^{-} \tag{2.28}$$

3 The (2.2) Lagrangian of a tensor product model and its moduli-dependent deformation

Consider now a tensor product model $(k_1, k_2, \ldots, k_N)$ corresponding to the weighted Fermat's curve (1.11). A world-sheet Lagrangian for this (2,2)-theory is given (in the superconformal gauge) by:

$$\mathcal{L}_o^{(2,2)}(z,\bar{z}) = \sum_{i=1}^{N} \frac{1}{2k_i+4} \Big[-\left(\beta_i \bar{\partial}\gamma_i + \tilde{\beta}_i \partial \tilde{\gamma}_i\right) + (2k_i + 3)\left(\gamma_i \bar{\partial}\beta_i + \tilde{\gamma}_i \partial\tilde{\beta}_i\right) \tag{3.1}$$

$$- (k_i + 3)\left(b_i \bar{\partial} c_i + \tilde{b}_i \partial \tilde{c}_i\right) - (k_i + 1)(c_i \bar{\partial} b_i + \tilde{c}_i \partial \tilde{b}_i)\Big]$$

In particular, for the proper Fermat's curve (1.9) we have:

$$\mathcal{L}_{F_n}^{(2,2)} = \frac{1}{2n+4} \sum_{i=1}^{n+2} \Big[-\left(\beta_i \bar{\partial}\gamma_i + \tilde{\beta}_i \partial \tilde{\gamma}_i\right) + (2n + 3)\left(\gamma_i \bar{\partial}\beta_i + \tilde{\gamma}_i \partial\tilde{\beta}_i\right) \tag{3.2}$$

$$-(n + 3)\left(b_i \bar{\partial} c_i + \tilde{b}_i \partial \tilde{c}_i\right) - (n + 1)\left(c_i \bar{\partial} b_i + \tilde{c}_i \partial \tilde{b}_i\right)\Big]$$

The Lagrangian (3.2) is invariant against the 2n + 4 SUSY transformations (2.25) of the β_i-γ_i-b_i-c_i variables and the analogous ones by the tilded fields.

Consider now an arbitrary homogeneous polynomial of degree n + 2 in the CP_{n+1}:

$$\Pi^{(n+2)}(Z) = c_{i_1,\ldots,i_{n+2}} Z^{i_1} \ldots Z^{i_{n+2}} \tag{3.3}$$

where $c_{i_1 \ldots i_{n+2}}$ is a symmetric tensor in (n + 2)-dimensions. The polynomial Π can always be rewritten (up to U(n + 2) rotation) as:

$$\Pi^{(n+2)}(Z) = F^{(n+2)}(Z) + M^{\alpha}P_{\alpha}^{n+3}(Z) \tag{3.4}$$

where $F_{(n+2)}(Z) = Z_1^{n+2} + Z_2^{n+2} + \ldots Z_{n+2}^{n+2}$ is the Fermat's polynomial and where $P_a^{n+2}(Z)$ is a basis of monomial for the non-trivial polynomial deformations, whose number

$$m(n) = \binom{2n+3}{n+1} - (n+2)^2 \tag{3.5}$$

is the number of algebraic moduli. In the first few cases we have $m(1) = 1$, $m(2) = 19$, $m(3) = 101$ and it is fairly easy to write the basis P_{α}^{n+2}. For instance in the well studied case of the quintic hypersurface ($n = 3$) the 101 P_{α}^{5} are given by [5]:

$$P_{\alpha}^{(5)}(Z) = \begin{cases} Z_i^2 Z_j^3 & i \neq j \quad 20 \\ Z_i Z_j Z_k^3 & i \neq j \neq k \quad 30 \\ Z_i Z_j^2 Z_k^2 & i \neq j \neq k \quad 30 \\ Z_i Z_j Z_k Z_\ell^2 & i \neq j \neq k \neq l \quad 20 \\ Z_1 Z_2 Z_3 Z_4 Z_5 & \quad 1 \end{cases} \tag{3.6}$$

$$\mathcal{L}_{(n+2)}^{(2,2)}(z,\bar{z},M) = \mathcal{L}_{F_n}^{(2,2)}(z,\bar{z}) + \sum_{\alpha=1}^{m(n)} M^{\alpha} b_i \tilde{b}_j P_i^{\alpha} \tilde{P}_j^{\alpha} \tag{3.9}$$

