Back-Reaction of Einstein's Gravitational-Waves as the Origin of Natal Pulsar Kicks

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Pulsars are observed to be born with mean spatial (3-D) velocities of ~ 450 kms^{-1} [1,2]. Since most stars in our galaxy are observed to possess mean velocities of ~ 30 kms^{-1} [3], it has been suggested such large velocities should be given to the proto-neutron star at its birth. The mechanism responsible for the impulses has not been properly identified, despite of interesting proposals have been advanced [4,5]. Here I identify such a mechanism to be linked to the *back-reaction force* of the *gravitational wave burst* released during the first millisecond after the core bounce of a supernova collapse. This behavior of the gravity wave is predicted by Einstein's general relativity theory. It turns that the overwhelming population of fast moving pulsars cataloged, compared to the tiny one of neutron star-neutron star binaries [6], makes these objects the most compelling evidence for the existence of gravitational waves, and correspondently points towards the Einstein theory of gravitation as the most correct one.

Key-words: Pulsars: observations; Supernovae: physics; General relativity: gravitational waves:: theory; Wave generation and sources; Gravitation.

Gravitational waves are ripples in the fabric of space-time generated by cataclysmic events such as supernova explosions or coalescences of binary neutron star and/or black holes. Its direct detection still awaits for firm confirmation. However, indirect evidences for their existence come from the observations by Taylor and Hulse of the dynamical evolution of the *binary pulsar* PSR1913+16 [7]. During a supernova core collapse, gravitational waves are produced by either the hydrodynamical evolution of the protoneutron star or the neutrino outflow after the core bounce [8–12].

Natal pulsar (recoil) kicks, as inferred from all-sky surveys, range over 100-1500 kms^{-1} [1,2], while their rotation periods pile up near 0.5 seconds [13,1,14]. These properties cannot be explained as an inheritance of late stellar evolutionary stages of their progenitor supergiant stars, since their cores rotate so slowly as to leave remnant pulsars with millisecond spin periods and high velocities. As a consequence, its origin should be unavoidably associated to the proto-neutron star formation process itself. Here I skip from *those conventional* mechanisms to kick the PNS as either asymmetric hydrodynamic explosions or neutrino (oscillating or not) convectively advected minijets [4,5]. Instead, I suggest that due to the *quadrupolar nature* of the *radiation-reaction force* [15] applied by the *gravitational wave burst* released just at the core bounce of a supernova collapse, recoil velocities and spins may be imparted to the PNS as kicks at birth. While the GW avances this radiation-reaction force (RRF) induced by each GW polarization mode may apply several (pair-folded) thrusts to the nascent neutron star. This effect could explain its rotation and motion as evidenced in pulsar surveys. Because the several RRF kicks from each polarization mode are applied in a plane orthogonal to the GW vector, the pulsar imparted spin and movement necessarily should correlate to each other [4,5].

It worths to stress that the direct gravitational wave energy and angular momentum released in the process is not the actual engine of the kicks. These quantities acquire very low values, and it has been demonstrated they cannot do the work to kick the star. It is well-known, from numerical simulations of gravitational waves generation during a supernova core collapse, that only about a fraction of $4\times10^{-6}\mathrm{M}_{\odot}c^2$ of the total binding energy of the proto-neutron star is released as gravitational waves. An inference from observations suggests this value too. [16] I shall demonstrate below that the induced reaction force of the gravitational radiation is the most likely mechanism behind the pulsars kicks. This is a very different argumentation. Approximately half of the enormous *gravitational wave luminosity* produced may be dissipated by this reaction force in the form of pulsar kicks. Thus, one can get from the collapse very few gravitational waves energy (and angular momentum) but the reaction force the waves induce is able to do the work. I would like to call to attention the fact that this gravitational radiation reaction force has been very recently suggested to be able to make young pulsars' precession to be damped out on a timescale of $\sim 4 \times 10^5$yr, too. [17] In the following I review the status of the current explanations for these pulsars properties, and then I introduce the gravity wave mechanism to show its feasibility.

