Magnetic Collapse of a Neutron Gas: No Magnetar Formation

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A degenerate neutron gas in equilibrium with a background of electrons and protons in a magnetic field exerts its pressure anisotropically, having a smaller value perpendicular than along the magnetic field. For critical fields the magnetic pressure may produce the vanishing of the equatorial pressure of the neutron gas, and the outcome could be a transverse collapse of the star. This fixes a limit to the fields to be observable in stable pulsars as a function of their density. The final structure left over after the implosion might be a mixed phase of nucleons and meson ($\pi^{\pm,0}, \kappa^{\pm,0}$) condensate (a strange star also likely) or a black string, but no magnetar at all.

Key-words: Nuclear matter aspects of neutron stars; Magnetic fields; Pulsars; Neutron stars.

Gravitational collapse occurs in a body of mass M and radius R when its rest energy is of the same order of its gravitational energy, $Mc^2 \sim GM^2/R$. We would like to argue that for a macroscopic magnetized body, e.g., composed by neutrons in an external field B, new physics appears and a sort of collapse occurs, when its internal energy density U is of the same order than its magnetic energy density $\mathcal{M}B$, where $\mathcal{M}$ is the magnetization. This problem is interesting in the context of astrophysical and cosmological objects, as neutron stars. Extremely magnetized neutron stars, or magnetars [1], have been attracting the attention of reseachers in the last years [2]. Their magnetic fields are estimated to be of order of 10^{15} G [2]. For fields of this order of magnitude, there are values of the density for which the magnetic energy of the system becomes of the same order than its total energy. For these physical conditions, superdense matter composed of neutral particles having a magnetic moment may undergo a transverse collapse since its pressure perpendicular to B vanishes. This implosion is driven by the same mechanism described in [4] for charged particles. As discussed below, the resulting object may be a hybrid or a strange star (depending on the macroscopic properties of the imploding neutron star) but no any magnetar.

In the present paper (a preliminar report of which was presented in [5]) we will approach this problem by starting from a model of a relativistic degenerate neutron gas, pervaded by an extremely strong magnetic field, in equilibrium with a background of electrons and protons. The last ones needed to prevent neutron beta decay $n \to p + e + \bar{\nu}$ through Pauli's principle. This configuration is maintained in approximate equilibrium through the equation $\mu_n = \mu_p + \mu_e + \mu_{\bar{\nu}}$ among their chemical potentials. However, as there is a flux of neutrinos escaping from the star its chemical potential would be taken as zero.

The Dirac equation for a charged particle having an anomalous magnetic moment and placed in a strong magnetic field was first derived by Pauli [7]. The free particle spectrum was obtained by Ternov *et. al* [8]. For neutral particles, it was presented first by Lostukov and Leventuiev, and reported by Klimenko [9]. We get for neutrons the eigenvalues,

$$E_n(p, B, \eta) = \sqrt{p_3^2 + (\sqrt{p_\perp^2 + m_n^2} + \eta q B)^2} \tag{1}$$

where p_3, $p_\perp$ are respectively the momentum components along and perpendicular to the external field $\mathbf{B}$, $q = 1.91 M_n$, where

M_n is the nuclear magneton, $\eta = 1, -1$ are the σ_3 eigenvalues corresponding to the two orientations of the magnetic moment (parallel and antiparallel) with regard to the field B. The expression (1) shows manifestly the change of spherical to axial symmetry with regard to momentum components. In the non-relativistic limit and for strong fields, this is reflected as small corrections to the spectrum as $E(p, B) = p^2/2m_n + \eta q B(1 + p_\perp^2/2m_n^2) + q^2B^2/2m_n$, valid for any neutral Fermi gas with zero electric dipole moment and nonzero magnetic moment.

The partition function $\mathcal{Z}$ which is obtained from the density matrix describing the model, leads to a thermodynamic potential $\Omega = -T \ln \mathcal{Z}$ involving the contributions from the species involved, which are considered to be in chemical equilibrium among themselves.

Having an equation relating the chemical potentials, and demanding conservation of both baryonic number $N_n + N_p = N_B$ and electric charge, $N_p + N_e = 0$, one may think to solve exactly the problem in terms of the external field as a parameter. In the present paper, however, we focus our discussion on the properties of the equation of state.* As it is known, the denser the Fermi gas, the better the ideal gas approximation [10]. This property is still valid in presence of external fields. We thus proceed to calculate the ideal gas thermodynamical quantities, and begin with the neutron thermodynamic potential.

