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The Velocity of Gravitational Waves

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Abstract

In this paper we discuss the propagation of gravitational waves in two theories: general relativity and NDL. We examine the propagation of the gravitational waves on a solvable case corresponding to a spherically symmetric static configuration. We show that in NDL theory the velocity of gravitational waves is lower than light velocity, a result that is contrary to the predictions of general relativity. We point out some consequencies of this result and a possible scenario for its verification.

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1 Introduction

Recently we have exploited some consequences of a field theoretical description of gravity adding a new ingredient: we do not require the extrapolation to the gravitational energy from the hypothesis of universality of the equivalence principle (EP). This means that gravity does not couple to itself as all others forms of energy. Although not identical, this NDL theory of gravity contains many of the ingredients of GR and is competitive with general relativity, as far as standard post-Newtonian tests and binary quadrupole emission are concerned (see ref. [1]).

The main lines of NDL approach can be synthesized in the following statements:

- Gravity is described by a symmetric second order tensor $\varphi_{\mu\nu}$ that satisfies a non-linear equation of motion;
- Matter couples to gravity in an universal way. In this interaction, the gravitational field appears only in the combination $\gamma_{\mu\nu} + \varphi_{\mu\nu}$, inducing us to define a quantity $g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}$. Such tensor $g_{\mu\nu}$ acts as an effective metric tensor of the spacetime as seen by matter or energy of any form except gravitational energy;
- The self interaction of the gravitational field break the universal modification of the spacetime geometry.

In this vein, there is a natural and direct way to test such NDL theory by the analysis of the gravitational waves. We start to undertake such task in this present paper.

We use units of light velocity ($c = 1$).

2 A Short Review of the NDL Theory of Gravity

2.1 General Features

In a previous paper [1] we have presented a modification of the standard Feynman-Deser approach of field theoretical derivation of Einstein's general relativity, arriving at a competitive gravitational theory . We shown that it is possible to obtain a theory that incorporates a great part of general relativity and can be interpreted in the standard geometrical way like GR, as far as the interaction of matter to gravity is concerned. The most important distinction of the new theory concerns the gravity to gravity interaction. This theory satisfies all standard tests of gravity and lead to new predictions about the propagation of gravitational waves. Since there is a large expectation that the detection of gravitational waves will occur in the near future, the question of which theory describes nature better will probably be settled soon.

We define a three-index tensor $F_{\alpha\beta\mu}$, which we will call the **gravitational field**, in terms of the symmetric standard variable $\varphi_{\mu\nu}$ (which will be treated as the potential) to describe spin-two field, by the expression[1]

$$F_{\alpha\beta\mu} = \frac{1}{2}(\varphi_{\mu[\alpha;\beta]} + F_{[\alpha}\gamma_{\beta]\mu}) \tag{1}$$

where F_α is the trace

$$F_\alpha = F_{\alpha\mu\nu}\gamma^{\mu\nu} = \varphi_{,\alpha} - \varphi_{\alpha\mu;\nu}\gamma^{\mu\nu}. \tag{2}$$

From the field variables we can construct the invariants:

$$\begin{aligned} A &\equiv F_{\alpha\mu\nu}F^{\alpha\mu\nu}, \\ B &\equiv F_\mu F^\mu. \end{aligned} \tag{3}$$

The Lagrangian density for the gravitational field is taken to be given by:

$$L = \frac{b^2}{\kappa}\left\{\sqrt{1 - \frac{U}{b^2}} - 1\right\}, \tag{4}$$

where b is a constant and U is defined by

$$U \equiv A - B. \tag{5}$$

The gravitational action is expressed as:

$$S = \int \mathrm{d}^4x\sqrt{-\gamma}\, L, \tag{6}$$

where γ is the determinant of the Minkowskian spacetime metric $\gamma_{\mu\nu}$ written in an arbitrary coordinate system. From the Hamilton principle we find the following equation of motion in the absence of material sources:

$$\left[L_U F^{\lambda(\mu\nu)}\right]_{;\lambda} = 0. \tag{7}$$

L_U represents the derivative of the Lagrangian with respect to the invariant U.

