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Some Points on Q.C.D. and String Theory

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Abstract

We clarify the applications of our previous study on the Q.C.D. String to Electro-Weak sector of Q.C.D.

1 The Q.C.D String

One of the most interesting problems in particle physics is that of understanding Q.C.D in terms of mesons and baryons as excitations of a collective color singlet variable. One of the most beautiful and compelling argument for the possibility of existence of such Physical approach consists by considering the Q.C.D Partional Functional with quarks integrated out and written as a dynamics of Fermionic Paths interacting with the gluon fields, the well-known Loop Space approach for Q.C.D ([1]). In this Loop Space approach, the central object is the Euclidean-Schwinger-Mandelstam-Feynman-Migdal-Polyakov Equation for the Non-Abelian Phase Factor defined by the bosonic trajectory of a pair quark-antiquark at the leading decoupling limit of large number of colors of t'Hoof ([1]).

$$\begin{aligned}
&\int_0^{2\pi} d\bar{\sigma}\left[-\frac{\delta^2}{\delta^2 c_\mu(\bar{\sigma})} - |c'_\mu(\bar{\sigma})|^2\right]\Phi[c_\mu(s); 0 \le s \le 2\pi] \\
&= \int_0^{2\pi} d\sigma \int_0^{2\pi} d\bar{\sigma} c'_\mu(\sigma)\delta^{(D)}[c_\mu(\sigma) - c_\mu(\bar{\sigma})]c'_\mu(\bar{\sigma}) \\
&\Phi[c_\mu(s); 0 \le s \le \sigma]\Phi[c_\mu(s); \sigma \le 2\pi]
\end{aligned} \tag{1}$$

where $\Phi[c_\mu(s), 0 \le s \le \sigma]$ denotes the (non-unitary) Euclidean quantum Q.C.D Wilson Loop ([2])

$$\begin{aligned}
&\Phi[c_\mu(s); 0 \le s \le \sigma] = \\
&\frac{1}{N_c}\left\langle Tr_c I\!P\left\{exp - \int_0^\sigma A_\mu(c^\alpha(\tilde{\sigma})c'_\mu(\tilde{\sigma})d\tilde{\sigma}\right\}\right\rangle
\end{aligned} \tag{2}$$

It is very important remark and implicitly used in our previous work ([2]) that the use of Feynman-Bosonic Paths for Feynman-Fermion Trajectories constraint the Bosonic Path $\{c_\mu(s)\}$ *to be smooth and possessing only isolated double point at path self-intersections as a consequence of Pauli-Exclusion occupation number principle for fermions* ([8]).

Another important point to remark is that Eq. (1) takes formally into account the Non-Perturbative nature of Yang-Mills Quantum Theory *by supposing the non-vanishing Gluon-Condensate factor normalized to unity multiplying the kinetic loop term* $|c'_\mu(\bar{\sigma})|^2$ *in*

Eq. (1).

$$\begin{aligned}
&\langle F_{\mu\nu}(A(x))F_{\mu\alpha}(A(x))\rangle \\
&\equiv \int \mathcal{D}^F[A(x)]exp\left\{-\frac{1}{4g^2}\int d^\nu x Tr_c(F^2_{\mu\nu}(A))(x)\right\}(F_{\mu\nu}(A(x))F_{\mu\alpha}(A(x)) \\
&= \delta^{\nu\alpha}\left\langle 0|F^2(A)|0\right\rangle = \delta^{\nu\alpha} \qquad (3)
\end{aligned}$$

In a series of author's papers ([2]) it was proposed a string path-integral solution for Eq. (1) with intrinsic two-dimensional Fermions Fields in the String World-Sheet and possessing intrinsic charges, a kind of generalized complex Elfin fields formed by a complex combination of majorana neutral fields and besides, possessing a intrisinc vertex interaction supported at the string world-sheet self-intersect points in order to reproduce the Non-Abelian character of the Non-Linear term of Eq. (1) - ([3]). We have, thus, considered the following string path integral to solve Eq. (1).