For the one-loop thermodynamical potential, Ω_n, we can write the expression

$$\Omega_n = -\Omega_0 \sum_{\eta=1,-1} [\frac{xf(x,\eta y)^3}{12} + \frac{(1+\eta y)(5\eta y - 3)xf(x,\eta y)}{24}$$

$$- \frac{(1+\eta y)^3(3-\eta y)}{24} L(x,\eta y) - \frac{\eta y x^3}{6} s(x,\eta y)], \qquad (2)$$

Above we have defined the variables $x = \mu_n/m_n$ and $y = qB/m_n$, and the functions

$$f(x,\eta y) = \sqrt{x^2 - (1+\eta y)^2},$$
$$s(x,\eta y) = (\pi/2 - \sin^{-1}(1+\eta y)/x,$$
$$L(x,\eta y) = \ln(x + f(x,\eta y))/(1+\eta y). \qquad (3)$$

For the density, defined as $N_n = -\partial\Omega_n/\partial\mu_n$, we can write

$$N_n = N_0 \sum_{\eta=1,-1} [\frac{f(x,\eta y)^3}{3} + \frac{\eta y(1+\eta y)f(x,\eta y)}{2}$$

$$- \frac{\eta y x^2}{2} s(x,\eta y)], \qquad (4)$$

while for the magnetization, given by $\mathcal{M}_n = -\partial\Omega_n/\partial B$, we get

$$\mathcal{M}_n = -\mathcal{M}_0 \sum_{\eta=1,-1} \eta[\frac{(1-2\eta y)xf(x,\eta y)}{6}$$

$$- \frac{(1+\eta y)^2(1-\eta y/2)}{3} L(x,\eta y) + \frac{x^3}{6} s(x,\eta y)], \quad (5)$$

where $N_0 = M_n^3/4\pi^2 \sim 2.04\times10^{39}$, $\Omega_0 = N_0 M_n \sim 3.0\times10^{36}$, and $\mathcal{M}_0 = M_n^2 e\mu/4\pi^2 \sim 2.92\times10^{16}$ and one can write $\mathcal{M}_n = \mathcal{M}_n^+(\eta=-1) - \mathcal{M}_n^+(\eta=+1)$, and obviously, $\mathcal{M}_n \geq 0$.

If we include the anomalous magnetic moment for electrons, one can give a common formula for the spectrum of electrons and protons in the external field B as [8]: $E = \sqrt{p_3^2 + (\sqrt{2eBn + m_{e,p}^2} + \eta q_{e,p} B)^2}$, where $q_e = \alpha e/8\pi m_e$, $q_p \sim 2.79 M_n$. For neutrons, the critical field at which the coupling energy of the anomalous magnetic moment equals the rest energy is $B_{cn} = 1.57\times10^{20}$. For protons $B_{cp} = 2.29\times10^{20}$G, while for electrons it is of order $B \sim e/r_0^2 \sim 10^{-16}$G, being r_0 the classical electron radius. However, these estimates must change if corrections depending on the field are made to the anomalous magnetic moment. At these critical fields vacuum becomes unstable under particle-antiparticle pair production. Such decays are, however, suppressed in the dense gases by Pauli's principle.

By defining $x_p = \mu_p/m_p$, $y_p = q_p/m_p$, $b = 2e/m_p^2$, $q_p = 2.79M_n$, then $y_p = 2.79e/2m_p^2$. We name also

$$g(x_p, B, n) = \sqrt{x_p^2 - h(B,n)^2}, \text{ and}$$
$$h(B,n) = (\sqrt{bBn+1} + \eta y_p B), \qquad (6)$$

to have for the star's proton thermodynamical potential

$$\Omega_p = -\frac{eBm_p^2}{4\pi^2} \sum_n \sum_{\pm\eta} [x_p g(x_p,B,n) - h(B,n)^2$$

