

ON KANT'S FIRST INSIGHT INTO THE PROBLEM OF DIMENSIONALITY AND ITS PHYSICAL FOUNDATIONS

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Abstract

In this article it is shown that a careful analysis of Kant's *Thoughts on the True Estimation of Living Forces* leads to the conclusion that — opposite to the usually accepted interpretation — Kant's reasoning, which supposedly establishes a relationship between the tridimensionality of space and Newton's law of universal gravitation, does not yield a satisfactory explanation of *space* dimensionality, actually restricting itself to justify the tridimensionality of *extension*.

Keywords: Space; Physical Space; Extension; Dimensionality; Kant; Newton.

I. Introduction

It was in 1747, during the flourishing period of the mechanist program, that Kant, in his first writing [1], sought to understand why space is tridimensional. It is largely accepted that his analysis establishes a clear framework for the discussion of space dimensionality as a problem in Physics, and represents its first physical solution [2-6]. Kant's contribution is generally summed up in the statement that *the reason for the tridimensionality of space can be found in Newton's law of gravitation, according to which the force between two bodies decays with the square of the distance separating them.*

However, it will be seen here that a more careful reading of Kant's *Thoughts about the True Evaluation of Living Forces* [1], leads us to conclude that — contrary to what is normally accepted — Kant's reasoning does not lead to a satisfactory understanding of the nature of space dimensionality, but limits itself to justify the tridimensionality of *extension* (*Ausdehnung*). In any case, it can be argued, as done by the authors [7], that the new approach conceived by Kant in his youth is related to the general causality concepts prevailing at the epoch.

Although its basic idea was abandoned during the critic period of Kantian philosophy, Kant's attempt to determine space dimensionality from a physical law is, unquestionably, a milestone in the modern discussion of dimensionality [6].

In spite of being part of a well known and discussed text, we feel that there are still some open questions which need to be clarified, namely:

- What are the basis of Kant's conjecture?
- Why does Kant ultimately limits himself, in his pre-critic phase, to explain the tridimensionality of *extension* rather than the one of *space*?
- Did the above fact have any repercussion in the concepts developed by him in the critic period?

In this essay Kant's first text is revisited, in an attempt to cast some light on aspects of these questions.

II. Kant and the Natural Philosophy of Space: between Newton and Leibniz

Probably under the influence of Knutzen, Kant developed, while at the university, a special interest in Physics and Mathematics. That interest evolved in such a way that his texts in the pre-critic period dealt essentially with Physics, Cosmology and the study of volcanoes. Two influences are felt in this phase: those from Newton and from Leibniz⁽¹⁾.

(1) Newton's influence over Kant actually transcends the pre-critic period, in the sense that Kant tried, along all his works, to build a *metaphysics as science*, in a way similar to the Newtonian system [8].

On one hand, the very title of Kant's first work, *Gedanken von der wahren Schätzung der lebendigen Kräfte* shows Leibniz's influence. On the other hand, Newtonian conceptions are the gist of his argument: the previous existence of substances — able to interact through forces — is crucial to the development of Kant's proof, as the title of the ninth paragraph of his text about *The Living Forces* [1] suggests. There he states his intention to discuss the tridimensionality of *space*:

[a.] “If the substances had no force whereby they can act outside themselves, there would be no extension, and consequently no space”⁽²⁾.

In fact, one of the first problem Kant was concerned to is related to the *corporeal matter* and to the interaction of physical substances. How to express this interaction in universal terms of cause and effect, and in which way is matter (the substance) “able to alter the state of the soul by means of the force it possess in its motion”, are issues about which he reflected [9]. Furthermore, he accepts Leibniz's idea that the bodies have something (*aliquid*) — an inherent, essential force —, prior even to extension itself:

“*In rebus corporeis esse aliquid praeter extensionem, imo extensione prius, alibi admonuimus*” [10].

It is clear that Kant admits here a relational space in a Leibnizian way, and not a Newtonian space conceived as a receptacle of bodies and phenomena.