If we perform the substitution (1.12) we obtain that each of the $m(n)$ operators $\Psi_{\alpha}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} \\ 1 & 1\end{pmatrix}(z,\bar{z}) = P_{\alpha}^{n+2}(\beta_i)\tilde{P}_{\alpha}^{n+2}(\tilde{\beta}_i)$ is a chiral-chiral primary field with $F^{n+2}(Z)$ with the correct weights and charges. Inserting these Ψ_{α} operators and the explicit for (2.21) of the supercurrents into eq. (1.2) (the right-moving supercurrent are identical) we obtain the expression for the deformations $\Phi_\alpha\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}(z,\bar{z})$ of the Lagrangian (2.13)

$$\Phi_\alpha\begin{pmatrix}1 & , & 1\\ 0 & , & 0\end{pmatrix}(z,\bar{z}) = b_i\tilde{b}_j P^\alpha_i(\beta)\tilde{P}^\alpha_j(\tilde{\beta}) \tag{3.7}$$

where we have introduced the notation

$$P^\alpha_{i_1\ldots i_m}(\beta) = \frac{\partial^m}{\partial\beta_{i_1}\ldots\partial\beta_{i_m}} P^\alpha(\beta) \tag{3.8}$$

and similarly for the tilded objects. In eq. (3.7) repeated indices are summed on, as usual. We conclude that for a polynomial Π_{n+2} infinitesimally close to $F_{n+2}(M_\alpha << 1)$ the corresponding $\mathcal{L}^{(2,2)}_{(n+2)}(z,\bar{z},M)$ lagrangian is given by:

$$\mathcal{L}^{(2,2)}_{(n+2)}(z,\bar{z},M) = \mathcal{L}^{(2,2)}_{F_n}(z,\bar{z}) + \sum_{\alpha=1}^{m(n)} M^\alpha b_i\tilde{b}_j P^\alpha_i\tilde{P}^\alpha_j \tag{3.9}$$

where $\mathcal{L}^{(2,2)}_{(Fn)}$ is defined by eq. (2.20).

We shall now prove that (3.9) describes the superconformal (2.2)-theories associated with the polynomial Π_{n+2} rewritten in the form (3.4).

In order to prove this result we have utilized the time honoured Noether coupling method, taking the moduli M^α as expansion parameter. Indeed the reader can verify that the Lagrangian (3.9) is invariant (up to a total divergences) against the following holomorphic supersymmetry transformations ($\bar{\partial}\varepsilon^\pm = 0$):

$$\delta\beta_i = 2\varepsilon^- b_i \tag{3.10a}$$

$$\delta b_i = \frac{1}{2}\,\varepsilon^+\partial\beta_i + \frac{1}{2n+4}\,\partial\varepsilon^+\beta_i \tag{3.10b}$$

$$\delta\gamma_i = \frac{1}{2}\,\varepsilon^+\partial c_i + \frac{n+1}{2n+4}\,\partial\varepsilon^+\beta_i \tag{3.10c}$$

$$\delta c_i = 2\varepsilon^-\gamma^- \tag{3.10d}$$

$$\delta\tilde{\beta}_i = 0 \tag{3.10e}$$

$$\delta\tilde{b}_i = 0 \tag{3.10f}$$

$$\delta\tilde{c}_i = \frac{1}{2}\,\varepsilon^+\sum_{\alpha=1}^{m(n)} M^\alpha P^\alpha \tilde{P}^\alpha_i \tag{3.10g}$$

$$\delta\tilde{\gamma}_i = \frac{1}{2}\,\varepsilon^+\sum_{\alpha=1}^{m(n)} M^\alpha P^\alpha \tilde{P}^\alpha_{ij} b_j \tag{3.10h}$$

