CBPF-NF-075/98

BRST Quantization of the Twisted $N = 2$ Super-Yang-Mills Theory in $4D$

A. Tanzini,

Dipartimento di Fisica, Universitá di Roma II,
"Tor Vergata", Italy

O. S. Ventura,

CBPF, Centro Brasileiro de Pesquisas Físicas
Departamento de Campos e Partículas
Rua Xavier Sigaud 150, 22290-180 Urca
Rio de Janeiro, Brazil

and

L.C.Q.Vilar, S.P. Sorella

UERJ, Universidade do Estado do Rio de Janeiro
Departamento de Física Teórica, Instituto de Física
Rua São Francisco Xavier, 524
20550-013, Maracanã, Rio de Janeiro, Brazil

UERJ/DFT - 06 - 98

November 20, 1998

Abstract

We perform the full quantization of the twisted $N = 2$ supersymmetric Yang-Mills theory in four dimensions, whose classical action is that of the topological Yang-Mills (TYM) theory. By means of the introduction of appropriate constant ghosts associated to the twisted generators of $N = 2$, we are able to quantize the model by taking into account both the gauge invariance and the topological symmetries of the TYM action. Concerning the usual BRST cohomology, we show that the twisted algebra can be useful in order to obtain the relevant cohomology classes. In particular, the requirement of analyticity in the constant ghosts will identify the topological sector of the twisted theory and the BRST nontrivial twisted action. This will lead us to suggest a possible approach in order to give an algebraic proof of the one-loop exactness of the $N = 2$ β-function.

I Introduction

The $N = 2$ super-Yang-Mills field theory has already a long history and many interesting properties, both at the perturbative and at the nonperturbative level, as for instance the one-loop exactness of its β-function.

Some time ago, after the work of Witten *[1]*, $N = 2$ super-Yang-Mills became known to be related to topological field theory, or, more precisely, to topological Yang-Mills theory (TYM). The TYM action, as it has originally been pointed out in *[1]*, can be seen indeed as the twisted version of the $N = 2$ supersymmetric Yang-Mills theory *[2]*. The supersymmetric charges of $N = 2$ then lead to a scalar, a vector and a self-dual tensor charge, which remain as symmetry generators of the twisted action. The final relationship of the twisted $N = 2$ theory with TYM can then be done by identifying the $\mathcal{R}$-charge of the twisted fields and generators as the ghost number. This means that the matter fields of $N = 2$ acquire the status of ghost fields and the scalar operator coming from the twist of the supersymmetric generators can then be interpreted as a "BRST-like" operator. The effect of this identification is, in the end, the independence of the theory from any scale, as the BRST cohomology becomes completely trivial. In fact, it was shown that TYM can be fully obtained from the gauge fixing of a surface term (the Pontryagin index) *[3, 4]*.

The problem at this point is that one looses contact with the usual BRST characterization of the observables. The emptiness of the cohomology forbids such a characterization for the topological invariants, for instance. The way out was found through the formalism of the equivariant cohomology. The observables in the equivariant sense were then shown to coincide with Witten's invariants *[5, 6]*.

More recently, an important advance on the algebraic quantization program of supersymmetric field theories in the Wess-Zumino gauge has been achieved. This goal was accomplished by the explicit introduction of the supersymmetry generators inside an extended BRST operator *[7]*. In fact, the case of $N = 2$ super-Yang-Mills has been also successfully worked out, with the result that its BRST cohomology was nontrivial, allowing for a nonvanishing beta function *[8]*. However, the origin of its nonrenormalization properties remains an open question from the algebraic point of view.

Our attitude in this paper will be to take into further analysis the twisted version of the conventional $N = 2$ supersymmetric euclidean Yang-Mills theory, without identifying the $\mathcal{R}$-charge with the ghost number. The twisting mechanism has the advantage of clarifying the topological structure which underlies the $N = 2$ super-Yang-Mills field theory. This will allow us to analyze the BRST structure of the $N = 2$ super-Yang-Mills within the framework of the descent equations, which opens the way for a better understanding of the finiteness properties displayed by the model.

