On the Finiteness of BF-Yang-Mills Theory in Three Dimensions

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Abstract

We show that the BF-Yang-Mills theory in 3 dimensions is finite. The main ingredient in the proof of this property is the validity of a trace identity that plays the role of a local form for the Callan-Symanzik equation to all orders in perturbation theory.

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1 Introduction

Topological field theories are a class of gauge model interesting from a physical point of view. In particular, their observables are of topological nature. An important topological model that has received much attention recently is the BF theory (see [1] for a general review and references). The latter represents a natural generalization of the Chern-Simons theory since it can be defined on manifolds of any dimensions whereas a Chern-Simons action exists only in odd-dimensional space-times. Moreover, the Lagrangian of the BF theory contains the quadratic terms needed for defining a quantum theory, whereas a Chern-Simons action shows this feature only in three dimensions.

On the other hand, recently also Yang-Mills gauge theories in the first order formalism have been re-interpreted as a deformation of a pure BF theory [2, 3, 4], named BF-Yang-Mills. The quantum equivalence between the latter and a pure Yang-Mills theory has been discussed in three dimensions by [5], and in four dimension by [6, 7]. This gives us the possibility to look at the pure YM theory as a perturbation of a topological theory [8].

The most peculiar property that topological field theories exihibit is their ultraviolet finiteness [9, 10, 11, 12]. This property relies on the existence of a topological vector supersymmetry [1, 13], whose origin is manifest in the case one chooses the Landau gauge. This topological vector supersymmetry plays moreover a crucial role in the construction of the explicit solutions of the BRS cohomology modulo d, giving therefore a systematic classification of all possible anomalies and physically relevant invariant conterterms [14].

In this letter, we shall be concerned in discussing the finiteness of the BF-Yang-Mills theory in three dimensions, using a different technique [15]. This one is suitable particularly in situations where all power of the topological vector supersymmetry is lost. Indeed, this is the case treated here, because the BF-Yang-Mills theory is not a topological one.

2 The BF-Yang-Mills Theory in Curved Space-Time

Following [15], we write the BF-Yang-Mills action on a curved manifold, as long as its topology remains that of flat $\mathcal{R}^3$, with asymptotically vanishing curvature. It is the latter two restrictions which allow us to use the general results of renormalization theory, established in flat space.

The classical BF-Yang-Mills theory in a curved manifold $\mathcal{M}$ is defined by the action

$$\Sigma_{\rm BFYM} = \int d^3x \, \left\{ \varepsilon^{\mu\nu\rho} B^a_\mu F^a_{\nu\rho} + e \, B^a_\mu B^{a\mu} \right\} \tag{2.1}$$

where $F^a_{\nu\rho} = \partial_\nu A^a_\rho - \partial_\rho A^a_\nu + g f_{abc} A^b_\nu A^c_\rho$ is the YM field strength, B^a_μ is an auxiliary field and e denotes the determinant of the dreibein field e^m_μ.

The action $\Sigma_{\rm BFYM}$ is invariant with respect to the gauge symmetry

$$\delta A^a_\mu \;\; = \;\; -D_\mu \theta^a \equiv - \left(\partial_\mu \theta^a + g \, f_{bc}{}^a A^b_\mu \theta^c \right) \, ,$$

$$\delta B^a_\mu \;=\; -g\, f^{abc} B^b_\mu \theta^c \,. \tag{2.2}$$

The BRS quantization is accomplished introducing one ghost, c^a, one antighost, $\bar{c}^a$, and one Lagrange multiplier, b^a, and defining a nilpotent s-operator as

$$\begin{aligned} sA^a_\mu &= -D_\mu c^a \,, \\ sB^a_\mu &= -g\, f^{abc} B^b_\mu c^c \,, \\ sc^a &= \frac{1}{2}\, g\, f_{bc}{}^a c^b c^c \,, \\ s\bar{c}^a &= b^a \,, \\ sb^a &= 0 \,. \end{aligned} \tag{2.3}$$

In the Landau gauge, the gauge-fixing term $\Sigma_{\rm gf}$ reads

$$\Sigma_{\rm gf} = -\, s \int d^3x \; e\, g^{\mu\nu} \partial_\mu \bar{c}_a A^a_\nu \;=\; - \int d^3x \; e\, g^{\mu\nu} \left(\partial_\mu b_a A^a_\nu + \partial_\mu \bar{c}_a D_\nu c^a \right) \,, \tag{2.4}$$

which is BRS-invariant.

