ISSN 0029-3865

CBPF-NF-002/87

A MODIFIED THEORY OF GRAVITY

by

M. Novello and N.P. Neto*

Centro Brasileiro de Pesquisas Físicas - CNPq/CBPF
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Universidade Federal do Rio de Janeiro - UFRJ
Instituto de Física
Cidade Universitária
21.910 - Ilha do Fundão - Rio de Janeiro, RJ - Brasil

ABSTRACT

We present a theory of gravity wich considers the topological invariant $I = R^{*}_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ as one of the basic quantities to be present in the description of the dynamics of gravitational interactions.

A cosmical scenario induced by this theory is sketched.

Key-words: Gravity; Topological invariant; Cosmology.

The development of cosmology in the last years has provoked the appearance of strong arguments in favor of some ancient suspictions about the behaviour of the Universe at large. Within this scheme we have been conducted to accept, in a broad sense, the truth of the two following statements:

(i) We live in a world represented by a (riemannian) metric which describes a non-stationary spatially homogeneous and isotropic (Friedmann-like) Universe of very slow expansion;

(ii) The Universe has not always been in a conformally flat configuration.

Although (i) does not require any further comment and seems to consist in a well stablished thruth accepted by the great majority of cosmologists, the assertion (ii) needs some explanation concerning its actual meaning.

Recently[1], many scientist have been very critical with respect to the so called standard cosmology [identified with the hot Big-Bang solution found by Friedmann and developped by many others] mainly due to the well-known difficulties which are inevitably present in this model. Among these we can quote, for instance, the question of the initial singularity (problem 1) and the explanation of the origin of the high degree of isotropy which is present in the 2.7^{o}K background radiation (problem 2) associated to the presence, in such geometry, of particle horizons.

Although it is, in principle, possible to find solutions to these difficulties without abandoning the condition of a conformally flat metric, some scientists have proposed the examination of models in which the Weyl tensor $W_{\alpha\beta\mu\nu}$ is non-

-null at some prior era. One of the main works which gave a real contribution to the clarification of this question and produced a severe critic to the standard model in the neighbourhood of the assumed singularity was undertaken by the russian cosmologists Lifshitz, Belinsky and Khalatnikov[2]. These scientists have shown (at first, in a limited scheme in which all matter in the world is identified to a perfect fluid in equilibrium) that a deep analysis of the coupled behaviour of geometry and matter in the very dense stage of the Universe yields the result that the Weyl tensor $W_{\alpha\beta\mu\nu}$ should vanish only in a later stage of the cosmic evolution, characterizing then the moment in which the Universe enters its present friedmannian era. More than this, from the analysis of Lifshitz et al., one concludes that Weyl curvature effects are, at these "early" times, much more important than the corresponding Ricci terms.

A similar result is obtained when the basic properties of the standard cosmological model are carefuly examined, at those regions of very high curvature, using a more general model for the matter content of the Universe. It has been shown that indeed, at those extreme regions ordinary matter has an unimportant role on the evolution of the geometry. This property has been used by Starobinsky[3] and others to create a model of an assymptotic de Sitter regime in which a cosmological constant Λ becomes the main agent for the isotropization of space-time. Such "vacum domain" made its appearance also in some recent models of cosmology — e.g., the so-called inflationary scenario. However, what seems to restricts strongly all these models is the difficulty of obtaining a mechanism by which the de Sitter era should be replaced by a friedmannian one.

In another context, quantum fluctuations and the properties of non-equilibrium thermodynamics in an intense gravitational field, with its intrinsic generation of entropy, has led some authors (Penrose[4], etc.) although in a more speculative way, to put in relief the role of Weyl conformal tensor in the evolution of the cosmological metric.

These considerations have provoked the study of a series of alternatives to the global behaviour of the Universe. Within the scope of General Relativity, however, the so called singularity theorems have induced a general belief that a true singularity should occur, inevitably, in the space-time. As a reaction to this position, some scientists started a program in order to propose alternative schemes which should modify drastically the behaviour of the Universe at those regions of very intense curvature and even eliminate the spectre of the singularity — in any case, giving arguments to sustain the truth of sentence (ii).