A recent review of current observations of pulsars [1,2] led Deshpande, Ramachandran & Radakrishnan (DRR'99) [18] to some crucial conclusions concerning the direction and magnitude of the proper motion, projected direction of the magnetic axis, magnitude of the magnetic field, direction of the rotation axis and its initial spin periods (which is not properly inferred from observations). It was also demonstrated that no significant correlation exists between the magnetic field strength and the magnitude of the spatial velocities of pulsars, or between the projected directions of the rotation axis (and/or the magnetic field axis) and the direction of the proper motion vector.

Such conclusions have fundamental implications for the mechanism triggering the asymmetric supernova kick velocities, as the observations do not support any mechanism producing net kick velocities parallel to the rotational axis. Consequently, their study rules out momentum impulses of any duration along the spinning axis, and any long duration (compared to the spin period) impulses along any one fixed axis, as the magnetic vector. In addition, their numerical simulations and confrontation with observations altogether lead to conclude that single impulses are also ruled out, but not *two or more impulses* of relatively short duration (DRR'99) [18]. Moreover, in order to avoid significant azimuthal averaging of the impulse' radial component the momentum transfer event should be very short-lived. Those conclusions apply to both the Spruit & Phinney (1998) [5] and Cowsik (1998) [4] mechanisms, thought to be the most prospective ones. Accordingly, all the mechanisms driving pulsar kicks from neutrino oscillations, such as those by Kusenko & Segrè (1997,1998) [19]; Akhmedov, Sciama & Lanza (1998) [20]; Grasso et al. (1998) [21]; Nardi & Zuluaga (2000) [22] ; Mosquera Cuesta (2000) [11]; and Mosquera Cuesta et al. (2000) [12], as well as the Harrison-Tademaru (1975) *rocket mechanism* [23,24], are definitely ruled out since for all of them to properly work, a dipolar magnetic field configuration pervading the PNS core is a critical assumption, and only one thrust is unavoidable. Conversely, only those scenarios in which impulses triggered in very short timescales and almost simultaneously applied to the PNS [5,4] are in principle capable of accelerating and spinning the nascent NS as observed in pulsar surveys [1,2]). Below we demonstrate that those constraints can be consistently satisfied by the mechanism proposed in this letter. As it will be discussed later on, the possibility of most pulsars to be born with a few millisecond periods is straightforward realized within our gravitational-wave radiation-reaction (force) driven pulsar kick mechanism.

In general relativity, the Einstein's theory of gravitation, the dynamics of a pulsating compact star is described according to the field equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$, where $R_{\mu\nu}$, R, $g_{\mu\nu}$ and $T_{\mu\nu}$, are correspondently the curvature tensor, curvature scalar and metric tensor of spacetime, and energy-momentum tensor of the matter fields constituting the body. $\kappa = 16\pi G/c^4$ is a constant. When linearized, for taking into consideration small perturbations ($h << 1$) of the Minkowski spacetime, the metric can be written as $g_{\mu\nu} = n_{\mu\nu} + h_{\mu\nu}$, and the field equations lead to the gravitational waveform $h_{ij}^{TT} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$,, a traceless, transverse (to the propagation direction z) and symmetric spatial tensor, with polarization tensor components as $e_{xx}^+ = -e_{yy}^+ = 1$ and $e_{xy}^\times = e_{yx}^\times = 1$, with any other combination of indexes null. These components $h_+ e_{ij}^+$ and $h_\times e_{ij}^\times$ are denominated the polarization modes of the gravitational wave, with h_+, $h_\times$ their corresponding GWs amplitudes. In general, h_+ is almost twice $h_\times$, and as the GW goes through a $\pi/4$ dephasing separates the modes inbetween. We will come back to this crucial property when discussing the pulsar kick mechanism from the GWs bursts at core bounce. The h_{ij}^{TT} tensor is analogous to the vector potential A in electrodynamics in the Lorentz gauge. In classical electrodynamics, by using the Maxwell's equations, the following vacuum relations can be cast: $A_0 = 0, A_{i,i} = 0, \partial^i \partial_j A_i = 0$. The correponding set for the case of the gravitational waves reads: $h_{0j}^{TT} = 0, h_{ij,j}^{TT} = 0$, and the Einstein's field equation (the GW equation): $\partial^i \partial_j h_{ij}^{TT} = 0$. To get these set of equations use has been done of the *traceless* property of the metric tensor.