$$\begin{aligned}
&Z^{AB}_{kp}[c_\mu(s), 0\le s\le 2\pi] = \int_{X^\mu(\sigma,0)=c^\mu(\sigma)} \mathcal{D}^F[X^\mu(\sigma,\tau)\mathcal{D}^F[\psi^{A'}_{(k')}(\sigma,\tau)] \\
&\mathcal{D}^F[\bar\psi^{B'}_{(p')}(\sigma,\tau)]\times(\bar\psi^A_{(k)}(0,0)\psi^B_{(p)}(2\pi,0)) \\
&exp\left\{-\frac{1}{2}\int_{-\infty}^{+\infty}d\tau\int_0^{2\pi}d\sigma[(\partial_\sigma X^\mu)^2+(\partial_\tau X^\mu)^2](\sigma,\tau)\right\} \\
&exp\left\{-\frac{1}{2}\int_{-\infty}^{+\infty}d\tau\int_0^{2\pi}d\sigma\sqrt{h(\sigma,\tau)}(\bar\psi\not{D}_{\{h_{ab}\}}\psi)(\sigma,\tau)\right\} \\
&exp\left\{-\beta\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{+\infty}d\tau'\int_0^{2\pi}d\bar\sigma\left\{\bar\psi_{(k)}\psi_{(k)}+\left(\bar\psi_{(k)}\psi_{(k)}\right)^2+\left(\bar\psi_{(k)},\gamma^\mu\psi_{(k)}\right)^2\right\}(\sigma,\tau)\right. \\
&\tau^{\mu\nu}(X(\sigma,\tau))\delta^{(D)}(X_\mu(\sigma,\tau)-X_\mu(\bar\sigma,\bar\tau))\tau_{\mu\nu}(X(\bar\sigma,\bar\tau)) \qquad (4)
\end{aligned}$$

The string surface fundamental parameter domain is taken to be the strip $\mathcal{D}=\{(\sigma,\tau); 0\le\sigma\le 2\pi,\ -\infty\le\tau\le\infty\}$ and the vector string field $X_\mu(\sigma,\tau)$ satisfies the periodicity condition $X_\mu(\sigma+2\pi,\tau)=X_\mu(\sigma,\tau)$ together with the boundary condition $X^\mu(\sigma,0)=c^\mu(\sigma)$. The induced surface metric tensor is denoted by $h_{ab}(\sigma,\tau)=(\partial_a X^\mu\partial_b X_\mu)(\sigma,\tau)$.

The intrisic two-dimensional Euclidean Dirac Spinors $\{\psi^A_{(k)}(\sigma,\tau); A=1,2; k=1,\cdots,N\}$ belong to a $U(N)$ fundamental representation. Finally, the area surface orientation tensor $\tau_{\mu\nu}(X(\sigma,\tau))=(\varepsilon^{ab}\partial_a X^\mu\partial_b X^\nu)(\sigma,\tau)$ are normalized to unity $\tau_{\mu\nu}(X(\sigma,\tau)\tau^{\mu\nu}(X(\sigma,\tau))=1$ but $\tau_{\mu\nu}(X(\sigma,\tau))\tau^{\mu\nu}(X(\bar\sigma,\bar\tau))\neq 1$ if $(\sigma\neq\bar\sigma)$.

It was proved in Ref. [2] that the Hamiltonian Operator associated to the Self-Avoiding string theory Eq. (4) with the identification $\beta = \lim_{N\to\infty} (g^2 N_c)$ *coincides with the Q.C.D* $(SU(\infty))$ *Eq. (1) if one imposes that all contributing string world-sheets* $\{X^\mu(\sigma,\tau)\}$ *for Eq. (4) are homotopical deformations of the Loop Boundary* $(c^\mu(\sigma))$. *Namelly, the self-intersecting equation* $X_\mu(\sigma,\tau) = X_\mu(\bar\sigma,\bar\tau)$ *has solutions if only* $\tau = \bar\tau$ *and* $\bar\sigma \in \{\sigma, \tilde\sigma\}$ *where* $\tilde\sigma \neq \sigma$. This imposed condition leads to the result that the interaction term Eq. (4) is a kind of a vertex, sewing along lines of world-sheet self-intersections the various non self intersecting pieces of the whole string world sheet. It is very important to remark that there is a induced four fermion interaction of Eq. (4) which comes from the trivial self-intersecting point $\bar\sigma = \sigma$ in Eq. (4) [Ref. 2]. Note that we have implicitly used the following normalization conditions for these generalized Elfin fields ($A = 1, 2$ denote World-Sheet Dirac indexes). The Neuman Condition:

$$\lim_{\tau\to 0} \partial_\sigma \psi^A_{(k)}(\sigma,\tau) = 0 \tag{5}$$

and the Loop renormalization for the String Propagator eq. (4)

$$Z^{AB}_{(k)(p)}[c_\mu(s) = c_\mu(0) = x_\mu] = \delta^{AB}\delta_{(k),(p)} \tag{6}$$

$$(1-\delta_{AB}) Z^{AB}_{(k)(p)}[c_\mu(s)] \equiv 0 \tag{7}$$

At this point it is worth remark that one is compelled to choose the number of Generalized Elfin Fields N in a such way that its contribution to the quantum geometrical measures conformal anomaly factor leads to the usual Feynman measure ([4]) (in order to the string theory be a well defined 2D Quantum Field Theory in the periodic strip

$$\mathcal{D} = \{(\sigma,\tau)\ ,\ 0 \le \sigma \le 2\pi\ ;\ -\infty \le \tau \le \infty\}$$

We have, thus, the condition $26 = D + 2N$ ([2],[4]).

The full Q.C.D ($SU(N)$) string for a finite number of colours is expected to be the extension of the Fermionic Self-Avoiding String Theory to non trivial topological sectors as in Ref. ([5]). Unfortunately, this result was not proved yet on base of Loops Equations. However, it may be a consequence of the Unitarization of the String Theory.

After describing brief the proposed (2D Renormalizable) Q.C.D ($SU(\infty)$) string we pass now to its modelling for the Strong Interaction of Mesons and Baryons.

2 Mésons and Baryons as String Excitations

The fundamental point on the String formulation for Q.C.D is related to interaction with the Electro-Weak sector flavor charges. In this sector one postulate that the vertex associated to the string theory Eq. (4) are related to the Physical Baryon-Meson sector and at this point *one identify the intrinsic Generalized Elfin Charges with flavor physical charges.* For instance, the scalar and vectorial flavored mesons are generated respectively by the vertex ([7]).

$$V_{scalar} = exp\left\{-\int_{-\infty}^{+\infty} d\tau \int_0^{2\pi} d\sigma \sqrt{h(\sigma,\tau)} J_{(i)}(X(\sigma,\tau))(\bar{\psi}_{(k)}(\tau^i)_{(k)(p)}\psi_{(p)})(\sigma,\tau)\right\} \tag{8}$$

$$\begin{aligned} V_{vectorial} = exp\Big\{ &-\int_{-\infty}^{+\infty} d\tau \int_0^{2\pi} d\sigma \sqrt{h(\sigma,\tau)}\; A^i_\mu(X(\sigma,\tau)) \\ &\partial_A X^\mu(\sigma,\tau)(\psi_{(k)}\gamma^A(\tau^i)_{(k)(p)}\psi_{(p)})(\sigma,\tau)\Big\} \end{aligned} \tag{9}$$

Note that $\{\psi_{(k)}, \bar{\psi}_{(k)}\}$ must belong to the fundamental representation of the Choosen Flavor Group with generators $\{\tau^i\}$.