The main approaches which have been followed by the different programs are:

(a) Modifications of Einstein's equations for gravity;

(b) Treatment of more sophisticated forms of representing matter (even dealing with non-equilibrium configurations);

(c) Examination of distinct, alternative types of interactions of matter and radiation with gravity (e.g., taking into account forms of non-minimal coupling of matter with gravity).

Suggestions (b) and (c) have been examined elsewhere[5]. We will concentrate our interest here on a specific model for case (a).

In the search of alternatives to the equations of gravity one must keep in mind those particular properties of the previous dynamics which have already overcomed the test of observation.

It is fairly true that among the many distinct geometries which have been displayed as exact solutions of Einstein's equations there are two (and only two) which have been either directly or indirectly observed, whose consequences have been already testued by experience. They are the solution generated by a compact static matter configuration yielding the so called Schwarzschild metric; and the solution of Friedmann for an homogeneous and isotropic Universe. We are certainly not exaggerating if we claim that any theory of gravity which exhibit those geometries as solutions of its equations of motion, can be thought of as a serious candidate for the description of gravitational phenomena. Unfortunately, this still leaves open a large assembly of alternatives. It is an interesting coincidence that for both the principal geometries (Schwarzschild and Friedmann) the topological invariant $I = R_{\alpha\beta\mu\nu}\overset{*}{R}^{\alpha\beta\mu\nu}$ vanishes. This seems to provide an orientation to guide our choice of the alternative equation of gravity: it should be constructed as a functional of I. Such idea is supported by the fact that at the quantum level topological invariants seem to perform an important role in the regularization of quantum processes.

However, this recognition does not solves our problem, since we still have an infinite set of possibilities for the dynamics. At this point we shall turn our attention to an old proposal of M. Born[6,7], developed later by him and his collaborator L. Infeld, of an extension of Maxwell's Electrodynamics, in order to guide us in the search of our present aimes.

In the early thirties these authors proposed to develop a (non-linear) electrodynamics based on the hypothesis that the basic fields of physics must be described by a dynamics comprising within itself some sort of limitation on the possible extreme values of the strenghths of the fields. In a subsequent stage, quantum physics has shown that non-linear effects of electrodynamics do appear in Nature and may be described by successive approximations which are in accordance with the expansion in series of the polynomial Lagrangian of Born-Infeld.

The above considerations led us to propose the following Lagrangian to describe gravity:

$$(1) \qquad L = \sqrt{-g}\left[\frac{1}{k}\left(R + \beta\sqrt{1-\left(\frac{I}{\beta^2}\right)^2} - \beta\right) + L_M\right]$$

in which constant β measures the maximum intensity admissible for the value of the topological invariant I. Let us make some comments on L . First we note that the choice of the radical term to be quadratic on I is, of course, not unique. However, had we chosen a linear term, e.g. $\sqrt{1-I}$, expanding this expression for small values of I the first contribution of I is the quadratic one. This is due precisely to the fact that I is a topological invariant and it contributes only with a surface integral to the action principle.

The term $\sqrt{-g}\ \beta$ is introduced in order to eliminate the contribution of the new part of the Lagrangian to the cosmological constant, when $I = 0$.

The equations of motion then are giving by:

$$R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu} - \frac{1}{2} \frac{\beta^3}{\sqrt{\beta^4 - I^2}} g_{\mu\nu} \mp \frac{4}{\beta}\left[\left(\frac{I}{\sqrt{\beta^4-I^2}}\right)_{,\lambda} R^{\beta}{}_{\mu\nu}{}^{*\lambda} \right]_{;\beta} = -KT_{\mu\nu} - \frac{1}{2}\beta g_{\mu\nu} \quad , \tag{2}$$

which reduces to Einstein's theory for $\frac{I}{\beta^2} << 1$. Using Bianchi identities we can show that eq. (2) guarantees the conservation of energy-momentum tensor.

COSMICAL SCENARIO

There are many well-known solutions of Einstein's theory which are solutions of our new set of equations. Among these we can quote the solutions found by Schwarzchild, Friedmann, Kasner, Gödel, Reisner-Nordstron. Thus, one should ask what the assumption of our new L is good for ? From a pratical point of view, we can answer this question by examining cosmology: we can perform a new cosmological scenario which could solve difficulties not only of the ancient standard program, but also of some new models like e.g. inflation.