At this stage, we will refer to this version as the twisted theory (TSYM) and leave the denomination of TYM for the cohomological theory obtained after the aforementioned identification. Notice that the classical action for the twisted theory will be just Witten's TYM action, but now with all fields interpreted as gauge or matter fields. In this way, the quantization of the twisted theory can proceed as for the case of supersymmetric models in the Wess-Zumino gauge *[7, 8]*. As we will see, this will require to take into account the full set of symmetries (gauge and supersymmetry) in the quantization procedure. The contact with the cohomological formulations of TYM *[3, 4]* will then be established. It is worth underlining that the requirement of analyticity in the constant ghosts will play a fundamental role in order to obtain nontrivial cohomology classes. In fact, we shall be able to show that one can move from a nontrivial theory to the cohomological TYM by giving up the analyticity condition. In this perturbative approach, we will have the chance to show that the invariant counterterm of the $N = 2$ theory is associated with solutions of the descent equations. These solutions turn out to be the topological invariants of TYM. This will allow us to conjecture on the origin of the one-loop exactness of the β-function in $N = 2$ supersymmetric Yang-Mills theory *[9]*.

In the next section we present a simple review of the twisting mechanism, making the connection of TYM with $N = 2$ super-Yang-Mills. In Section three we proceed with the quantization of the twisted theory in analogy with the $N = 2$ theory. Section four is devoted to the renormalization of the model. Finally, we shall make contact with the results already existing in the literature and we shall draw a possible path toward the algebraic proof of the nonrenormalization theorem for the β-function.

II The Twisted Action

Following *[1]*, the TSYM classical action is given by

$$
\begin{aligned}
\mathcal{S}_{TSYM} = \frac{1}{g^2} tr \int d^4x \, \Big(& \frac{1}{2} F^{+}_{\mu\nu} F^{+\mu\nu} + \frac{1}{2} \overline{\phi} \left\{ \psi^{\mu}, \psi_{\mu} \right\} \\
& - \chi^{\mu\nu} (D_{\mu}\psi_{\nu} - D_{\nu}\psi_{\mu})^{+} + \eta D_{\mu}\psi^{\mu} - \frac{1}{2} \overline{\phi} D_{\mu} D^{\mu} \phi \\
& - \frac{1}{2} \phi \left\{ \chi^{\mu\nu}, \chi_{\mu\nu} \right\} - \frac{1}{8} [\phi, \eta] \, \eta - \frac{1}{32} \left[\phi, \overline{\phi} \right] \left[\phi, \overline{\phi} \right] \Big) \, ,
\end{aligned}
\tag{1}
$$

where g is the *unique* coupling constant and $F^{+}_{\mu\nu}$ is the self-dual part of the Yang-Mills field strength. The three fields $(\chi_{\mu\nu}, \psi_{\mu}, \eta)$ in the expression (1) are anticommuting, with $\chi_{\mu\nu}$ self-dual, and $\left(\phi, \overline{\phi} \right)$ are commuting complex scalar fields, $\overline{\phi}$ being assumed to be the complex conjugate of ϕ. Of course, TSYM being a gauge theory, is left invariant by the gauge transformations

$$
\begin{aligned}
\delta^{g}_{\epsilon} A_{\mu} &= -D_{\mu}\epsilon \, , \\
\delta^{g}_{\epsilon} \lambda &= [\epsilon, \lambda] \, , \qquad \lambda = \chi, \psi, \eta, \phi, \overline{\phi} \, .
\end{aligned}
\tag{2}
$$

It is easily checked that the kinetic terms in the action (1) corresponding to the fields $\left(\chi, \psi, \eta, \phi, \overline{\phi} \right)$ are nondegenerate, so that these fields have well defined propagators. The only degeneracy is that related to the pure Yang-Mills term $F^{+}_{\mu\nu} F^{+\mu\nu}$. Therefore, from eq.(2) one is led to interpret $\left(\chi, \psi, \eta, \phi, \overline{\phi} \right)$ as ordinary matter fields, in spite of the unconventional tensorial character of $(\chi_{\mu\nu}, \psi_{\mu})$. We assign to $(A, \chi, \psi, \eta, \phi, \overline{\phi})$ the dimensions $(1, 3/2, 3/2, 3/2, 1, 1)$ and $\mathcal{R}$-charges $(0, -1, 1, -1, 2, -2)$, so that the TSYM action (1) has vanishing total $\mathcal{R}$-charge. Let us emphasize once more that we do not identify the $\mathcal{R}$-charge with the ghost number, so that we avoid the cohomological interpretation which leads to TYM theory. The action (1) has to be regarded just as the twisted version of $N = 2$ super-Yang-Mills theory.