To express the BRS invariance as a Slavnov-Taylor identity we couple the nonlinear variations of the quantum fields to antifields (or external sources) $A^{*\mu}_a$, c^*_a, $B^{*\mu}_a$:

$$\Sigma_{\rm ext} = \int d^3x \sum_{\Phi = A^a_\mu,\, c^a,\, B^a_\mu} \Phi^* s\Phi \,. \tag{2.5}$$

The total action,

$$\Sigma = \Sigma_{\rm BFYM} + \Sigma_{\rm gf} + \Sigma_{\rm ext} \,, \tag{2.6}$$

obeys the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) \;=\; \int d^3x \sum_{\Phi = A^a_\mu,\, c^a,\, B^a_\mu} \frac{\delta\Sigma}{\delta\Phi^*} \frac{\delta\Sigma}{\delta\Phi} + b\,\Sigma = 0 \,, \quad \text{with} \;\; b = \int d^3x \; b^a \frac{\delta}{\delta\bar{c}^a} \,. \tag{2.7}$$

For later use, we introduce the linearized Slavnov-Taylor operator

$$\mathcal{B}_\Sigma \;=\; \int d^3x \sum_{\Phi = A^a_\mu,\, c^a,\, B^a_\mu} \left(\frac{\delta\Sigma}{\delta\Phi^*} \frac{\delta}{\delta\Phi} + \frac{\delta\Sigma}{\delta\Phi} \frac{\delta}{\delta\Phi^*} \right) + b \,. \tag{2.8}$$

$\mathcal{S}$ and $\mathcal{B}$ obey the algebraic identity

$$\mathcal{B}_{\mathcal{F}} \mathcal{B}_{\mathcal{F}} \mathcal{F}' + (\mathcal{B}_{\mathcal{F}'} - b)\, \mathcal{S}(\mathcal{F}) = 0 \,, \tag{2.9}$$

$\mathcal{F}$ and $\mathcal{F}'$ denoting arbitrary functionals of ghost number zero. From the latter follows

$$\mathcal{B}_{\mathcal{F}}\, \mathcal{S}(\mathcal{F}) = 0 \quad , \quad \forall\, \mathcal{F} \quad , \tag{2.10}$$

$$(\mathcal{B}_{\mathcal{F}})^2 = 0 \quad \text{if} \quad \mathcal{S}(\mathcal{F}) = 0 \,. \tag{2.11}$$

In addition to the Slavnov-Taylor identity (2.7), the total action (2.6) turns out to be characterized by the following additional constraints:

- the Ward identity for the diffeomorphisms

$$\mathcal{W}_{\rm diff}\Sigma = \int d^3x \sum_\Phi \delta^{(\varepsilon)}_{\rm diff}\Phi \frac{\delta\Sigma}{\delta\Phi} = 0\,, \tag{2.12}$$

with

$$\begin{array}{rcl} \delta^{(\varepsilon)}_{\rm diff}F_\mu &=& \varepsilon^\lambda\partial_\lambda F_\mu + \left(\partial_\mu\varepsilon^\lambda\right)F_\lambda\,, \quad F_\mu = \left(A^a_\mu,\ B^a_\mu,\ e^m_\mu\right)\,, \\ \delta^{(\varepsilon)}_{\rm diff}\Phi &=& \varepsilon^\lambda\partial_\lambda\Phi\,, \quad \Phi = \left(b^a,\ c^a,\ \bar{c}^a\right)\,, \\ \delta^{(\varepsilon)}_{\rm diff}F^{*\mu}_a &=& \partial_\lambda\left(\varepsilon^\lambda F^{*\mu}_a\right) - \left(\partial_\lambda\varepsilon^\mu\right)F^{*\lambda}_a\,, \quad F_\mu = \left(A^{a*}_\mu,\ B^{a*}_\mu\right)\,, \\ \delta^{(\varepsilon)}_{\rm diff}c^{a*} &=& \partial_\lambda\left(\varepsilon^\lambda c^{a*}\right)\ ; \end{array}$$