The action (3.9) is also invariant under the anti-holomorphic transformations $(\partial\bar{\varepsilon}^\pm = 0)$ one obtain from eq.s (3.10) exchanging tilded with untilded fields $(\phi \leftrightarrow \phi)$ and holomorphic with antiholomorphic derivatives $(\partial \leftrightarrow \tilde{\partial})$. In particular when the parameters $\varepsilon^\pm$ are constant the total divergence into which varies the interacting Lagrangian (3.9) is the sum of n + 2 copies of the total divergence (2.19) into which varies the free action. Explicitly we have:

$$\delta\mathcal{L}^{(2,2)}_{(n+2)}(z,\bar{z},M) = \varepsilon^{+}\left(\bar{\partial}f^{+}_{z} + \partial f^{+}_{\bar{z}}\right) + \varepsilon^{-}\left(\partial f^{-}_{z} + \partial f^{-}_{\bar{z}}\right) \qquad (3.11)$$

$$f^{+}_{z} = \frac{1}{2}\left(\frac{1}{2n+4}\,\beta_i\partial c_i - \frac{n+1}{2n+4}\,\partial\beta_i c_i\right) \qquad (3.12a)$$

$$f^{+}_{\bar{z}} = -\left(\frac{1}{2n+4}\,\beta_i\bar{\partial}c_i - \frac{n+1}{2n+4}\,\bar{\partial}\beta_i c_i\right) \qquad (3.12b)$$

$$f^{-}_{z} = b_i\gamma_i \qquad (3.21c)$$

$$f^{-}_{\bar{z}} = 0 \qquad (3.12d)$$

Inserting the result and eqs. (3.10) into the general formulae (2.19) we obtain the Noether supercurrents:

$$G^{-}_{z} = 2\gamma_i b_i \qquad (3.13a)$$

$$G^{-}_{\bar{z}} = 0 \qquad (3.13b)$$

$$G^{+}_{z} = \frac{n+1}{2n+4}\,c_i\partial\beta_i - \frac{1}{2n+4}\,\beta_i\partial c_i \qquad (3.13c)$$

$$G^{+}_{\bar{z}} = \frac{1}{2n+4}\,\beta_i\bar{\partial}c_i - \frac{n+1}{2n+4}\,\bar{\partial}\beta_i c_i - \frac{1}{2}\sum_{\alpha=1}^{m(n)} M_\alpha P^{\alpha}\tilde{P}^{\alpha}_{j}\tilde{b}_j \approx 0 \qquad (3.13d)$$

The component $G^{+}_{\bar{z}}$ is weakly zero as a consequence of the field equations:

$$\bar{\partial}\beta_i = \bar{\partial}b_i = 0 \tag{3.14a}$$

$$\partial\tilde{\beta}_i = \partial\tilde{b}_i = 0 \tag{3.14b}$$

$$\bar{\partial}c_i = \sum_{\alpha=1}^{m(n)} M^\alpha P^\alpha_i \tilde{b}_j \tilde{P}^\alpha_j \tag{3.14c}$$

$$\bar{\partial}\gamma_i = \sum_{\alpha=1}^{m(n)} M^\alpha b_j P^\alpha_{ij} \tilde{b}_\ell \tilde{P}^\alpha_\ell \tag{3.14d}$$

$$\partial\tilde{c}_i = \sum_{\alpha=1}^{m(n)} M^\alpha \tilde{P}^\alpha_i b_j P^\alpha_j \tag{3.14e}$$

$$\partial\tilde{\gamma}_i = \sum_{\alpha=1}^{m(n)} M^\alpha \tilde{b}_j \tilde{P}^\alpha_{ij} b_l P^\alpha_l \tag{3.14f}$$

In the proof of this fact one has to remember that P^α is a homogeneous polynomial so that

$$\beta^i P^\alpha_{ij_1\ldots j_m} = (n - m + 2)P^\alpha_{j_1\ldots j_m} \tag{3.15}$$