For a better understanding of this point, let us briefly review the twisting procedure of the $N = 2$ supersymmetric algebra in flat euclidean space-time *[2]*. In the absence of central extension, the $N=2$ supersymmetry in the Wess-Zumino gauge is characterized by 8 charges $\left(\mathcal{Q}^i{}_\alpha, \overline{\mathcal{Q}}^j{}_{\dot\alpha}\right)$ obeying the following relations

$$\begin{aligned} \left\{\mathcal{Q}^i{}_\alpha, \overline{\mathcal{Q}}_{j\dot\alpha}\right\} &= \delta^i_j \partial_{\alpha\dot\alpha} + \text{gauge transf.} + \text{eqs. mot.}\ , \\ \left\{\mathcal{Q}^i{}_\alpha, \mathcal{Q}^j{}_\beta\right\} &= \left\{\overline{\mathcal{Q}}^i_{\dot\alpha}, \overline{\mathcal{Q}}^j_{\dot\beta}\right\} = \text{gauge transf.} + \text{eqs. mot.}\ . \end{aligned} \tag{3}$$

where $(\alpha,\dot\alpha)=1,2$ are the spinor indices, $(i,j)=1,2$ the internal $SU(2)$ indices labelling the different charges of $N=2$, and $\partial_{\alpha\dot\alpha} = (\sigma^\mu)_{\alpha\dot\alpha}\,\partial_\mu$, σ^μ being the Pauli matrices. The special feature of $N=2$ is that both spinor and internal indices run from 1 to 2, making it possible to identify the index i with one of the two spinor indices $(\alpha,\dot\alpha)$. This corresponds to replace the $SU(2)_L$ factor of the Lorentz symmetry group of the theory by the diagonal subgroup $SU(2)'_A = \text{diag}\,(SU(2)_L \times SU(2)_A)$. It is precisely this identification which defines the twisting procedure *[2]*. Identifying therefore the internal index i with the spinor index α, we can construct now the following twisted generators $(\delta, \delta_\mu, \delta_{\mu\nu})$,

$$\begin{aligned} \delta &= \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta}\mathcal{Q}_{\beta\alpha}\ ,\ \delta_\mu = \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}_{\alpha\dot\alpha}\ (\overline{\sigma}_\mu)^{\dot\alpha\alpha}\ , \\ \delta_{\mu\nu} &= \frac{1}{\sqrt{2}}(\sigma_{\mu\nu})^{\alpha\beta}\mathcal{Q}_{\beta\alpha} = -\delta_{\nu\mu}\ . \end{aligned} \tag{4}$$

Notice that the generators $\delta_{\mu\nu}$ are self-dual due to the fact that the matrices $\sigma_{\mu\nu}$ are self-dual in euclidean space-time. In terms of these generators, the $N=2$ susy algebra reads now

$$\begin{aligned} \delta^2 &= \text{gauge transf.} + \text{eqs. of motion}\ , \\ \{\delta,\delta_\mu\} &= \partial_\mu + \text{gauge transf.} + \text{eqs. of motion}\ , \\ \{\delta_\mu,\delta_\nu\} &= \text{gauge transf.} + \text{eqs. of motion}\ , \end{aligned} \tag{5}$$

$$\begin{aligned} \{\delta,\delta_{\mu\nu}\} &= \{\delta_{\mu\nu},\delta_{\rho\sigma}\} = \text{gauge transf.} + \text{eqs. of motion}\ , \\ \{\delta_\mu,\delta_{\rho\sigma}\} &= -\varepsilon_{\mu\rho\sigma\nu}\partial^\nu - g_{\mu[\rho}\partial_{\sigma]} + \text{gauge transf.} + \text{eqs. of motion}\ . \end{aligned} \tag{6}$$

The algebraic structure realized by the generators (δ,δ_μ) in eq.(5) is typical of the topological models *[10, 11]*. In this case the vector charge δ_μ, usually called vector supersymmetry, is known to play an important role in the derivation of the ultraviolet finiteness properties of the topological models and in the construction of their observables *[10]*.