- the Ward identity for the local Lorentz transformations

$$\mathcal{W}_{\rm Lorentz}\Sigma = \int d^3x \sum_\Phi \delta^{(\lambda)}_{\rm Lorentz}\Phi \frac{\delta\Sigma}{\delta\Phi} = 0\,, \tag{2.13}$$

where

$$\delta^{(\lambda)}_{\rm Lorentz}\Phi = \frac{1}{2}\lambda_{mn}\Omega^{mn}\Phi\,, \quad \Phi = \text{any field}\,,$$

with $\Omega^{[mn]}$ acting on Φ as an infinitesimal Lorentz matrix in the appropriate representation;

- the Landau gauge condition

$$\frac{\delta\Sigma}{\delta b_a} = \partial_\mu\left(eg^{\mu\nu}A^a_\nu\right)\ ; \tag{2.14}$$

- the ghost equation of motion

$$\mathcal{G}^a\Sigma = \frac{\delta\Sigma}{\delta\bar{c}_a} + \partial_\mu\left(eg^{\mu\nu}\frac{\delta\Sigma}{\delta A^{*\nu}_a}\right) = 0\,, \tag{2.15}$$

this implies that the theory depends on the field $\bar{c}$ and on the antifield $A^{*\mu}$ through the combination

$$\hat{A}^{*\mu}_a = A^{*\mu}_a + eg^{\mu\nu}\partial_\nu\bar{c}_a\,; \tag{2.16}$$

- the antighost equation, peculiar to Landau gauge [16]

$$\bar{\mathcal{G}}^a\Sigma = \int d^3x\left(\frac{\delta}{\delta c^a} - g\,f^{abc}\bar{c}_b\frac{\delta}{\delta b^c}\right)\Sigma = \Delta^a_{\rm cl}\,, \tag{2.17}$$

with

$$\Delta^a_{\rm cl} = g\int d^3x\,f^{abc}\left(A^{*\mu}_b A_{c\mu} - c^*_b c_c + B^{*\mu}_b B_{c\mu}\right)\,;$$

(The right-hand side of (2.17) being linear in the quantum fields will not get renormalized.)

- the Ward identity for the rigid gauge invariance following from (2.7) and (2.17)

$$\mathcal{W}^a_{\rm rigid} = \int d^3x\left(\sum_{\Phi\,=\,\text{all fields}} f^{abc}\Phi_b\frac{\delta}{\delta\Phi^c}\right)\,. \tag{2.18}$$

3 Cohomology and Renormalizability

We face now the problem of showing that all the constraints defining the classical theory also hold at the quantum level, i.e., that we can construct a renormalized vertex functional,

$$\Gamma = \Sigma + \mathcal{O}(\hbar) \ , \tag{3.1}$$

obeying the same constraints and coinciding with the classical action at zero order in $\hbar$.

As announced in Section 2, the proof of renormalizability will be valid for manifolds which are topologically equivalent to a flat manifold and admit an asymptotically flat metric. It is only in this case that one can expand in powers of $\bar{e}^m_\mu = e^m_\mu - \delta^m_\mu$, considering $\bar{e}^m_\mu$ as a classical background field in flat $\mathcal{R}^3$, and thus make use of the general theorems of renormalization theory actually proved for flat space-time [19, 20].

Following [5], we add to the total action (2.6) a Chern-Simons term [17]

$$\Sigma_{\rm CS} = \int d^3x \, m \, \varepsilon^{\mu\nu\rho} \left(A^a_\mu \partial_\nu A^a_\rho + \frac{g}{3} f_{abc} A^a_\mu A^b_\nu A^c_\rho \right) \ , \tag{3.2}$$

where m is a topological mass. This will safe the theory from IR divergences; the zero limit, which formally recovers the massless theory, is argued to be smooth for resummed quantities [5]. Moreover, the Chern-Simons term does not change neither the algebraic structure nor the form of the operators entering the algebraic analysis.

- *Power-Counting*

The first point to be checked is the power-counting renormalizability - in fact super-renormalizability. It follows from the dimension of the action being bounded by three. The ultraviolet dimension, as well as the ghost number and the Grassmann parity of all fields and antifields are collected in Table 1.