They are the vertex (and theirs Lorentz-Flavor tensor-like generalizations) which generate the méson spectrum. One remark now to be pointed out is that these vertex are coupled to *unphysical fields* $\{J^i(x); A^i_\mu \tau_i\}$ *in the External Space-Time.*

The Coupling of the theory with the physical Electro-Weak Fields should be made only at the Boundary (after evaluating the String Path Integral Eq. (4) with external vertex Eqs. (8)-(9) which by its turn, is a functional of the loop $\{c_\mu(s)\}$); by means of the Boundary Interactions Phase Factors:

$$\mathbb{P}_{flavor}\{exp - \int_0^{2\pi} d\sigma(\sigma^i(c_\mu(\sigma))\tau_i)\sqrt{|c'_\mu(\sigma)|^2}\} \tag{10}$$

$$\mathbb{P}_{flavor}\{exp - \int_0^{2\pi} d\sigma(W^i_\mu(c_\alpha(\sigma))\tau_i)c'_\mu(\sigma)\} \tag{11}$$

which are obtained after reformulating the Electro-Weak Sector in the Quark-Lepton Space Appendix 1. Here $\sigma^i(x)\tau_i$ and $A^i_\mu(x)\tau_i$ *are physical flavored fields.*

Scattering Amplitudes (S – Matrix) of the Mesons-Leptons-Higgs-Flavor Gauge fields are, thus, evaluated in the Loop Space ([1]).

In order to consider Baryons on this Geometrical Functional Integral Approach, *one must build External Fermion Vertex* and add to those Eqs. (8) – (9), which still to be an open problem in this approach although there is a obscure propose by Migdal which considers directly in Loop Space a new kind of Q.C.D Baryon Loop Wave Equation ([1]).

Finally, we strees our hope that this Quantum Geometric framework does not have any inconsistency and lead to the observable Strong S – Matrix ([6]) with flavor interaction.

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Appendix A – The Electro-Weak Sector as Dynamics of Loops and Strings

Let us consider a set o microscopical quark-leptons fermi fields interacting with a color Yang-Mills field $A^e_\mu \lambda_e$ and a flavor Yang-Mills field $W^i_\mu \tau_i$

$$\begin{aligned}
Z = & \int \mathcal{D}^F[\psi(x)]\mathcal{D}^F[\bar{\psi}(x)]\mathcal{D}^F[e(x)]\mathcal{D}^F[\bar{e}(x)]\mathcal{D}^F[A_\mu(x)]\mathcal{D}^F[W_\mu(x)] \\
& exp\left\{-\frac{1}{4g^2}\int d^\nu x Tr_{color}(F^2_{\mu\nu}(A))(x) - \frac{1}{4e^2}\int d^\nu x Tr_{flavor}(F^2_{\mu\nu}(W))(x)\right\} \\
& exp\left\{-\frac{1}{2}\int d^\nu x(\bar{\psi}\not{D}(A\otimes W)\psi)(x)\right\} \\
& exp\left\{-\frac{1}{2}\int d^\nu x(\bar{e}\not{D}(W)e)(x)\right\} \\
& exp\left\{-\frac{1}{2}\int d^\nu x(\bar{\eta}e + \bar{e}\eta)(x)\right\}
\end{aligned} \tag{A.1}$$

where $\{\psi_{(x)}, \bar{\psi}\}$; $\{e(x), \bar{e}(x)\}$ denotes the quark and lepton fields respectively.

After integrating out the quark-lepton fields and writing the effective action in Loop Space ([1],2), we obtain the following expression

$$\begin{aligned}
Z = & \int \mathcal{D}^F[A_\mu(x)]\mathcal{D}^F[W_\mu(x)] \\
& exp\left\{-\frac{1}{4g^2}\int d^\nu x Tr_c(F^2_{\mu\nu}(A))(x) - \frac{1}{4e^2}\int d^\nu x Tr_F(F^2_{\mu\nu}(W))(x)\right\} \\
& exp\left\{-\sum_{\{c_\mu(\sigma)\}}(\Phi[c_\mu(\sigma), 0\le\sigma\le s](A_\mu)\Phi[c_\mu(\sigma), 0\le\sigma\le s](W_\mu)\right\} \\
& exp\left\{-\sum_{\{L_\mu(\bar{\sigma})\}}\Phi[L_\mu(\bar{\sigma}), 0\le\bar{\sigma}\le\bar{s}](W_\mu\right\} \\
& exp\left\{-\sum_{\{\tilde{L}_\mu(\bar{\sigma})\}}^{(xy)}\int d^\nu x d^\nu y \eta_\alpha(x)\psi^{(ab)}_{(xy)}[\tilde{L}_\mu(\bar{\sigma}), 0\le\bar{\sigma}\le\bar{s}](W_\mu)\bar{\eta}_b(\delta)\right\}
\end{aligned} \tag{A.2}$$