$$\times \ln\frac{x_p + g(x_p,B,n)}{h(B,n)}, \qquad (7)$$

and for its density

$$N_p = \frac{eBm_p}{2\pi^2} \sum_n \sum_{\pm\eta} g(x_p,B,n). \qquad (8)$$

The magnetization is given by

$$\mathcal{M}_p = \frac{em_p^2}{4\pi^2} \sum_n \sum_{\pm\eta} [x_p g(x_p,B,n) - [h(B,n)^2 + (\eta y_p$$

$$+ (bn/2\sqrt{bBn+1}))] \ln\frac{x_p + \sqrt{x_p^2 - g(x_p,B,n)}}{h(B,n)}, \quad (9)$$

where the coefficents of these formulae are $N_0 = em_pB/2\pi^2 \sim 4.06\times10^{19}B$, $\Omega_0 = N_0 m_p B \sim 6.1\times10^{16}B$, and $\mathcal{M}_0 = N_0 m_p = \Omega_0/B$.

For low magnetic fields n_{max} is very large, and one can approximate the sum over Landau quantum numbers by an integral. This would lead to a neutron to proton ratio $N_n/N_p \geq 8$, similarly as the zero magnetic field case [6]. The maximum occupied Landau quantum number n may be given as $n_{max} = (x_p - \eta y_p B)^2 - 1/bB$.

For $B \ll B_{cp}$, so that $y_p B \ll 1$, and $x_p \geq 1$, one can take approximately $n_{max} \sim (x_p^2 - 1)/bB$, and for fields large enough $n_{max} = 0$. As $x_p \sim x_n$, the proton density decreases with increasing B, favoring the inverse beta decay. In the extreme case of confinement to the Landau ground state we can estimate the bound $N_p = N_e \leq$

*In concluding this paper, the authors got awared of a recent paper [15] on neutron gas in a magnetic field, containing expressions similar to ours for the spectra and densities of neutrons and protons (already reported in [5]), but different equations of state. The behavior of proton-neutron fraction and pion condensation are studied in detail in that paper.

$2.5 \times 10^9 B N_n^{1/3}$. For this limit it is roughly $\Omega_n/\Omega_p \geq 4 \times 10^{-10} N^{2/3}/B$. For fields $B \sim m_p/q_p$ and $x_p \gg 1$, $n_{max} \geq 1$, and thus large Landau numbers are again occupied. However, the dominant longitudinal pressure and magnetization comes from the neutron gas.

The lost of rotational symmetry of the particle spectrum determines an anisotropy in the thermodynamic properties manifesting in different equations of state for directions parallel and perpendicular to the external field, as is seen from the energy-momentum tensor in the constant magnetic field $\mathcal{T}_{\mu\nu}$ [11], [4]. Its spatial components $\mathcal{T}_{11} = \mathcal{T}_{22} = P_\perp$ and $\mathcal{T}_{33} = P_3$, which contain the sum of the partial pressures of the several species involved, are

$$P_\perp = -\Omega - B\mathcal{M} \qquad P_3 = -\Omega, \tag{10}$$

where $\Omega = \sum_i \Omega_i$ is the total thermodynamical potential, $i = n, p, e$, and $\mathcal{M} = \sum_i \mathcal{M}_i$. By calling $S = \sum_i S_i$ and $N = \sum_i N_i$, where both the partial entropy and density are $S_i = \partial\Omega_i/\partial T$, $N_i = \partial\Omega_i/\partial\mu_i$, one can write the internal energy density as $U = -\mathcal{T}_{44} = \mu N + TS + \Omega$ (we take $T = 0$ in the present degenerate case), where the total quantities U, μN, Ω are of similar order. For positive magnetization, which is our case, the transverse pressure exerted by the magnetized particles is smaller than the longitudinal one in the amount $B\mathcal{M}$. If we assume that the body is in equilibrium under the balance of neutron and gravitational pressures, being the latter of order $P_{grav} \sim GM^2/R^4$, where R is the geometric average radius of the star, the body stretch along the direction of the magnetic field. (This effect can be figured out from looking at the spectrum described by Eq.(1), since the contribution from $\eta = -1$ terms are dominant. Then, if one approximates the second term inside the square root as $[m_n + (p_\perp^2/2m_n - qB)]^2$, the term in parenthesis accounts for the transverse kinetic energy. This term decreases as the magnetic field increases. Equating it to zero, by taking $p_\perp \sim p_F$, where p_F is the Fermi momentum, we obtain a functional relation between μ_n and B leading to the vanishing of the transverse kinetic energy, and in consequence of the transverse pressure. Observe that for $x = 1.001$, $y \sim 10^{-6}$. A more accurate result is obtained, however, from $\mathcal{T}_\perp = 0$).