	A_μ	B_μ	b	c	$\bar{c}$	$A^{*\mu}$	$B^{*\mu}$	c^*	g
d	$1/2$	$3/2$	$3/2$	$-1/2$	$3/2$	$5/2$	$3/2$	$7/2$	$1/2$
$\Phi\Pi$	0	0	0	1	-1	-1	-1	-2	0
GP	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$

Table 1: Ultraviolet dimension, d, ghost number, $\Phi\Pi$, and Grassmann parity, GP.

In order to explicitely find the possible renormalizations and anomalies of the theory, we can use the following result [21, 15]: The degree of divergence of a 1-particle irreducible Feynman graph, γ, is given by

$$d(\gamma) = 3 - \sum_\Phi d_\Phi N_\Phi - \frac{1}{2} N_g \ . \tag{3.3}$$

Here N_Φ is the number of external lines of γ corresponding to the field Φ, d_Φ is the dimension of Φ as given in Table 1, and N_g is the power of the coupling constant g in the

[5]According to [18] the observables shoud be independent of the mass m.

integral corresponding to the diagram γ. The dependence on the coupling constant is a characteristic of super-renormalizable theories.

The equivalent expression

$$d\,(\gamma) = 4 - \sum_{\Phi} \left(d_\Phi + \frac{1}{2} \right) N_\Phi - L \ , \tag{3.4}$$

where L is the number of loops of the diagram, shows that only graphs up to two-loop order are divergent.

In order to apply known results on the quantum action principle [20, 14] to the present situation, one may consider g as an external field of dimension $\frac{1}{2}$. Including it in the summation under Φ, (3.3) gets the same form as in a strictly renormalizable theory:

$$d\,(\gamma) = 3 - \sum_{\tilde{\Phi}=\Phi,\, g} d_{\tilde{\Phi}} N_{\tilde{\Phi}} \ , \quad \text{with} \quad d_g = \frac{1}{2} \ . \tag{3.5}$$

Thus, including the dimension of g into the calculation, we may state that the dimension of the counterterms of the action is bounded by 3. But, since they are generated by loop graphs, they are of order 2 in g at least. This means that, not taking now into account the dimension of g, we can conclude that their real dimension is bounded by 2. The same holds true for the possible breakings of the Slavnov-Taylor identity.

The gauge condition (2.14), ghost equation (2.15), antighost equation (2.17) as well as rigid gauge invariance (2.18) can be easily shown to hold at all orders, i.e., they are not anomalous [14]. The validity to all orders of the Ward identities of diffeomorphisms and local Lorentz will be assumed in the following: the absence of anomalies for them has been proved in refs. [22, 23] for the class of manifolds we are considering here. Therefore, we shall be working in the space of diffeomorphism and local Lorentz invariant functionals.

It remains now to show the possibility of implementing the Slavnov-Taylor identity (2.7) for the vertex functional Γ. As it is well known [14], this amounts to study the cohomology of the nilpotent operator $\mathcal{B}_\Sigma$, defined by (2.8), in the space of the local integral functionals Δ of the various fields involved in the theory. The cohomology classes of $\mathcal{B}_\Sigma$ are defined such that Δ and $\Delta + \mathcal{B}_\Sigma \hat{\Delta}$ belong to the same equivalence class. The set of these classe is called the cohomology group $\mathcal{H}^p\left(\mathcal{B}_\Sigma\right) = \mathcal{Z}^p\left(\mathcal{B}_\Sigma\right)/\mathcal{Q}^p\left(\mathcal{B}_\Sigma\right)$; $\mathcal{Z}^p\left(\mathcal{B}_\Sigma\right)$ being the space of cocycles - the nontrivial part of the general solution - and $\mathcal{Q}^p\left(\mathcal{B}_\Sigma\right)$ being the space of coboundaries - BRS-variation - both of ghost number p. The cohomological group $\mathcal{H}^0\left(\mathcal{B}_\Sigma\right)$ constitute the invariants of the theory, and $\mathcal{H}^1\left(\mathcal{B}_\Sigma\right)$ the possible anomalies.