In all this cognitive process, *forces* play a fundamental role. Kant's viewpoint reminds us the stoic idea, of great impact during the last three centuries b.C. [11], that there exists a force — which permeates everything —, due to the interaction of *pneuma* and ponderable matter, and that this force creates a well-ordered continuum, called *space*.

In Kant's opinion, it is through these forces that connections and relationships among bodies can be established, from which the necessary *order*⁽³⁾ to the existence of *space* is achieved. This can be seen from the following passage:

[b.] “It is easily proved that there would be no space and no extension, if substances had no force whereby they can act outside themselves. For without a force of this kind there is no connection, without this connection no order, and without this order no space” [12].

From this quotation we see that without this *force* there would be no *relation* between things, no *order* and no *space* (in this sequence).

As Handyside well remarks in his *Introduction* to the English translation cited in [1], in this phase “Kant considers the space as a *subsidiary phenomenon*⁽⁴⁾,

(2) The underlining is ours.

(3) A kind of astronomical order.

(4) The emphasis is ours.

which depends on the intelligible relations of these substances”. Such relations being expressed by *force laws*, it seems evident to us that, although Kant already accepts the core of the Newtonian scientific program [7], he (at least during this period) diverges from Newton in a crucial point of his system, namely the essence of space.

Absolute space and time are, according to Koyré,

“...réalités que Newton acceptait sans hésiter — puisqu’il pouvait les appuyer sur Dieu et les fonder en Dieu...” [13].

But for the young Kant, space *is not* the divine sensorium. On the contrary, it is a substance-dependent construction, able to express and emphasize particularly the role Reason plays. Naturally this is not the Cartesian identification of *space*, *quantity* and *corporeal substance* [14], for here *forces* cause *extension*. This pre-critic analysis puts Man (and not God) at the center of the discussion about space and its qualities. However, this does not mean that God is not an essential part of his argument. The role played by God is discussed in Ref. [15]. It is important to emphasize this point, since — as we shall see below — Kant’s justification of tridimensionality depends strongly on *this* concept of space, opposed to the one he will adopt in his critic period.

The ability with which Kant extracts from Newton the ideas of matter, gravitation *etc.*, and from Leibniz the ideas of relational space and *vis viva*, articulating them in his own argument, reveals the great originality that will characterize his thought.

III. Kant and the Tridimensionality

Although Kant announces in his first text the intention to discuss the tridimensionality of *space*, his conception of space (in the pre-critic phase) allows him to lay only the basis for the tridimensionality of the *extension*. Indeed, in the ninth paragraph, Kant states that

[c.] “*the ground of the threefold dimension of space is still unknown.*”

and, in the title of the next paragraph, he suggests a possible relation between the tridimensionality of space and the law of attraction between different bodies:

[d.] “*It is probable that the threefold dimension of space is due to the law according to which the forces in the substances act upon one another*” [16].

However, in throughout the text that follows the above quotation, corresponding to the very demonstration of the statement, Kant actually refers to the dimensionality of *extension* which, of course, has an ontological *status* completely different from that of *space*.

Kant’s reasoning, as seen in the previous section, encompasses the following points: first, the idea that there exists a force inherent to the substances (that is

to say, to the bodies), without which there would be no extension and no relation. Second, this force is necessary to establish the relations among the things, necessary to the order, and finally, without this order, space does not exist.

The fundamental role that concept of *force* — the first essence of matter and of its extension — plays in Kant’s explanatory system as is corroborated by the following quotation:

[e.] “*Since everything which is to be found among the qualities of a thing must be capable of being derived from that which contains in itself the most complete ground of the thing itself, the qualities of the extension, and subsequently their threefold dimension, will be grounded in the qualities of the force which the substances possess in respect of the things with which they are connected.*” [16].