This extreme case of vanishing of $P_\perp = -\Omega - B\mathcal{M}$ means that the transverse gravitational and Fermi gas pressures cannot compensate each other and an instability appears leading to a transverse collapse. We do not enter in the quantitative study of this problem, which would lead to a special sort of hybrid stars [19,18] or black strings [13,14]. In Figure 1 we have drawn the equation $P_{\perp n} = 0$ in terms of the variables x, y. We observe that there is a continuous range of values of the chemical potential, starting from values $x = 1.001$ and magnetic field intensities, from $y = 0.000001$, for which the collapse takes place. The latter value of y means fields on order

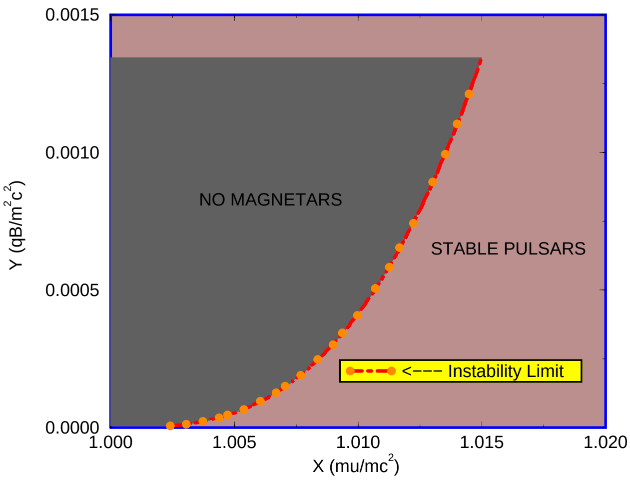

FIG. 1. The instability condition: $P_\perp = 0$. A neutron star having a configuration such that its dynamical stage would be represented by a point above the central curve in this plot would be unstable to transversal collapse, since $P_\perp \leq 0$ there.

of $B \sim 10^{14}$ G. To these ranges of x, y corresponds a continuous range of densities, from $10^{-4} N_0$ (10^{11} g/cm^{-3}) onwards. Thus, although stable neutron stars start to occur for densities $N \sim 10^{-2} N_0$ [17], the mechanism of decreasing the transverse pressure may act in the external regions of them, where densities of order $10^{-4} N_0$ may occur. This would lead to transverse compression of the whole mass of the star, concentrating the magnetic lines of force and increasing B. For $B \sim B_{cp,cn}$, the magnetic coupling of quarks with B becomes of the order of their binding energy through the colour field.

This might lead to a deconfinement phase transition leading to a quark (q)-star, a pressure-induced transition to uds-quark matter via ud-quark condensates, as discussed in Refs.([18,19]).

Observations of GRB980827 from SGR 1900+14 evidenced the existence of a stable pulsation with period 5.16 s [3]. In analogy to the case for SGR 1806-20 [2], the observed spin-down rate of the pulse period, $\dot{P} = 1.1 \times 10^{-10}$ s s^{-1}, led Kouveliotou et al. [2] to announcing the discovery of a *magnetar* in the source SGR 1900+14. These findings apparently lead to verify the Duncan and Thompson [1] magnetar model for SGRs. Kouveliotou et al. [2] claimed that spin down may be explained by the emission of dipolar radiation from a NS endowed with a very strong magnetic field $B \sim [2{-}8]10^{14}$ G. Next we briefly review the standard theory of magnetars. We shall show why they cannot survive after reaching the claimed superstrong magnetic fields, and then we present prospectives for a hybrid or strange star to appear as a remnant of the quantum collapse.