It has been proved in [5, 6], that there is no candidate to such an anomaly, i.e., $\mathcal{H}^1\left(\mathcal{B}_\Sigma\right) = \emptyset$. However, $\mathcal{H}^0\left(\mathcal{B}_\Sigma\right) \neq \emptyset$ and contains all invariant operators; which include $F^a_{\mu\nu} F^{\mu\nu}_a$ and the Chern-Simons term. But, since the counterterm is at least of order g^2, the power-counting theorem selects only the Chern-Simons term. We can just state that the radiative corrections can be reabsorbed through a redefinition of the topological mass only. This concludes the proof of the renormalizability of the theory: all functional identities hold without anomaly and the renormalizations might only affect the topological mass m.

4 Quantum Scale Invariance

Here, the argument is similar to the one presented in [15]. In order to get more deeply into the scaling properties of the present theory, we need a local form of the Callan-Symanzik equation. This will allow us to exploit the fact that the integrand of the Chern-Simons term in the action is not gauge invariant, although its integral is so. Such a local form of the Callan-Symanzik equation is provided by the "trace identity".

In order to derive the latter, let us first introduce the energy-momentum tensor, defined as the following tensorial quantum insertion obtained as the derivative of the vertex functional with respect to the dreibein:

$$\Theta_\nu{}^\mu \cdot \Gamma = e^{-1} e_\nu{}^a \, \frac{\delta \Gamma}{\delta e_\mu{}^a} \ . \tag{4.1}$$

¿From the diffeomorphism Ward identity (2.12), there follows the covariant conservation law of the energy-momentum tensor:

$$e \, \nabla_\mu \left[\Theta_\nu{}^\mu(x) \cdot \Gamma \right] = w_\nu(x) \, \Gamma + \nabla_\mu w_\nu{}^\mu(x) \, \Gamma \, , \tag{4.2}$$

where ∇_μ is the covariant derivative with respect to the diffeomorphisms[6], with the differential operators $w_\lambda(x)$ and $w_\lambda{}^\mu(x)$ acting on Σ representing contact terms:

$$w_\lambda(x) = \sum_{\text{all fields}} (\nabla_\lambda \Phi) \frac{\delta}{\delta \Phi} \ , \tag{4.3}$$

(becoming the translation Ward operator in the limit of flat space), and

$$w_\lambda{}^\mu(x) = A^{*\mu} \frac{\delta}{\delta A^{*\lambda}} + B^{*\mu} \frac{\delta}{\delta B^{*\lambda}} - A_\lambda \frac{\delta}{\delta A_\mu} - B_\lambda \frac{\delta}{\delta B_\mu} - \delta_\lambda^\mu \left[c^* \frac{\delta}{\delta c^*} + A^{*\alpha} \frac{\delta}{\delta A^{*\alpha}} + B^{*\alpha} \frac{\delta}{\delta B^{*\alpha}} \right] \tag{4.4}$$

The integral of the trace of the tensor $\Theta_\lambda{}^\mu$,

$$\int d^3x \, e \, \Theta_\mu{}^\mu = \int d^3x \, e_\mu{}^a \frac{\delta \Sigma}{\delta e_\mu{}^a} \equiv N_e \Sigma \ , \tag{4.5}$$

turns out to be an equation of motion – which means that $\Theta_\lambda{}^\mu$ is the improved energy-momentum tensor. This follows from the identity, which is easily checked by inspection of the classical action

$$N_e \Sigma = \left(\frac{1}{2} N_B - \frac{1}{2} N_{B^*} - \frac{1}{2} N_A + \frac{1}{2} N_{A^*} + \frac{3}{2} N_b + \frac{3}{2} N_{\bar c} - \frac{1}{2} N_c + \frac{1}{2} N_{c^*} \right) \Sigma + \left(m \partial_m + \frac{1}{2} g \partial_g \right) \Sigma \ , \tag{4.6}$$

[6] For a tensor T, such as, e.g., A_μ or $\delta / \delta A^{*\mu}$:

$$\nabla_\lambda T^{\mu \cdots}_{\nu \cdots} = \partial_\lambda T^{\mu \cdots}_{\nu \cdots} + \Gamma_\lambda{}^\mu{}_\rho T^{\rho \cdots}_{\nu \cdots} + \cdots - \Gamma_\lambda{}^\rho{}_\nu T^{\mu \cdots}_{\rho \cdots} - \cdots \, ,$$