We follow the standard procedure [2, 3] to define an effective Riemannian metric tensor in terms of the potential $\varphi_{\alpha\beta}$, by the expression

$$g_{\mu\nu} = \gamma_{\mu\nu} + \varphi_{\mu\nu}. \tag{8}$$

This relation has a deep meaning, once for all forms of non-gravitational energy the net effect of the gravitational field is felt precisely as if gravity was nothing but a consequence of changing the metrical properties of the spacetime from the flatness structure to a curved one. The definition of the associated metric tensor is provided precisely by the above expression. This means that any material body (including photons) follows along geodesics as if the metric tensor of spacetime was given by the above expression.

[1]We are using the anti-symmetrization symbol like $[x, y] \equiv xy - yx$ and the symmetrization symbol $(x, y) \equiv xy + yx$. Note that indices are raised and lowered by the background metric $\gamma_{\mu\nu}$. The covariant derivative is denoted by a semicomma ';' and it is constructed with this geometry.

2.2 The Wave Propagation

Let us review briefly the result set up in [1] for the velocity of the gravitational wave in NDL theory. We represent by the symbol $[J]_\Sigma$ the discontinuity of the function J through the surface Σ. Following Hadamard [4] we impose the discontinuity conditions for the field:

$$[F_{\mu\nu\alpha}]_\Sigma = 0 \tag{9}$$

and

$$[F_{\mu\nu\alpha;\lambda}]_\Sigma = f_{\mu\nu\alpha} k_\lambda, \tag{10}$$

where k_α represents the normal vector to the surface of discontinuity Σ. The coeficientes $f_{\alpha\beta\gamma}$ has the same symmetries of the field $F_{\alpha\beta\gamma}$. Taking the discontinuity of the equation of motion (7) we obtain:

$$f^\mu{}_{(\alpha\beta)} k_\mu + 2\frac{L_{UU}}{L_U}(\eta - \zeta) F^\mu{}_{(\alpha\beta)} k_\mu = 0 \tag{11}$$

in which we defined $\eta \equiv F_{\alpha\beta\mu} f^{\alpha\beta\mu}$ and $\zeta \equiv F_\mu f^\mu$. After some algebraic manipulations results:

$$k_\mu k_\nu [\gamma^{\mu\nu} + \Lambda^{\mu\nu}] = 0, \tag{12}$$

in which the quantity $\Lambda^{\mu\nu}$ is written in terms of the gravitational field:

$$\Lambda^{\mu\nu} \equiv 2\frac{L_{UU}}{L_U}[F^{\mu\alpha\beta} F^\nu{}_{(\alpha\beta)} - F^\mu F^\nu]. \tag{13}$$

Note that the discontinuities of the gravitational fields propagate in a modified geometry, changing the background geometry $\gamma^{\mu\nu}$, into an effective one $g^{\mu\nu}_{\rm eff}$,

$$g^{\mu\nu}_{\rm eff} \equiv \gamma^{\mu\nu} + \Lambda^{\mu\nu} \tag{14}$$

which depends of the distribution of the field $F_{\alpha\beta\mu}$ and also of the dynamics. This fact shows that such a property stems from the structural form of the Lagrangian. Thus, in the NDL theory the characteristic surfaces of the gravitational waves propagate on the null cone of an effective geometry. We remark that this geometry is distinct of that observed by all other forms of energy and matter, which differs from the general relativity predictions. This result gives a possibility to choose between these two theories just by observations of the gravitational waves. In the next section we will present a synthesis of the main properties of the solutions of the gravitational field in NDL theory. Then we will evaluate the velocity of the gravitational wave in such background and compare with the GR result.

2.3 The Static Spherically Symmetric Solution

We set for the auxiliary metric[2] of the background the form

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{15}$$

[2]We note that this metric is non-observable by any form of energy.

This means that all operations of raising and lowering indices are made by this Minkowski metric $\gamma_{\mu\nu}$. We remark the fact that matter[3] feels a modified geometry given by Eq. (8).

