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One-loop divergences of quantum gravity using conformal parametrization

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Abstract. *We calculate the one-loop divergences for quantum gravity with cosmological constant, using new parametrization of quantum metric. The conformal factor of the metric is treated as an independent variable. As a result the theory possesses an additional degeneracy and one needs an extra conformal gauge fixing. We verify the on shell independence of the divergences from the parameter of the conformal gauge fixing, and find a special conformal gauge in which the divergences coincide with the ones obtained by t'Hooft and Veltman (1974). Using conformal invariance of the counterterms one can restore the divergences for the conformal metric-scalar gravity.*

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1 Introduction

The renormalization of quantum gravity and in particular the calculation of the one-loop divergences for quantum General Relativity is considered as a problem of special interest. The non-renormalizability of quantum gravity has been established after the pioneer one-loop calculation by t'Hooft and Veltman [1] and Deser and van Nieuwenhuizen [2], who derived the divergences for pure gravity and also for the gravity coupled to scalar, vector and spinor fields. In both [1] and [2] the background field method has been used such that the splitting of the metric was performed according to $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$.

Later, the derivations of the one-loop divergences have been carried out many times, using different parametrizations of quantum metric and non-minimal gauge fixing conditions. The calculations were also done for gravity coupled to various kinds of matter fields. One can mention: the first calculation for the pure gravity in a non-minimal gauge [3]; the calculations using plane Feynman diagrams with various parametrizations of the quantum metric [4]; in [5] the result identical to the one of [1] has been achieved using local momentum representation technique; the calculation for gravity coupled to Majorana spinor using the (slightly modified) Schwinger-DeWitt technique [6]; the calculations in the first order formalism (with affine connection independent on the metric) using plane Feynman diagrams [7] and background field method and Schwinger-DeWitt technique [8]. Ref. [8] contains also the one-loop result for the $g^{\mu\nu} \to g^{\mu\nu} + h^{\mu\nu}$ parametrization, different from the one of [1]. The generalized Schwinger-DeWitt technique has been applied in [9] to confirm the gauge fixing dependence found in [3]. The calculation for gravity with cosmological constant has been done in [10] and for the Einstein-Cartan theory with external spinor current in [11]. Recently, the one-loop calculations for the pure gravity has been performed in [12] where the parametrizations like $g_{\mu\nu} \to g_{\mu\nu} + (-g)^r\, h_{\mu\nu}$ have been applied. The parametrizations of [12] are more general than the ones used in both [1] and [8], so that [12] reproduces both results in the limiting cases.

The interest to the gauge fixing dependence of the in quantum gravity has been revealed in the last years, when some more complicated linear gauges have been studied [13] (one can consult this paper for the list of references concerning the problem of gauge dependence in quantum field theory and quantum gravity). The purpose of the present letter is to report about the calculation of the one-loop divergences in quantum gravity, in some new parametrization which is different from those which have been used before. This parametrization is based on the separation of the conformal factor from the metric and is related to the well known conformal structure of gravity (see, for example, [14, 15]). In part, our parametrization resembles the one which has been applied in [16] for the derivation of the divergences in $2+\epsilon$ space-time dimensions. As usual, there is the possibility to conduct an efficient auto-verification of the result, using the on shell gauge fixing independence. One has to notice that the study of conformal gauge in four dimensions has some special importance, since its use permits partial verification of the gauge fixing procedure $h^\mu_\mu = 0$, which is usually applied in conformal quantum gravity [21, 22]. It is worth to notice that the divergences for the Weyl gravity calculated in [21] and [22] differ unlike one uses the so-called conformal regularization introduced in [21]. The result of our calculation, which is intended to check the applicability of the conformal gauge $h^\mu_\mu = 0$, can be relevant in the general context of conformal quantum gravity theories in four dimensions.

The present letter is organized as follows. In the next section we present the details of the one-loop calculations. The analysis of the results, including the on-shell gauge fixing independence is performed in section 3, and in the last section we draw our conclusions.

2 One-loop calculation in a conformal gauge

Our starting point is the gravity action with the cosmological constant

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,, \tag{1}$$

In order to illustrate how the degeneracy related to the conformal symmetry appears, let us briefly repeat the consideration of [14, 15].