From this sentence, independently of the subjacent *force* definition (Cartesian, Leibnizian, or Newtonian), it is clear that this is the force through which the substances act upon one another; the one which is responsible for the collective relations which will, in Kant’s view, define space. As to the nature of this *force*, Kant states that

[f.] “*The force, whereby a substance acts in union with others, cannot be thought apart from a determinate law which reveals itself in the mode of its action. Since the character of these laws according to which a whole collection of substances (that is, a space) is measured, in other words, the dimension of extension⁽⁵⁾, will likewise be due to the laws according to which the substances by means of their essential forces seek to unite themselves*”.

After these considerations Kant stresses that the law of forces to which he refers is Newton’s law of attraction which depends on the inverse of the square of the distance. At this point, however, he expresses himself with double caution, omitting whether the tridimensionality to which he refers is related to *space* or to *extension* and concluding that it *seems to result* from the form of Newton’s law of attraction, as the text below shows:

[g.] “*The threefold dimension seems to arise from the fact that substances in the existing world so act upon one another that the strenght of the action holds inversely as the square of the distances*” [16].

It must be noticed that, except for the title, in no part of this tenth paragraph does Kant explicitly use the word *space* when referring to the tridimensionality, alluding to it only three times. The first and second, quoted above, only reinforce the idea of *space* defined from that of *physical substance*. The third, which also seems relevant to the theme treated here, is when Kant concludes his speculations

⁽⁵⁾ The underlining is ours.

by referring to *space*, that is, to the various types of spaces, as objects of study of Geometry:

[h.] “*A science of all these possible kinds of space would undoubtedly be the highest enterprise which a finite understanding could undertake in the field of geometry.*” [17].

This fact can be considered as an indication of the degree to which Kant does not share Galilean ideas about the geometrization of Physics. His causal explanatory system is essentially Newtonian [7], and his entire argumentation is built upon the laws of force. Even though his result refers to the dimensions of *extension*, Kant had to consider the possibility of the existence of spaces with a different number of dimensions, before any formal theory for these types of space. It will be the nineteenth century discovery of non-Euclidean geometries that will give impulse to the discussion of these issues [2]. It seems to us that not only was Kant conscious, already in 1747, that the road to the comprehension of the dimensionality of space should involve both Physics and Mathematics [18], but, most importantly, that he set the basis for modern discussions of this fascinating theme.

Certainly the epistemologically most important fact in the contribution of the young Kant to this theme is the rupture with the Aristotelian view of the issue — both in its general realm (the cause of the space) and in its particular aspect (the cause of dimensionality) —, through the introduction of *force* as the *causa efficiens* of space, through the concept of *order*. Although in a certain sense he is Aristotelic, considering the role played by the concept of *substance* used in his discussion of the dimensionality of space, it should be noticed that, contrary to Aristotle — in whose system *force* (*dynamis*) leads to the rupture of cosmic order — Kant considers, in his first text, *force* as a generator of *order*.

IV. Final Considerations and Conclusions

Therefore, we conclude that Kant actually proposes a justification for the tridimensionality of *extension* and not of *space*, since he considers the latter as non-perceptible, as the product of an intellectual effort to seek to establish a kind of *order* from the intelligible things. This *space* appears as the object of study of Geometry and not of Physics⁽⁶⁾. What is perceptible, what really impresses the soul are the spatially extense objects, the *matter* which causes effects on other *substances*:

⁽⁶⁾ It was Riemann — a mathematician — who first contributed to join what Kant had separated. By speculating that matter could determine the metric structure of space, Riemann anticipated a certain correlation between the *substance* on one hand and *physical* and *geometrical spaces*, on the other. This idea, which, according to Jammer, did not resonate among the majority of the physicists and mathematicians in Riemann’s time, was largely discussed by Einstein in his theory of relativity.

[i.] “...matter, by means of the force which it has in its motion, changes that state of the soul whereby the soul represents the world to itself.”[9].

It is possible to extract intelligible relations from substances starting from causal and universal laws of force like, particularly, Newton’s law⁽⁷⁾.