According to Duncan and Thompson [1], neutron stars (NSs) with very high dipole magnetic field strength, $B_D \sim [10^{14} - 10^{15}]$ G, may form when (classical) conditions for a helical dynamo action are efficiently met during the seconds following the core-collapse in a supernova (SN) explosion [1]. A newly-born NS may undergo vigorous convection during the first 30 s following its formation. If the NS spins (differentially) sufficiently fast ($P \sim 1$ms) the conditions are created for the $\alpha - \Omega$ dynamo action to be built. Collapse theory shows that some presupernova stellar cores could endow enough spin so as to rotate near their Keplerian equatorial velocity (the break-up spin) $\Omega_K \geq ([\frac{2}{3}]^3 GM/R^3)^{1/2}$ after core bounce. Under these conditions, fields as large as $B \sim 10^{17}(\frac{P}{1\text{ms}})$ G may be generated as long as the differential rotation is dragged out by the growing magnetic stresses [1]. For this process to efficiently operate the ratio between the spin rate (P) and the convection overturn time scale (τ_{con}), the Rossby number R_0, should be ≤ 1 ($R_0 \gg 1$ should induce less effective mean-dynamos [1]). In this case, amplification of the magnetic field strength by these helical motions is not precluded, since the α^2 or $\alpha - \Omega$ dynamos may survive depletion due to turbulent diffusion. An ordinary dipole $B_{sat} \sim [10^{12} - 10^{13}]$ G may be built by incoherent superposition of small dipoles of characteristic size $\lambda \sim [\frac{1}{3} - 1]$km and $B_{sat} = (4\pi\rho)^{1/2}\lambda/\tau_{con} \simeq 10^{16}$ G. At such fields, the huge rotational energy of a $f \geq 1$kHz NS is leaked out via *magnetic braking*, and an enormous energy is injected into the SN remnant. This energy may power a *plerion* in the SN remnant.

As shown above, at the end we are left with a NS with an extremely high field strength and a huge matter density $\rho \sim [10^{14} - 10^{12}]$ gcm^{-3}. As illustrated in Figure 1, those are the conditions for the quantum instability to start to dominate the dynamics of the young pulsar. At this stage, the magnetic pressure inwards may overpass the star's energy density at its equator, as defined in Eq.(10), and the collapse becomes unavoidable. As the collapse proceeds, higher and higher densities are reached till the point the supranuclear density may reverse the direction of implosion (hybrid star). From that moment, the sound wave generated at the core bounce builds itself into a shock wave traveling through the star at $V_{SW} = c/\sqrt{3}$ kms^{-1}. Although the magnetic field strength could be quite large as the collapse advances, the kinetic energy ($E \sim 10^{51}$ erg, the mean energy obtained in simulations of SN driven by the *prompt shock* [16]) carried away by the shock wave may counterbalance it, and even surpase it, i.e., its *ram* pressure will equal the magnetic pressure at r_A, the Alfvén radius given by

$$\rho_{eject} V_{SW}^2 \geq \frac{B^2}{8\pi\mu_0}\left(\frac{R}{r_A}\right)^6, \; r_A = \left(\frac{2\pi^2}{G\mu_0^2}\right)^{1/7}\left[\frac{B^4 R^{12}}{M\dot{m}^2}\right]^{1/7}, \tag{11}$$

Then the magnetic field lines are pushed out, and finally broken, from r_A onwards, into the SN remnant surroundings as a violent explosion that dissipates a large part of the magnetic flux ($\Phi \sim B^2 r_A^2$) trapped inside the magnetar magnetosphere. This is analogous to the mechanism operating during a solar flare or a coronal mass-ejection, where the very high B in the "sun-spot" is drastically diminished. Although the process is quite fast, the large amount of matter ejected from the star at such large velocities drains out the dipole field of the remnant below the quantum electrodynamic limit of $B_{QED} \sim 10^{13}$G. Further, since all the differential rotation has been dragged up to build up the superstrong magnetic field, then nothing else remains to make it to grow to its precollapse value. Thus no such ultra high B should reappear. We may be left with a submillisecond strange star [18] or a hybrid star [19] with "canonical" field strength.

We conclude that if a degenerate neutron gas is under the action of a strong magnetic field $B \lesssim B_c$, for adequate values of its density its transverse pressure vanishes, the outcome being a transverse collapse. This phenomenon establishes a limit to the magnetic field expected to be observable in a neutron star, as a function of its density, and suggests a possible end in the evolution of the highly magnetized neutron star as a mixed phase of nucleons and $\pi^{\pm,0}$, $\kappa^{\pm,0}$ meson condensate or a black string. Thus this result implies that most of the observed pulsars should have some either strange matter or meson condensate in their interiors.

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