Performing conformal transformation $g_{\mu\nu} \to \hat{g}_{\mu\nu} = g_{\mu\nu}\cdot e^{2\sigma(x)}$, one meets relations between geometric quantities of the original and transformed metrics:

$$\sqrt{-\hat{g}} = \sqrt{-g}\;e^{4\sigma}\,, \qquad \hat{R} = e^{-2\sigma}\left[R - 6\Box\sigma - 6(\nabla\sigma)^2\right]. \tag{2}$$

Substituting (2) into (1), after integration by parts, we arrive at:

$$S = \int d^4x\sqrt{-g}\,\left\{\frac{6}{\kappa^2}\,e^{2\sigma}\,(\nabla\sigma)^2 + \frac{1}{\kappa^2}\,e^{2\sigma}\,R + \frac{2}{\kappa^2}\Lambda e^{4\sigma}\right\}, \tag{3}$$

where $(\nabla\sigma)^2 = g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma$. If one denotes

$$\varphi = \sqrt{12/\kappa^2}\,\cdot\,e^{\sigma}\,, \tag{4}$$

the action (1) becomes

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,(\nabla\varphi)^2 + \frac{1}{12}\,R\,\varphi^2 + \frac{\kappa^2}{72}\,\Lambda\,\varphi^4\right\}, \tag{5}$$

that is the action of conformal metric-scalar theory. This theory is conformally equivalent to General Relativity with cosmological constant. Contrary to General Relativity, the theory (5) possesses extra local conformal symmetry, for it is invariant under the transformation

$$g'_{\mu\nu} = g_{\mu\nu} \cdot e^{2\rho(x)}\,, \qquad \varphi' = \varphi \cdot e^{-\rho(x)}\,. \tag{6}$$

This symmetry compensates an extra (with respect to (1)) scalar degree of freedom.

Let us now discuss the relation between two theories on quantum level. In case of renormalizable field theory the difference between two conformally equivalent theories appears on quantum level because of conformal anomaly. For quantum gravity one can not go so far because both theories are non-renormalizable and therefore anomaly is ambiguous [4]. At the same time, we can investigate the difference in quantization of two theories and the resulting difference in divergences. One has to notice that, despite the derivation of divergences in the theory (5) is possible using the techniques developed in [9] and [17], such a calculation would be quite difficult. Technically it is much more cumbersome than similar derivation for the non-minimal, non-conformal metric-scalar theory [18, 19]. In this paper we do not try to perform this calculation directly, but instead consider the derivation of the one-loop divergences in the theory (1) using special conformal parametrization.

Since the theory (1) is diffeomorphism invariant, it should be quantized as a gauge theory. On the other hand, the theory (5) has an extra conformal symmetry, and thus its quantization requires an extra gauge fixing which is called to remove corresponding degeneracy. As we shall see later, this is also true for the quantization of (1) in conformal variables.

In the framework of the background field method, let us consider the following shift of the metric

$$g_{\mu\nu} \to g'_{\mu\nu} = e^{2\sigma}\left[g_{\mu\nu} + h_{\mu\nu}\right], \tag{7}$$

where $h_{\mu\nu}$ and σ are quantum fields and $g_{\mu\nu}$ is the background metric. All raising and lowering of indices is done through $g_{\mu\nu}$. The parametrization (7) resembles the conformal transformation which led to the conformal form of the action (3). Then one can expect to meet an additional degeneracy for the quantum field, related to the conformal symmetry.

For the one-loop divergences, one needs only the bilinear, in the quantum fields $h^{\mu\nu}$ and σ,

[4] For instance, the divergences of (1) vanish for the special gauge fixing, and then anomaly vanishes.

part of the action. This part can be presented in the symbolic form:

$$S^{(2)} = \int d^4x \sqrt{-g} \begin{pmatrix} h^{\mu\nu} & \sigma \end{pmatrix} \hat{H} \begin{pmatrix} h^{\alpha\beta} \\ \sigma \end{pmatrix}. \tag{8}$$

Now, one has to introduce the gauge fixing for the diffeomorphism. We choose the gauge fixing term in the form

$$S_{GF} = -\frac{1}{\alpha} \int d^4x \sqrt{-g}\, \chi_\mu \chi^\mu \tag{9}$$

with

$$\chi_\mu = \nabla_\alpha h^\alpha{}_\mu + \beta\, \nabla_\mu h - \gamma\, \nabla_\mu \sigma, \tag{10}$$

where $h = h^\mu{}_\mu$ and $\alpha,\ \beta, \gamma$ are gauge fixing parameters. It is useful to choose them in such a way that the bilinear form becomes minimal second order operator.