The static spherically symmetric solution of NDL theory has only two non-null gravitational components of $\varphi^{\mu\nu}$:

$$\varphi_{00} = \varphi^{00} = \mu(r) \tag{16}$$
$$\varphi_{11} = \varphi^{11} = -\nu(r). \tag{17}$$

Consequently the gravitational field $F_{\alpha\beta\mu}$ reduces to the non-null quantities:

$$F_{100} = -\frac{\nu}{r} \tag{18}$$
$$F_{122} = \frac{F_{133}}{\sin^2\theta} = \frac{1}{2}\nu r - \frac{1}{2}\mu' r^2 \tag{19}$$
$$\tag{20}$$

in which a prime $'$ symbolizes the derivative with respect to the radial variable r. The unique component of the trace that remains is F_1,

$$F_1 = \mu' - 2\frac{\nu}{r}. \tag{21}$$

From these we can evaluate the invariant U:

$$U = \frac{\nu^2}{r^2} - \frac{2\nu\mu'}{r}. \tag{22}$$

From the equations of motion (7) and under the hypothesis of symmetry of the solution we obtain

$$\nu = \frac{2M}{r}\left\{1 - (\frac{r_c}{r})^4\right\}^{-\frac{1}{2}} \tag{23}$$
$$\mu = \frac{1}{2}\sqrt{bM}\left\{F(\alpha, \sqrt{2}/2) + \mu_0\right\} \tag{24}$$

in which the constant r_c appearing in Eq. (23) is given by

$$r_c{}^2 \equiv \frac{2M}{b}. \tag{25}$$

In Eq. (24) $F(\alpha, \sqrt{2}/2)$ is the elliptic function and the constant μ_0 must be chosen to yield the correct assymptotic limit. The quantity α is given by

$$\alpha \equiv \arcsin\left[1 - \left(\frac{r_c}{r}\right)^2\right]^{\frac{1}{2}}. \tag{26}$$

[3]Massive or massless particles — photons, for instance — that is, any form of non-gravitational energy

3 Electromagnetic and Gravitational Waves in the Spherically Symmetric Solution

From what we have seen, in the NDL theory all kind of matter and non-gravitational energy couple to gravity differently than gravity to gravity interaction. In order to compare the propagation of electromagnetic and gravitational waves we will proceed as follows.

The metric that defines the structure of the spacetime in which the discontinuities of the electromagnetic field propagate, is provided by Eq. (8). We define a 4-vector l_μ such that

$$l_\mu l_\nu g^{\mu\nu} = 0 \tag{27}$$

where $g^{\mu\nu}$ is the inverse of the metric $g_{\mu\nu}$, defined by

$$g^{\mu\rho} g_{\mu\sigma} = \delta^\rho{}_\sigma. \tag{28}$$

Analogous to the section (2.2), this 4-vector is a gradient of the hypersurface of discontinuities. The background gravitational field we are considering here, corresponds to a spherically symmetric and static configuration. Thus, the relation (27) reduces in this case to

$$\frac{(l_0)^2}{1+\mu} - \frac{(l_1)^2}{1+\nu} - \frac{(l_2)^2}{r^2} - \frac{(l_3)^2}{r^2\sin^2\theta} = 0. \tag{29}$$

Gravitational waves propagate as null geodesic in an effective geometry $g^{\mu\nu}_{\rm eff}$ given by Eq. (14). The quantity $\Lambda^{\mu\nu}$ is defined by Eq. (13) and have only two non null components:

$$\Lambda^{00} = -\frac{4M^2}{b^2 r^4} \tag{30}$$

$$\Lambda^{11} = -\Lambda^{00}. \tag{31}$$

Correspondingly, the effective metric is provided by

$$g^{00}_{\rm eff} = 1 - \frac{4M^2}{b^2 r^4} \tag{32}$$

$$g^{11}_{\rm eff} = -1 + \frac{4M^2}{b^2 r^4} \tag{33}$$

$$g^{22}_{\rm eff} = -\frac{1}{r^2} \tag{34}$$

$$g^{33}_{\rm eff} = -\frac{1}{r^2\sin^2\theta} \tag{35}$$

Inserting these results in Eq. (12) the equation of propagation of gravitational waves became:

$$\left(1 - \frac{4M^2}{b^2 r^4}\right)(k_0)^2 - \left(1 - \frac{4M^2}{b^2 r^4}\right)(k_1)^2 - \frac{(k_2)^2}{r^2} - \frac{(k_3)^2}{r^2\sin^2\theta} = 0. \tag{36}$$

We can summarize this situation as:

- Electromagnetic waves propagate in null cone of the geometry $g_{\mu\nu}$;

- Gravitational waves propagate in null cone of the geometry $g^{\text{eff}}_{\mu\nu}$.