From the physical point of view a deeper comprehension of Kant’s conjecture can only be reached by means of the field concept in Physics [7]. It is through the solution of Laplace–Poisson equation in n -dimensional Euclidean space that the relation between the exponent of the Newtonian potential and the dimensionality of space is established. But it is only in the context of contemporary unified field theories that the problem of space dimensionality achieves the status of a central problem in Physics: from this point of view one can realize the full meaning of Kant’s contribution.

As far as we know — with the concordance of Brittan [3] — there is no other attempt by Kant aimed at giving a physical basis to the dimensionality issue. It is known that Kant returned to this question, as certified by the manuscripts collected in the *Opus Postumum* [20], but, ironically enough, there is an interruption in a key part of the text which makes it impossible for us to discover how the mature Kant would return to the dimensionality problem from the point of view of Physics. We will therefore conclude this essay with this reticent quotation by Kant:

“The quality of space and time, for instance, that the first has 3 dimensions, and the second only one, that the revolution is ruled by the square of the distances are principles that...[interruption]” [20].

Acknowledgments

We would like to thank our colleagues Martha Cristina and Helio da Motta for reading and commenting the first draft of the manuscript. F.C. is in debt with the CNPq of Brazil for financial support.

⁽⁷⁾ Such a causal relation was elaborated later by Überweg [19].

Appendix

This appendix includes the original German text from Kant's *Werke* [1], corresponding to the aforementioned quotations.

- [a.] *“Wenn die Substanzen keine Kraft hätten, außer sich zu wirken, so würde keine Ausdehnung, auch kein Raum sein”.*
- [b.] *“Es ist leicht zu erweisen, daß kein Raum und keine Ausdehnung sein würden, wenn die Substanzen keine Kraft hätten, außer sich zu wirken. Denn ohne diese Kraft ist keine Verbindung, ohne diese keine Ordnung, und ohne diese endlich kein Raum.”.*
- [c.] *“Der Grund von der dreifachen Dimension des Raumes ist noch unbekannt.”.*
- [d.] *“Es ist wahrscheinlich, daß die dreifache Abmessung des Raumes von dem Gesetze herrühre, nach welchem die Kräfte derer Substanzen in einander wirken”.*
- [e.] *“Weil alles, was unter den Eigenschaften eines Dinges vorkömmt, von demjenigen muß hergeleitet werden können, was den vollständigen Grund von dem Dinge selber in sich enthält, so werden sich auch die Eigenschaften der Ausdehnung, mithin auch die dreifache Abmessung derselben, auf die Eigenschaften der Kraft gründen, welche die Substanzen, in Absicht auf die Dinge, mit denen sie verbunden sind, besitzen.”.*
- [f.] *“Die Kraft, womit eine Substanz in der Vereinigung mit andern wirkt, kann nicht ohne ein gewisses Gesetze gedacht werden, welches sich in der Art seiner Wirkung hervortut. Weil die Art des Gesetzes, [nach | welchem die Substanzen in einander wirken, auch die Art der Vereinigung und Zusammensetzung vieler derselben bestimmen muß, so wird das Gesetz], nach welchem eine ganze Sammlung Substanzen (das ist ein Raum) abgemessen wird, oder die Dimension der Ausdehnung, von den Gesetzen herrühren, nach welchen die Substanzen vermöge ihrer wesentlichen Kräfte sich zu vereinigen suchen.”.*
- [g.] *“Die dreifache Abmessung scheint daher zu rühren, weil die Substanzen in der existierenden Welt so in einander wirken, daß die Stärke der Wirkung sich wie das Quadrat der Weiten umgekehrt verhält”.*
- [h.] *“Eine Wissenschaft von allen diesen möglichen Raumes-Arten wäre ohnfehlbar die höchste Geometrie, die ein endlicher Verstand unternehmen könnte.”.*
- [i.] *“... dahero ändert die Materie, mittelst ihrer Kraft, die sie in der Bewegung hat, den Zustand der Seele, wodurch sie sich die Welt vorstellt.”.*

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