One can find that this can be achieved by taking $\alpha = 2$, $\beta = -1/2$ and $\gamma = 2$. Then the bilinear form of the action with the gauge fixing term becomes

$$S^{(2)} + S^{(2)}_{GF} = \int d^4x \sqrt{-g} \left\{ h^{\mu\nu} \left[K_{\mu\nu\,,\,\alpha\beta}(\Box - 2\Lambda) + M_{\mu\nu\,,\,\alpha\beta} \right] h^{\alpha\beta} + \right.$$

$$\left. + \sigma\, (-4\Box + 2R + 16\Lambda\,)\, \sigma + h^{\mu\nu}\, (-g_{\mu\nu}\Box - 2R_{\mu\nu} + g_{\mu\nu}R + 4\Lambda g_{\mu\nu}\,)\, \sigma \right\} , \tag{11}$$

where

$$K_{\mu\nu\,,\,\alpha\beta} = \frac{1}{4} \left(\delta_{\mu\nu\,,\,\alpha\beta} - \frac{1}{2} g_{\mu\nu} g_{\alpha\beta} \right) \tag{12}$$

and

$$M_{\mu\nu\,,\,\alpha\beta} = -\frac{1}{4}\, \delta_{\mu\nu\,,\,\alpha\beta} R + \frac{1}{8}\, (g_{\nu\alpha} R_{\mu\beta} + g_{\mu\alpha} R_{\nu\beta} + g_{\mu\beta} R_{\nu\alpha} + g_{\nu\beta} R_{\mu\alpha}\,) -$$

$$-\frac{1}{4}\, (g_{\alpha\beta} R_{\mu\nu} + g_{\mu\nu} R_{\alpha\beta}\,) + \frac{1}{8}\, (R_{\mu\alpha\nu\beta} + R_{\nu\alpha\mu\beta} + R_{\nu\beta\mu\alpha} + R_{\mu\beta\nu\alpha}\,) + \frac{1}{8} g_{\mu\nu} g_{\alpha\beta} R\,, \tag{13}$$

where we have used standard notation $\delta_{\mu\nu\,,\,\alpha\beta} = \frac{1}{2}(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha})$.

It proves useful to separate the field $h^{\mu\nu}$ into the trace and the traceless part, $h^{\mu\nu} = \bar{h}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} h$. Then the bilinear form (11) becomes

$$\begin{aligned} S^{(2)} + S^{(2)}_{GF} \;\; &= \;\; \int d^4x \sqrt{-g} \left\{ \bar{h}^{\mu\nu} \left[\frac{1}{4}\, \bar{\delta}_{\mu\nu\,,\,\alpha\beta}\, (\Box - 2\Lambda) + M_{\mu\nu\,,\,\alpha\beta} \right] \bar{h}^{\alpha\beta} + \right. \\ &+ \;\; \bar{h}^{\mu\nu}\, [-2\, R_{\mu\nu}\,]\, \sigma + h \left[-\frac{1}{16}\, \Box + \frac{1}{8}\, \Lambda \right] h + \\ &+ \;\; h \left[-\Box + \frac{1}{2}\, R + 4\Lambda \right] \sigma + \sigma\, (-4\Box + 2R + 16\Lambda\,]\, \sigma \left. \right\} . \end{aligned} \tag{14}$$

Here

$$\bar{\delta}_{\mu\nu,\alpha\beta} = \delta_{\mu\nu,\alpha\beta} - \frac{1}{4}\, g_{\mu\nu} g_{\alpha\beta}$$

is the projector to the traceless states. The expression (14) exhibits the degeneracy in the mixed $h-\sigma$ sector, and hence further calculation requires some additional restriction on the quantum fields. This degeneracy is a direct consequence of the conformal symmetry (6) and thus we have to fix this symmetry. Let us choose the conformal gauge fixing in the form $\sigma = \lambda h$ with λ being the gauge fixing parameter. Then (14) becomes:

$$\begin{aligned} S^{(2)} + S^{(2)}_{GF} &= \int d^4x\sqrt{-g}\, \Big\{ \bar{h}^{\mu\nu} \left[\frac{1}{4}\, \bar{\delta}_{\mu\nu\,,\,\alpha\beta}\, (\Box - 2\Lambda) + M_{\mu\nu\,,\,\alpha\beta} \right] \bar{h}^{\alpha\beta} + \\ &+ \bar{h}^{\mu\nu}\, [\, -2\lambda R_{\mu\nu}\,]\; h + h\; [\; b_1\Box + 2b_2\Lambda + b_3 R\;)\; h\; \Big\} \end{aligned} \tag{15}$$

where we introduced the notations

$$b_1 = -\frac{1}{16} - \lambda - 4\lambda^2; \qquad b_2 = \frac{1}{16} + 2\lambda + 8\lambda^2; \qquad b_3 = \frac{1}{2}\lambda + 2\lambda^2. \tag{16}$$