The stress-energy tensor for GWs propagating in the z-direction has non-null components $T^{00} = \frac{T^{0z}}{c} = \frac{T^{zz}}{c^2} = \frac{c^2}{16\pi G} < (\dot{h}_+)^2 + (\dot{h}_\times)^2 >$, where the brackets denote averaging over several wavelengths. Here T_{00} is the energy density, T^{z0} the energy flux, and T^{zz} the momentum flux. In our picture, they are produced during the first millisecond after the supernova core bounce. The gradient of this pressure exerts a force onto the object acted upon, as shown next.

When a GW passes through a compact body (the PNS, in the case) the star oscillates in a way described by the usual equations of motion but the oscillations themselves are driven by the GW force $F_i = \frac{1}{2} m \ddot{h}_{ij}^{TT} \delta_j$ acting on each element of the mass m, with δ_j the relative displacement of that element. Therefore, the GW carries energy and momentum, i. e., it does some work. However, this direct force is unable to give the nascent pulsar the expected thrusts.

On the other hand, due to their quadrupolar nature (in general relativity) the energy and momentum densities cannot be localized at a given point but distributed over a few wavelengths (λ_{GW}) around the star. We shall see next that for the case of nascent neutron star this wavelength λ_{GW} will correlate roughly with the impact parameter for the GWs quadrupoled-shaped back-reaction force, conjectured here to be the trigger of the PNS natal kicks.

In a very simplified picture, the mechanism of a supernova explosion invokes the transfer of energy from the stellar core (the progenitor of the PNS remnant) and the supergiant star mantle that is blown away. The source of the huge energy transfer required to eject the envelope is assumed to be a neutrino flash. Nearly 3×10^{53}erg are released during the outburst, almost all of that energy escapes as neutrinos [9], while nearly 1% flows as gravitational waves and photons. In spite of being so small an energy, the gravitational wave luminosity is a huge quantity $L_{GW} \sim 6 \times 10^{56}\mathrm{ergs}^{-1}$ (see details below). Since neutron matter cannot be packed beyond the nuclear density the star core bounces. During the next first millisecond the central core density gets so high that no radiation nor even neutrinos can escape. They get themselves frozen-in. The whole PNS dynamics is governed then by gravity alone. Over that timescale, the sound wave generated at bounce builds up itself into a shock wave, which pushes out the infalling envelope matter, driven the final explosion. It is over this postbounce timescale that the gravitational wave burst produced will fully dominate the PNS evolution in such way so as to give it the recoil and spin at birth via its back-reaction force. Since no other known process can do as much, I suggest here that this is the most fundamental mechanism responsible for the observed recoil velocities and spin periods of pulsars.

The non-linear character of Einstein equations implies the radiated gravitational waves act backwards onto their source. This property is the key idea making it possible to explain the natal pulsar motions and spins by the GWs produced at core bounce. As is clearly expected, the radiation reaction force can impart to the PNS several (pair-folded) thrusts as far as the gravitational wave passes through a given location of the star. The total *radiation-reaction force* corresponding to the GWs energy loss given above is defined as $\vec{F}^{(react)} = -M\nabla\Phi^{(react)}\hat{k},$, with $\hat{k}$ the gravitational-wave vector in the planewave limit. The radiation-reaction potential is given by $\Phi^{(react)} = \frac{G}{5c^5}\bar{I}^{(5)}_{ij}x_i x_j$. Here the term $\bar{I}^{(5)}_{ij}$ represents the fifth time derivative of the traceless quadrupole mass-tensor, M the star total mass, and $x_i = \bar{d}$ (see below) the position at which the potential is being considered. So, to obtain order of magnitude estimates, the gravitational wave-induced potential may be recast as

$$\Phi^{(react)}_{GW} \sim \frac{G}{c^5}\left(\frac{MR^2}{T^5_{dyn}}\right)\bar{d}.\bar{d}, \qquad (1)$$

where R defines the NS star radius, and the dynamical timescale for the whole process is defined by (see Figure 6 in Ref. [8]) $T_{dyn} = \left(R^3/GM\right)^{1/2} \sim 1\mathrm{ms}$, as demonstrated by numerical simulations [10].