Our new scenario is based on the assumption that during its history the Universe has experienced the whole spectra of permissible values for the topological invariant I. In order to fix our ideas, let us concentrate in a specific configuration and assume that the evolution of I can be represented as in Figure 1.

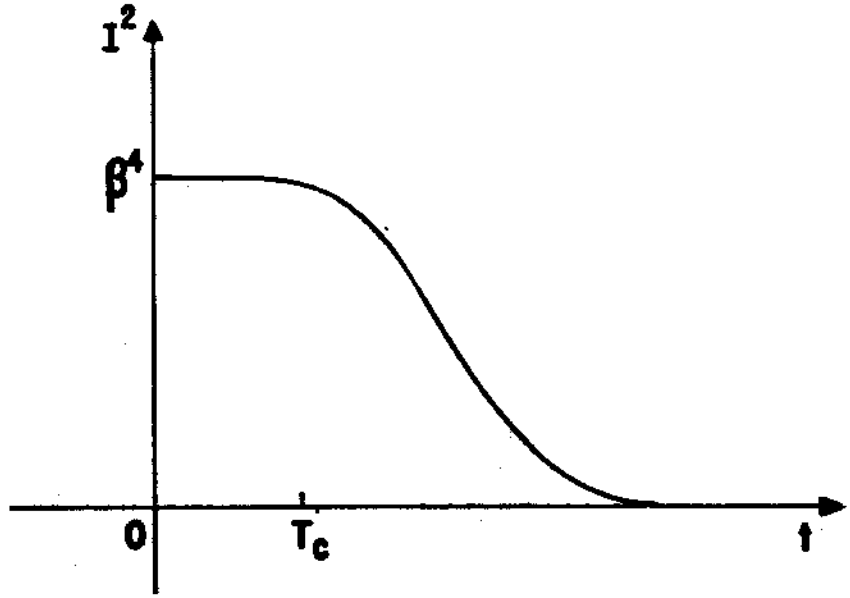

Fig. 1 - Representation of the evolution of the topological invariant $I = R^*_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ during the cosmic history for an expanding Universe.

In this model, during a period $\Delta\tau = \tau_0$ the Universe experienced a value ${I_0}^2$ very near its admissible maximum ${I_0}^2 \approx \beta^4$. At this stage the net effect of the I-term in L is to induce a very large cosmological constant (remark that $\lim_{I\to\beta^2} \Lambda_{eff} = \lim_{I\to\beta^2} \frac{\beta^3}{\sqrt{\beta^4 - I^2}} \approx \infty$) which could modify drastically the primordial behaviour of the Cosmos, even avoiding the singularity. The Universe at this regime $[I^2 = cte \approx \beta^4]$ is in a de Sitter like state that is, a region in which the main responsable for the curvature of space-time is the effective cosmological constant. After τ_0 (the time in which I leaves its constant value and starts to diminish) the Universe leaves smoothly this phase, pass through a region in which matter becomes more and more important and finally enters the actual Friedmann era in which I vanishes and, besides, geometry becomes conformally flat.

This simplified scenario, which is in principle allowed by our set of equations, exhibits some features very similar to other sophisticated mechanisms which have been examined in the last years to produce some alternative to the standard cosmological scenario (e.g. cosmic spontaneous breakdown of symmetry of a given scalar field, phase transitions, etc). Furthermore, the equations of motion generated by our Lagrangian L reproduces all observable effects of classical gravity.

REFERENCES

[1] M. Novello - Cosmos et Contexte (ed. Masson, Paris, 1987).

[2] E.M. Lifshitz - I.M. Khalatnikov - in II Escola de Cosmologia e Gravitação CBPF, Brasil, (1982) ed. M. Novello.

[3] A.A. Starobinskii - JETP Letters, 37 (66)1983.

[4] R. Penrose - Singularities and time-asymmetry (Cambridge University Press, ed. S. Hacoking and W. Israel, 1978).

[5] M. Novello - L.A.R. Oliveira - To appear in Journal of Modern Physics (1987).

[6] M. Born - L. Infeld - Nature 132, 970(1933).
- Proc. Roy. Soc. A150, 141(1935).

[7] J. Plebanski - Lectures on Non Linear Electrodynamics (ed. Nordita, 1968).