The total one-loop divergences will be given by

$$\Gamma^{(1)}_{\rm div} = \frac{i}{2} Tr \ln \hat{H}_{\rm grav}|_{\rm div} - i Tr \ln \hat{M}|_{\rm div} \tag{17}$$

where the last term is the contribution from the ghost fields, and $\hat{H}_{\rm grav}$ is the operator corresponding to eq. (15). The standard Schwinger-DeWitt algorithm enables one to derive

$$\frac{i}{2} Tr \ln \hat{H}_{\rm grav}|_{\rm div} = -\frac{1}{\varepsilon}\int d^4x\sqrt{-g}\, \left\{ \frac{19}{18} R^2_{\rho\lambda\sigma\tau} + \left(\frac{4}{b_1}\lambda^2 - \frac{55}{18} \right) R^2_{\rho\lambda} + \right.$$

$$\left. \left(\frac{59}{36} - \frac{\lambda^2}{b_1} + \frac{b_3}{6b_1} + \frac{b_3^2}{2b_1^2} \right) R^2 + \left(\frac{2b_2b_3}{b_1^2} + \frac{b_2}{3b_1} + 9 \right) R\Lambda + \left(\frac{2b_2^2}{b_1^2} + 18 \right) \Lambda^2 \right\} \tag{18}$$

where $\varepsilon = (4\pi)^2(n-4)$. Also, the operator of the ghost action $\hat{M}$ is

$$\hat{M}^{\nu}_{\mu} \;=\; -\,\delta^{\nu}_{\mu}\,\Box \;-\; R^{\nu}_{\mu}\,. \tag{19}$$

We remark that the ghost operator does not depend on the gauge transformation of the field σ, because at the one-loop level, in the background field method, the generator of the gauge transformations which enters into the expression for $\hat{M}^{\nu}_{\mu}$ is the one for the background (not quantum!) fields [1] (see also [20]) and in case of σ this operator is zero.

Calculation of the ghost contribution yields standard result [1]

$$-i Tr \ln \hat{M}|_{\rm div} \;=\; \frac{1}{\varepsilon}\int d^4x\sqrt{-g}\, \left\{ -\frac{11}{90}\, E + \frac{7}{15}\, R^2_{\mu\nu} + \frac{17}{30}\, R^2 \right\}, \tag{20}$$

where $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2$. Finally, one arrives at the following one-loop divergences:

$$\Gamma^{(1)}_{\rm div} = -\frac{1}{\varepsilon}\int d^4x\sqrt{-g}\,\left\{ p_1(\lambda)E + p_2(\lambda)C^2 + p_3(\lambda)R^2 + p_4(\lambda)R\Lambda + p_5(\lambda)\Lambda^2\right\} \tag{21}$$

where C^2 is the square of the Weyl tensor $C^2 = E + 2(R^2_{\mu\nu} - \frac{1}{3}\,R^2)$ and

$$\begin{aligned}
p_1(\lambda) &= \frac{1}{180}\,\frac{149 + 2384\lambda + 15296\lambda^2}{(1+8\lambda)^2},\\
p_2(\lambda) &= \frac{1}{20}\,\frac{7 + 112\lambda - 192\lambda^2}{(1+8\lambda)^2},\\
p_3(\lambda) &= \frac{1}{12}\,\frac{3 + 80\lambda + 1152\lambda^2 + 6144\lambda^3 + 10240\lambda^4}{(1+8\lambda)^4},\\
p_4(\lambda) &= \frac{2}{3}\,\frac{13 + 432\lambda + 5696\lambda^2 + 31744\lambda^3 + 63488\lambda^4}{(1+8\lambda)^4} \quad \text{and}\\
p_5(\lambda) &= 4\,\frac{5 + 176\lambda + 2368\lambda^2 + 13312\lambda^3 + 26624\lambda^4}{(1+8\lambda)^4}.
\end{aligned} \tag{22}$$

The above expression (21), (22) contains complicated dependence on the gauge fixing parameter λ. Besides, the one-loop divergences may depend on others gauge fixing parameters α, β, γ from (10). Here we are interested only in the dependence on λ, and keep α, β, γ fixed as before.