where the $\Gamma_{\cdot}{}^{\cdot}{}_{\cdot}$'s are the Christoffel symbols corresponding to the connexion ω_μ^{mn}. The covariant derivative of a tensorial density $\mathcal{T}$, such as, e.g., $A^{*\mu}$ or $\delta / \delta A_\mu$, is related to that of the tensor $e^{-1} \mathcal{T}$ by:

$$\nabla_\lambda \mathcal{T}^{\mu \cdots}_{\nu \cdots} = e \, \nabla_\lambda \left(e^{-1} \mathcal{T}^{\mu \cdots}_{\nu \cdots} \right) \ .$$

where N_e is the counting operator of the dreibein field e^a_μ. It is interesting to note that (4.5) is nothing but the Ward identity for rigid Weyl symmetry [25] – broken by the mass terms and dimensionful couplings.

The trace $\Theta_\mu{}^\mu(x)\cdot\Gamma$ turns out to be vanishing, up to total derivatives, mass terms and dimensionful couplings, in the classical approximation, due to the field equations, which means that (4.1) is the improved energy-momentum tensor. It is easy to check that for the classical theory, the following equation holds

$$w(x)\Sigma \equiv \left(e^a_\mu(x)\frac{\delta}{\delta e^a_\mu(x)} - w^{\rm trace}(x)\right)\Sigma = \Lambda(x)\ , \tag{4.7}$$

or, equivalently:

$$e\Theta_\mu{}^\mu(x) = w^{\rm trace}(x)\Sigma + \Lambda(x)\ , \tag{4.8}$$

with

$$\begin{aligned} w^{\rm trace}(x) &= -\frac{1}{2}\left(A_\mu\frac{\delta}{\delta A_\mu} - A^{*\mu}\frac{\delta}{\delta A^{*\mu}} - B_\mu\frac{\delta}{\delta B_\mu} + B^{*\mu}\frac{\delta}{\delta B^{*\mu}} + c\frac{\delta}{\delta c} - c^*\frac{\delta}{\delta c^*}\right) + \\ &+ \frac{3}{2}\left(\bar c^a\frac{\delta}{\delta\bar c^a} + b^a\frac{\delta}{\delta b^a}\right)\ , \end{aligned} \tag{4.9}$$

and

$$\begin{aligned} \Lambda &= \frac{m}{2}\varepsilon^{\mu\nu\lambda}A^a_\mu F^a_{\nu\lambda} - \frac{g}{2}f_{abc}\left(\varepsilon^{\mu\nu\lambda}B^a_\mu A^b_\nu A^c_\lambda + \hat A^{*\mu\,a}A^b_\mu c^c - \frac{1}{2}c^{*\,a}c^b c^c + B^{*\mu\,a}B^b_\mu c^c\right) + \\ &+ \ \text{total derivative terms}\ , \end{aligned} \tag{4.10}$$

Λ being invariant under $\mathcal{B}_\Sigma$. The latter is the effect of the breaking of scale invariance due to the dimensionful couplings. The dimension of Λ – the dimensions of g and m not being taken into account – is lower than three: it is a soft breaking.

Let us now look for the quantum version of the trace identity (4.7) or (4.8). We first observe that the following commutation relations holds:

$$\begin{aligned} \left[\frac{\delta}{\delta b^a(y)}, w(x)\right] &= -\frac{3}{2}\delta(x-y)\frac{\delta}{\delta b^a(x)}\ , \\ \left[\mathcal{G}^a(y), w(x)\right] &= -\frac{3}{2}\delta(x-y)\,\mathcal{G}^a(x) + \frac{3}{2}\partial^x_\mu\delta(x-y)\left(eg^{\mu\nu}\frac{\delta}{\delta A^{*\nu}_a}\right)(y)\ , \\ \left[\bar{\mathcal{G}}^a, w(x)\right] &= \frac{1}{2}\frac{\delta}{\delta c^a(x)}\ . \end{aligned} \tag{4.11}$$

Now the relations (4.11) applied to the vertex functional Γ yield for insertion $w(x)\Gamma$ the properties

$$\frac{\delta}{\delta b_a(y)}w(x)\Gamma = -\frac{3}{2}\partial^x_\mu\delta(x-y)\left(e\,g^{\mu\nu}A^a_\nu\right)(y)\ ,$$

$$\mathcal{G}^a(y)w(x)\Gamma = \frac{3}{2}\partial^x_\mu\delta(x-y)\left(eg^{\mu\nu}\frac{\delta\Gamma}{\delta A^{*\nu}_a}\right)(y) , \qquad (4.12)$$
$$\bar{\mathcal{G}}^a w(x)\Gamma = \frac{1}{2}\frac{\delta\Gamma}{\delta c_a(x)} .$$

where we again use the fact that the constraints (2.14), (2.17) and (2.15) can be maintained at the quantum level.