Here, we have kept the subscript (A_μ) or (W_μ) to remind the reader about the gauge field kind entering in the definition of the Mandelstam-Feynman Phase Factors, namelly

$$\Phi[c_\mu(\sigma), 0\le\sigma\le s](A_\mu) = \frac{1}{N_c}\, Tr_{color}\mathbb{P}\{exp(-\int_0^s d\sigma A_\mu(c^\mu(\sigma))\dot{c}_\mu))\} \tag{A.3}$$

$$\psi^{ab}_{(xy)}[\bar{L}_\mu(\sigma), 0\le\sigma\le\bar{s}](W_\mu) = \mathbb{P}\left\{exp(-\int_0^{\bar{s}} W_\mu(\tilde{L}_\mu(\bar{\sigma}))\dot{\tilde{L}}_\mu(\bar{\sigma}))\right\} \tag{A.4}$$

The Bosonized sum over the *closed fermionic quark trajectories* are given by the following path integral ([1],[2]) which comes from the procedure of writting fermionic functional determinants in Loop Space ([1])

$$\sum_{\{c_\mu(\sigma)\}} \equiv -\int_0^\infty \frac{ds}{s} \int d^\nu x \int_{c_\mu(0)=c_\mu(s)=x_\mu} \mathcal{D}^F[c_\mu(\sigma)]\mathcal{D}^F(\pi_\mu(\sigma)]$$
$$exp(i\int_0^s d\sigma(\pi^\mu(\sigma)\dot{c}_\mu(\sigma))I\!P_{Dirac}[exp(i\int_0^s d\sigma(\pi^\mu(\sigma)\gamma_\mu))] \qquad (A.5)$$

In relation to the Bosonized sum over the *open Fermionic Lepton Feynman trajectories* which enters into the definition of the Lepton Propagator, it differs slightly for Eq. (5-A) and is given by

$$\sum_{(\tilde{L}_\mu(\sigma))}^{(xy)} \equiv \int_0^\infty d\bar{s} \int_{\tilde{L}_\mu(0)=x_\mu;\tilde{L}_\mu(\bar{s})=y_\mu} \mathcal{D}^F[L_\mu(\sigma)]\mathcal{D}^F[\pi_\mu(\sigma)]$$
$$exp(i\int_0^{\bar{s}} d\sigma(\pi^\mu(\sigma)\cdot\dot{\tilde{L}}_\mu(\sigma))I\!P_{Dirac}(exp(i\int_0^{\bar{s}} d\sigma(\pi^\mu(\sigma)\cdot\gamma_\mu(\sigma))) \qquad (A.6)$$

By using now the result Eq. (4) to evaluate exactly the Gluon Functional Integral in terms of a String Theory in Eq. (A.2) we get our main result, which express the Strong-Electro-Weak Quantum Field Theory Eq. (A.1) in terms of a Dynamics of Interacting Contours

$$\{c_\mu(\sigma), 0 \le \sigma \le s\}\ ;\ \{L_\mu(\bar{\sigma}), 0 \le \bar{\sigma} \le \bar{s}\}\ ;\ \{\tilde{L}_\mu(\bar{\sigma})0 \le \sigma \le s\}$$

and our proposed string theory Eq. (4) for all topological sectors added with appropriate vertex for Mesons and Baryons Excitations (see Eq. (8) – Eq. (11) in the text).

Perturbative Calculations in terms of these Geometrical Path Integrals are implemented in the String Path Integral in terms of the Genus String World-Sheet expansion together with a expansion of the Q.C.D vertex in Eq. (4) in the number of non-trivial self-intersecting double points or equivalently in a power expansion in the sweing Q.C.D coupling constant.

Appendix B – The Self-Avoiding Elfin String Hamiltonian

In this appendix we present the formal evaluation of the Quantum Hamiltonian associated to our proposed String Path Integral.