Because part of the GWs luminosity emitted during the core-collapse should be dissipated through the RRF $\vec{F}^{(react)}$, we can derive the pulsar recoil velocity $\vec{V}_{kick}$ from the dynamical relation $L_{GW} = \frac{G}{c^5} < \bar{I}^{(3)}\bar{I}^{(3)} > |_{1\mathrm{ms}} \equiv F^{(react)}V_{\mathrm{recoil}} \times \cos(\vec{F}^{(react)}, \vec{V}_{\mathrm{recoil}})$, where the third time derivative of the quadrupolar mass-tensor can be recast as [15] $\bar{I}^{(3)} \sim MR^2/T^3_{dyn} \sim MV_f^3/R$. The internal velocity of the star mass V_f can be defined recalling that during the first millisecond after core bounce all of the matter constituting the PNS is pervaded by a *shock wave* traveling through it with velocity $V_f = c/\sqrt{3}$. Thus, for a canonical neutron star: $M = 1.44M_\odot$, $R_{NS} = 10\mathrm{km}$, with $V_f \sim V_{sound} = c/\sqrt{3}$, we can get the characteristic recoil velocities as observed in pulsar surveys, ($V_{\mathrm{recoil}} \lesssim 1500\ \mathrm{kms}^{-1}$) as

$$V_{\mathrm{recoil}} \sim 1.0 \times 10^9 \cos(\vec{F}^{(react)}, \vec{V}_{\mathrm{recoil}})\ \mathrm{cms}^{-1}, \qquad (2)$$

for values of the angle between the GWs reaction force and the effective pulsar velocity: $\cos(\vec{F}^{(react)}, \vec{V}_{\mathrm{kick}}) = \cos^{-1}(0.15) = 8.15$ degrees. Note that no reference to any particular axis on the NS has been pointed out. The actual principal axes configuration will depend on the specific features of the supernova core collapse, which should include the overall degree of asymmetry, precollapse spin, radius and core mass, etc.

This radiation-reaction force should act upon the PNS during the gravitational wave damping timescale [26]

$$\Delta T^{(react)}_{GWs} \sim 0.40\mathrm{s}\left(\frac{1.4M_\odot}{\mathrm{M}}\right)\left(\frac{10^6 cm}{\mathrm{R_{NS}}}\right)^2\left(\frac{\mathrm{P}}{1\mathrm{ms}}\right)^4. \qquad (3)$$

Such a time interval is sufficiently short to satisfy the constraint put forward by (DRR'99), as quoted above. It also fits quite well the duration $\Delta t \sim 0.32$s for the four thrusts supposed to be applied to the PNS in the Spruit & Phinney kick model [5].

In order to derive a *natal* pulsar spin, we need to understand firstly how the gravitational radiation-reaction force acts upon the nascent neutron star ($R_{PNS} \sim 30\mathrm{km}$ [5]). Gravitational waves carry off angular momentum J_i from a spinning PNS at the rate $\frac{dJ_i}{dt} = \sum_A \varepsilon_{ijk}x^A_j F^{A(react)}_k = \frac{2G}{5c^5}\varepsilon_{ijk} < \bar{I}^{(2)}_{jm}\bar{I}^{(3)}_{km} >$. But as quoted above this is not enough to make the star spinning as required. Thus it follows that perhaps only the reaction force, with an appropriate lever-arm, is the one responsible for the initial rotation of the star. For compact objects, i. e., black holes or neutron stars, the GWs wavelength $\lambda_{GW} \sim$ a few kilometers. A GW with frequency of 10^3Hz possess an associated wavelength $\lambda_{GW} \sim 3 \times 10^7$cm, which for a particular equation of state of the PNS matter it is comparable to its typical precollapse radius [8,10]. If the RRF acts at the centroide of the force per unit length distribution (the GW field on each quadrant in the planewave approximation), defined by the distance $x^A_j = \bar{d}$ from the star center (corresponding to a given phase of the emitted GW, i. e., $\varphi = \pi/4 \longrightarrow \bar{d} \sim 25\mathrm{km}$), then it should produce a torque on the star which is directly proportional to that lever-arm. The standard relation bewteen the torque applied by a given force to a rotating object and the angular acceleration $\vec{\alpha}$ induced on it turns out to be