The quantum version of (4.7) or (4.8) will be written as

$$w(x)\Gamma = \Lambda(x)\cdot\Gamma + \Delta(x)\cdot\Gamma , \qquad (4.13)$$

where $\Lambda(x)\cdot\Gamma$ is some quantum extension of the classical insertion (4.10), subjected to the same constraints (4.12) as $w(x)\Gamma$. It follows that the insertion $\Delta\cdot\Gamma$ defined by (4.13) obeys the homogeneous constraints

$$\frac{\delta}{\delta b_a(y)}\left[\Delta(x)\cdot\Gamma\right] = 0 , \quad \bar{\mathcal{G}}^a\left[\Delta(x)\cdot\Gamma\right] = 0 , \quad \mathcal{G}^a\left[\Delta(x)\cdot\Gamma\right] = 0 , \qquad (4.14)$$

beyond the conditions of invariance or covariance under $\mathcal{B}_\Gamma$, $\mathcal{W}_{\rm diff}$, $\mathcal{W}_{\rm Lorentz}$ and $\mathcal{W}_{\rm rigid}$.

By power-counting the insertion $\Delta\cdot\Gamma$ has dimension 3, but being an effect of the radiative corrections, it possess a factor g^2 at least, and thus its effective dimension is at most two – let us recall that we have attributed power-counting dimension 1/2 to g. It turns out that there is no insertion obeying all these constraints – the power-counting select the Chern-Simons Lagrangian density, but the latter is not BRS invariant. Hence $\Delta\cdot\Gamma = 0$: there is no radiative correction to the insertion $\Lambda\cdot\Gamma$ describing the breaking of scale invariance, and (4.13) becomes

$$e\,\Theta_\mu{}^\mu(x)\cdot\Gamma = w^{\rm trace}(x) + \Lambda(x)\cdot\Gamma . \qquad (4.15)$$

This local trace identity leads to a Callan-Symanzik equation (see e.g. Section 6 of [15]:

$$\left(m\partial_m + \frac{1}{2}g\partial_g\right)\cdot\Gamma = \int d^3x\ \Lambda(x)\cdot\Gamma , \qquad (4.16)$$

but now with no radiative effect at all: the β-functions associated to the parameters g and m both vanish, scale invariance remaining affected only by the soft breaking Λ.

We have thus shown that there is no renormalization at all: the BF-Yang-Millstheory in three dimensions is UV finite. Indeed the finiteness of the BF-Yang-Mills in 3 dimensions is a consequence of the cohomology of the theory which is exactly as in the standard Yang-Mills Chern-Simons theory.

– *Note*: The action (2.1) is named Gaussian formulation in [5]. Another formulation equivalent to the Gaussian one of the BF-Yang-Mills (see [5, 6, 7]) is obtained by introducing a pure gauge field η^a and replacing (2.1) by action

$$\Sigma_{\rm BF-Yang-Mills} = \int d^3x\ \left\{\varepsilon^{\mu\nu\alpha}B^a_\mu F^a_{\nu\alpha} + e\ \left(B^a_\mu - D_\mu\eta^a\right)\left(B^{a\mu} - D^\mu\eta^a\right)\right\} ,$$

which is named extended BF-Yang-Mills theory. This one is easily seen to be left invariant by standard gauge transformations (2.2). Moreover this action has the additional topological invariance

$$\delta_t A^a_\mu = 0 \ , \ \ \delta_t B^a_\mu = -D_\mu \Lambda^a \ , \ \ \delta_t \eta^a = \Lambda^a \ .$$

This fact is the starting point for interpretation of the BF-Yang-Mills theory as a deformation of the pure BF theory [2, 4, 8] . It has been showed in [5, 6] that both formulations possess the same perturbative quantum properties. This result implies that the extended formulation is finite too.

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