As a first step we consider the two-dimensional string time (see [2]), τ in the range $0 \le \tau \le A$ in the Path-Integral eq. (4). As a consequence, the Euclidean Quantum Hamiltonian will be given by the expression

$$H^{AB}_{(k)(p)} = -\lim_{A\to 0} \frac{\partial}{\partial A}\left\{ Z^{AB}_{(k)(p)}[c_\mu(\sigma); 0 \le \sigma \le 2\pi, A]\right\} \tag{B.1}$$

and should act on the Theory's Hilbert Space composed by Loop Functionals.

Evaluation of the A-derivative is straightforward since the 2D-Quantum Field Theory defining the Path-Integral Eq. (4) is renormalizable in the strip $0 \le \tau \le A$; $0 \le \sigma \le s = 2\pi$ due to double self-intersecting point condition of the loop functionals (see the remarks below Eq. (4) in the paper).

This evaluation results into the Path Integral Average of the A-derivative of the full interacting action in Eq. (4) at zero time

$$\begin{aligned}
&\left\langle\left\langle \lim_{\tau\to 0}\int_0^{2\pi} d\sigma \left[-\frac{1}{2}\left(\frac{2X^\mu}{2\sigma}\right)^2 - \frac{1}{2}\left(\frac{2X^\mu}{\delta\tau}\right)^2\right](\sigma,\tau)\right\rangle\right\rangle \\
&\lim_{\tau\to 0}\beta\left\langle\left\langle\int_0^{2\pi} d\sigma\int_0^{2\pi} d\bar\sigma(\bar\psi_{(k)}\psi_{(k)}) + (\bar\psi_{(k)}\psi_{(k)})^2 + (\bar\psi_{(k)}\gamma^\mu\psi_{(k)})^2\right.\right. \\
&\{c'_\mu(\sigma)c_\nu(\sigma) - c'_\nu(\sigma)c_\mu(\sigma)\}/\sqrt{|c'_\mu(\sigma)|^2}\delta^{(\nu)}(c_\mu(\sigma) - c_\mu(\bar\sigma)) \\
&\{c'_\mu(\bar\sigma)c_\nu(\bar\sigma) - c'_\nu(\bar\sigma)c_\mu(\bar\sigma)\}/\sqrt{|c'_\mu(\sigma)|^2}\ \rangle\rangle
\end{aligned} \tag{B.2}$$

By using now the usual Heisenberg Commutation Relation for Euclidean 2D Massless Free Field

$$\frac{\delta}{\delta c_\mu(\sigma)} = \lim_{\tau\to 0}\left\langle\left\langle\frac{\partial}{\delta\tau}X^\mu(\sigma,\tau)\right\rangle\right\rangle \tag{B.3}$$

and the Boundary conditions Eq. (5) we get the result:

$$\begin{aligned}
&\int_0^{2\pi} d\sigma\left\{\left[-\frac{\delta^2}{\delta^2 c_\mu(\sigma)} - |c'_\mu(\sigma)|^2\right]\right\}\ Z^{AB}_{(k)(p)}[c_\mu(s), 0 \le s \le 2\pi] = \\
&= \beta\int_0^{2\pi} d\sigma\int_0^{2\pi} d\bar\sigma c'_\mu(\sigma)\delta^{(D)}[c_\mu(\sigma) - c_\mu(\bar\sigma)] \\
&Z^{AD}_{(k)(m)}[c_\mu(s), 0 \le s \le \sigma]Z^{DE}_{(m)(n)}[c_\mu(s), s = \sigma]Z^{EF}_{(n)(p)}[c_\mu(s), \sigma \le s \le 2\pi]
\end{aligned} \tag{B.4}$$

By using now the String Propagator condition Eq. (6) in Eq. (B.4) we get the result expressing that the String Path-Integral Eq. (4) satisfies the flavor-like Q.C.D ($SU(\infty)$) Loop Wave Equation (1) after the identification of the Theories's Coupling Constants

$$\lim_{N_i \to \infty} g^2_{Q.C.D}\; N = \beta_{string} \tag{B.5}$$