3 Analysis of the results

The expression (21), (22) looks quite cumbersome and somehow chaotic because of the complicated dependence on the gauge fixing parameter λ. But, in fact, there are a few possibilities to check and analyze it. First of all, for the value $\lambda = 0$, all the σ-field contributions drop and we arrive at the well-known result [1, 10]

$$\Gamma^{(1)}_{\rm div} = -\frac{2}{\varepsilon}\int d^4x\sqrt{-g}\,\left\{ \frac{1}{120}R^2 + \frac{7}{20}R^2_{\mu\nu} + \frac{53}{90}E + \frac{13}{3}\,R\Lambda + 10\,\Lambda^2 \right\}. \tag{23}$$

For other values of λ the divergences are different and one can check that the λ-dependence can not be compensated by the change of other gauge fixing parameters α, β or by the change of parameter r introduced in [12].

If we take a limit $\lambda \to \infty$, the result is not conformal invariant, as one could naively expect. Let us give some additional comment on this point. The above calculation can be regarded as a particular case of the much more complicated derivation of the one-loop divergences in the theory (5), which was mentioned in the Introduction. In general calculation one is supposed to shift both fields φ (or σ, this is equivalent) and $g_{\mu\nu}$, while in this paper we took the background scalar to be constant. Let us imagine, for a moment, that we shifted both fields

$$\sigma \to \chi + \sigma\,, \qquad g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}\,. \tag{24}$$

As far as we believe into conformal invariance of the one-loop divergences[5], the result for conformal metric-scalar theory is [23]

$$\Gamma_{div} = \frac{1}{\epsilon}\int d^4x\sqrt{-g}\,\{p_1\,E + p_2\,C^2 +$$

$$+p_3\left[R - \frac{3\Box\zeta}{\zeta} + \frac{3(\nabla\zeta)^2}{2\zeta^2}\right]^2 + p_4\zeta\,\left[R - \frac{3\Box\zeta}{\zeta} + \frac{3(\nabla\zeta)^2}{2\zeta^2}\right] + p_5\zeta^2\Bigg\}\,, \tag{25}$$

where $\zeta = \zeta(\chi)$ is some function of χ. The procedure accepted in this paper is equivalent to taking $\chi = const$, and therefore (22) should be regarded as (25) with constant ζ. Obviously, constant ζ does not transform and the conformal invariance is lost.

In order to verify the result of the calculation, one can use classical equations of motion $R_{\mu\nu} = -\Lambda\, g_{\mu\nu}$. On shell the divergences become

$$\Gamma^{(1)\,\mathrm{on-shell}}_{\mathrm{div}} = -\frac{1}{\varepsilon}\int\, d^4x\sqrt{-g}\,\left\{\frac{53}{45}E - \frac{58}{5}\Lambda^2\right\}\,, \tag{26}$$

independent on the gauge fixing parameter λ. As a consequence, the on shell renormalization group equation for the dimensionless cosmological constant $\kappa^2\Lambda$ [21] is gauge fixing independent in our conformal parametrization. One has to notice that the coefficients of (26) are linear combinations of all five functions (22), and thus the complete cancellation of the λ-dependence, together with (23), provide a very confident verification of the result (21).

4 Conclusions

We have studied the equivalence between General Relativity and conformal metric-scalar theory on quantum level. The one-loop divergences were calculated for quantum gravity, for the first time this was done in the conformal parametrization for quantum metric. We have found that the dependence on the new gauge fixing parameter disappears on shell. This gives an efficient check to the whole procedure based on fixing the conformal symmetry by using the trace of

[5] Some remarkable example of the opposite one can meet in the Weyl gravity, where the results of two one-loop calculations [21] and [22] coincide only after the use of the so-called conformal regularization [21]. The lack of equivalence between the results of [18] and [19] may indicate to the similar problem.

quantum metric $h = h^\mu_\mu$. The results of our work show that the source of the discrepancy in the results for the quantum Weyl gravity is not caused by this conformal gauge fixing. Finally, the supposition of conformal invariance of the counterterms enables one to restore the divergences for the gravity coupled to conformal scalar field (25).

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