A question then appears: *Which wave propagates faster?* In order to investigate this problem we can run by two equivalent ways: one can either evaluate the norm of the vector l_μ in the geometry $g^{\text{eff}}_{\mu\nu}$:

$$l_\mu l_\nu g^{\mu\nu}_{\text{eff}} \tag{37}$$

or else evaluate the norm of the vector k_μ in the geometry $g_{\mu\nu}$:

$$k_\mu k_\nu g^{\mu\nu}. \tag{38}$$

In this vein we evaluate the quantity (37) and investigate the character of l_μ in the geometry $g^{\mu\nu}_{\text{eff}}$. For the solution we are considering here, results:

$$||l||_{g_{\text{eff}}} = \left(1 - \frac{4M^2}{b^2 r^4}\right)(l_0)^2 - \left(1 - \frac{4M^2}{b^2 r^4}\right)(l_1)^2 - \frac{(l_2)^2}{r^2} - \frac{(l_3)^2}{r^2 \sin^2\theta} \tag{39}$$

Substituting expression (29) in the last two terms in the right hand side of the above equation, we obtain

$$||l||_{g_{\text{eff}}} = \left(1 - \frac{4M^2}{b^2 r^4} - \frac{1}{1+\mu}\right)(l_0)^2 - \left(1 - \frac{4M^2}{b^2 r^4} - \frac{1}{1+\nu}\right)(l_1)^2 . \tag{40}$$

The solutions of the functions μ and ν are given by (24) and (23). Since we are interested here just on the sign of the norm of the 4-vector l_μ in the geometry determined by $g^{\text{eff}}_{\mu\nu}$, it is enough to consider only the main terms. Thus, expanding the field solutions as:

$$\frac{1}{1+\mu} \approx 1 + \frac{2M}{r} + O\left(r^{-2}\right) \tag{41}$$

$$\frac{1}{1+\nu} \approx 1 - \frac{2M}{r} + O\left(r^{-2}\right), \tag{42}$$

and substituting in Eq. (40) results:

$$||l||_{g_{\text{eff}}} = -\frac{2M}{r}(l_0)^2 - \frac{2M}{r}(l_1)^2 . \tag{43}$$

Hence, $||l||_{g_{\text{eff}}} < 0$. We thus conclude that l_μ is a space-like vector in the geometry $g^{\text{eff}}_{\mu\nu}$, i.e., in the NDL theory gravitational waves travel with velocity lower than light.

4 Conclusion

The NDL theory forecast that gravitational waves travel slower than light has some interesting consequences. The first one is the possibility to exist gravitational Chêrenkov Radiation, that is the emission of gravitational radiation when a massive particle exceeds locally the "graviton" speed [5]. Evidently, this phenomenom will put limits to how far from their sources cosmic rays can be found with ultra high energies, such as the ones searched by the AUGER Project (E $\geq 10^{20}$eV) [6]. How restringent these limits are will be addressed in future work.

One question naturally stands up: can we obtain observational evidence that gravitational waves travel at speeds below the light velocity in the presence of gravitational potentials? Very high gravitational potentials can be found in the vicinities of neutron stars, black holes and supernova cores. However, unless we are talking about very massive black holes, all these gravitational sources are not sufficiently extended in size to allow the integration of a large enough difference in the arriving times at Earth for photons and gravitons supposely generated at the same place and instant. On the other hand, black holes of 10^6 to 10^9 solar masses, as it is believed to exist at the center of galaxies such as M87, M51 and others [7], present much better conditions to delay the gravitational waves relatively to the electromagnetic waves. After all, the Schwarszchild radius of a 10^9 solar mass black hole has the size of two Astronomical Units, a distance that light takes about 1000 seconds to cross if traveling outside the horizon. This may give us enough time for the potential to act on the gravitational waves, slowing them down for sufficient time, in order to accumulate a time delay possible to be measured with the technology of the next generations of gravitational wave observatories and electromagnetic telescopes in the few decades to come.

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