$\vec{d} \times \vec{F}^{(react)} = \bar{I}\vec{\alpha}$. Here $\vec{d}$ is the lever-arm vector for the torque applied. Thus we can rewrite the magnitude of the angular acceleration as $\alpha \equiv \Delta\omega/\Delta t \sim \omega/T_{dyn}$, to finally get (same parameters as above) the *natal* pulsar rotation frequency

$$\bar{\omega} \sim 0.6 \times 10^4 \mathrm{rads}^{-1}, \tag{4}$$

from which we obtain the pulsar spin frequency $f_{spin} \sim 10^3\mathrm{s}^{-1}$, and consequently, the natal pulsar spin period $P \sim 10^{-3}$s, for $\bar{d} \sim 25\mathrm{km} < R_{PNS}$ [5]. It is easy to see that the lowest recoil velocities (and spins) observed may be obtained for a lever-arm $\bar{d}$ half order of magnitude lower. Thus the GW RRF kick mechanism provides a consistent picture agreeing quite well with the observational initial spin periods and recoil velocities of pulsars, as quoted ealier.

To these rapidly spinning nascent neutron stars the nonaxisymmetric instability driven by gravitational waves, so called *r-modes* instability, may be relevant and able to reduce the pulsar *natal* initial spin period to its current *observational* limit $\sim$ 20ms, for nascent very hot ($T \sim 10^9$K) neutron stars [25]. The new results imply that if the NS is born rotating at its maximum frequency (Kepler frequency $\Omega_K \sim 0.67[\pi G\bar{\rho}]^{1/2}$), the r-mode instability will force the stars to spin down to periods of roughly 15-20 ms essentially in the first year following its birth [25]. We note in passing that Madsen [27] has shown that for the case of sub-milliseconds spin periods, bulk viscosity effects the early evolution of the young pulsar making it impossible for the *r-mode* instability to grow up. Accordingly, submillisecond pulsars may be encountered in future surveys. However, our analysis shows that the expected initial rotation periods naturally pile up around 1-2 ms as far as a NS (not a strange star [27]) is formed, which allows for the r-mode instability to grow up.

On the possible realization of this mechanism in astrophysical sources: two observational and one theoretical insights may be pertinent. Firstly, there is strong evidence that some pulsars appear anomalously displaced from the center of its apparent parental supernova remnant. It seems as if the pulsar had moved from the instant of the explosion with a velocity so large compared to typical observed ones [1]. Velocities of the order of 5000 kms^{-1} have been inferred for these anomalous running pulsars. [3] Thence, if our mechanism is actually acting on those early stages of the postbounce evolution, it is not difficult to see from Eq.(2) that because higher velocities are predicted (upto one order of magnitude), these anomalous displacements can be well fitted into our picture for the pulsars kicks.

The other peculiar observational piece of discussion relates to the expected *inverse* correlation between pulsar velocities and spin periods suggested by Cowsik [4]. It is claimed here that, despite of being quite difficult to test within the context of conventional ideas for the natal neutron star kicks, as mentioned ealier, our model predicts explicitly such a correlation, since as shown above the quadrupolar nature of the radiation-reaction force, in general, allows for special combinations of the parameter space: for a large spatial velocity a small magnitude of the reaction force vector $|\vec{F}^{(react)}|$ is obtained, which implies a small angular frequency, and conversely. These possibilities can easily be verified through direct inspection of expressions for the GW power and RRF torque.

From the theoretical point of view, there is strong evidence for acceleration of black holes that have been impinged by a powerful gravitational wave. [28] If such a strong gravitational wave is able to kick a black hole to the sustantial velocities as discussed in Ref. [28], then it is expected a similar acceleration impulse to occur onto a PNS during the supernova postbounce due to GWs kicks at birth.

To sumarize, if this mechanism actually works during the postbounce of a supernova collapse, then the large population of high velocity pulsars turn these objects the more compelling evidence for the existence of gravitational waves, and for the Einstein's gravity theory to be the most likely realized in nature.

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