By the same token we can also verify that G^+_z is holomorphic ($\bar{\partial}G^+_z = 0$). As we see, the $\{\beta_i, b_i\}$-fields remain holomorphic (respectively antiholomorphic) also in presence of interaction. On the other hand the $\{\gamma_i, c_i\}$-fields have no longer a definite holomorphic character. Notwithstanding this fact, canonical quantization leads to the same fundamental OPEs as in the free case (2.12), that is we find

$$\beta_i(z)\gamma_j(\omega,\bar{\omega}) = -\frac{\delta_{ij}}{z-\omega} + \text{reg} \tag{3.16}$$

and similarly for the other cases. Relying on these OPEs, we can retrieve the transformations (3.10) by inserting the Noether currents (3.13) into the general formula (2.27). The holomorphic supercurrents

$$G^+(z) = \frac{n+1}{2n+4}\, c_i\partial\beta_i - \frac{1}{2n+4}\,\beta_i\partial c_i = G^+_z \tag{3.17a}$$

$$G^-(z) = 2\gamma_i b_i = G^-_z \tag{3.17b}$$

close the OPEs of the N = 2 superconformal algebra (eqs. (2.24)) together with the U(1) current

$$J(z) = -\frac{n+1}{n+2}\, b_i c_i + \frac{1}{n+2}\,\beta_i\gamma_i \tag{3.18}$$

and with the holomorphic part of the stress-energy tensor:

$$T(z) = T_{zz} = -\frac{1}{2n+4}\,\beta_i\partial\gamma_i + \frac{2n+3}{2n+4}\,\gamma_i\partial\beta_i - \frac{n+3}{2n+4}\, b_i\partial c_i - \frac{n+1}{2n+2}\, c_i\partial b_i \tag{3.19}$$

the central charge is c = 3n, as expected. The mixed component of this latter

$$\begin{aligned} T_{z\bar{z}} = &-\frac{1}{2n+4}\,\beta_i\bar{\partial}\gamma_i + \frac{2n+3}{2n+4}\,\gamma_i\bar{\partial}\beta_i \\ &-\frac{n+3}{2n+4}\, b_i\bar{\partial}c_i - \frac{n+1}{2n+4}\, c_i\bar{\partial}b_i + \sum_{\alpha=1}^{m(n)} M^{\alpha} b_i\tilde{b}_j P^{\alpha}_i\tilde{P}^{\alpha}_j \approx 0 \end{aligned} \tag{3.20}$$

is weakly zero upon use of the field equations (3.14).

4 Conclusions

We have exhibited a (2.2)-theory that generalizes the Gepner tensor product construction to an arbitrary point in the moduli space of complex structure deformations.

Hopefully our moduli-dependent realisation of the (2,2) superconformal algebra will provide new tokens for the evaluation of th Zamolodchikov metric. In this respect it should be noted that the moduli are hidden in the mode expansion of the β_i-γ_i-b_i-c_i fields. Indeed from the field equations (3.14) we learn that β_i-b_i, ($\tilde{\beta}_i$-$\tilde{b}_i$) are holomorphic (respectively antiholomorphic) and admit the same mode expansion as in the free case. On the other hand, the general solution of the field equations for c_i, γ_i is given by

$$\gamma_i(z) = \mathring{\gamma}_i(z) - \oint_{\bar{z}} \frac{d\bar{\omega}}{2i\pi} \ \ell n(\bar{z} - \bar{\omega}) \sum_{\alpha=1}^{m(n)} M^{\alpha}\tilde{\bar{b}}^{\alpha}_j(\bar{\omega})\tilde{P}^{\alpha}_{ij}(\bar{\omega})b_i(z)P^{\alpha}_i(z) \qquad (4.1a)$$

$$c_i(z) = \mathring{c}_i(z) - \oint_{\bar{z}} \frac{d\bar{\omega}}{2i\pi} \ell n(\bar{z} - \bar{\omega}) \sum_{\alpha=1}^{m(n)} M^{\alpha}\tilde{\bar{P}}^{\alpha}_i(\bar{\omega})b_j(z)\tilde{P}^{\alpha}_j(z) \qquad (4.1b)$$

where $\mathring{\gamma}_i(z)$ and $\mathring{c}_i(z)$ admit the same mode expansion as the corresponding free fields. The modes $\{\mathring{\gamma}^n_i, \mathring{c}^n_i\}$ are those having the standard commutation relations with the modes $\{\beta^n_i, b^n_i\}$. Using eqs. (4.1) in eqs. (3.17) we obtain an explicitly moduli dependent representation of the (2,2) algebra involving only the free oscillators β^n_i, γ^n_i, b^n_i, c^n_i and their tilded analogous.

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