Let us now turn to the relationship between Witten's TYM and the $N=2$ Yang-Mills theory, and show, in particular, that TYM has the same field content of the $N=2$

Yang-Mills theory in the Wess-Zumino gauge. The minimal $N = 2$ supersymmetric pure Yang-Mills theory is described by a gauge multiplet which, in the Wess-Zumino gauge, contains *[8, 2]*: a gauge field A_μ, two spinors ψ^i_α $i = 1, 2$, their conjugate $\overline{\psi}^i_{\dot\alpha}$, and two scalars $\phi, \overline{\phi}$ ($\overline{\phi}$ being the complex conjugate of ϕ). All these fields are in the adjoint representation of the gauge group. We proceed by applying the previous twisting procedure to the $N = 2$ Wess-Zumino gauge multiplet $(A_\mu, \psi^i_\alpha, \overline{\psi}^i_{\dot\alpha}, \phi, \overline{\phi})$. Identifying then the internal index i with the spinor index α, it is very easy to see that the spinor $\overline{\psi}^i_{\dot\alpha}$ can be related to an anticommuting vector ψ_μ, i.e

$$\overline{\psi}^i_{\dot\alpha} \stackrel{twist}{\longrightarrow} \overline{\psi}_{\alpha\dot\alpha} \longrightarrow \psi_\mu = (\overline{\sigma}_\mu)^{\dot\alpha\alpha}\overline{\psi}_{\alpha\dot\alpha} \ . \tag{7}$$

Concerning now the fields ψ^i_β we have $\psi^i_\beta \stackrel{twist}{\longrightarrow} \psi_{\alpha\beta} = \psi_{(\alpha\beta)} + \psi_{[\alpha\beta]}$, $\psi_{(\alpha\beta)}$ and $\psi_{[\alpha\beta]}$ being respectively symmetric and antisymmetric in the spinor indices α, β. To $\psi_{[\alpha\beta]}$ we associate an anticommuting scalar field η, while $\psi_{(\alpha\beta)}$ turns out to be related to an antisymmetric self-dual field $\chi_{\mu\nu}$ through

$$\begin{aligned} \psi_{[\alpha\beta]} &\longrightarrow \eta = \varepsilon^{\alpha\beta}\psi_{[\alpha\beta]} \ , \\ \psi_{(\alpha\beta)} &\longrightarrow \chi_{\mu\nu} = \widetilde{\chi}_{\mu\nu} = (\sigma_{\mu\nu})^{\alpha\beta}\psi_{(\alpha\beta)} \ . \end{aligned} \tag{8}$$

Therefore, the twisting procedure allows to replace the $N = 2$ Wess-Zumino multiplet $(A_\mu, \psi^i_\alpha, \overline{\psi}^i_{\dot\alpha}, \phi, \overline{\phi})$ by the twisted multiplet $(A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \overline{\phi})$ whose field content is precisely that of the TSYM action (1). The same holds for the $N = 2$ pure Yang-Mills action *[2]*,

$$\mathcal{S}^{N=2}_{YM}(A_\mu, \psi^i_\alpha, \overline{\psi}^i_{\dot\alpha}, \phi, \overline{\phi}) \stackrel{twist}{\longrightarrow} \mathcal{S}_{TSYM}(A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \overline{\phi}).$$

Thus the TSYM comes in fact from the twisted version of the ordinary $N = 2$ Yang-Mills in the Wess-Zumino gauge. This important point, already underlined by Witten in his original work *[1]*, deserves a few clarifying remarks in order to make contact with the results on topological field theories obtained in the recent years.

The first observation is naturally related to the existence of further symmetries of the TSYM action (1). According to the previous analysis, we conclude that the TSYM will be left invariant by the twisted generators $(\delta, \delta_\mu, \delta_{\mu\nu})$. In fact, it is easy to check that the twisted scalar generator δ corresponds to Witten's fermionic symmetry $\delta_{\mathcal{W}}$ *[1]*

$$\begin{array}{lll} \delta_{\mathcal{W}} A_\mu = \psi_\mu \ , & \delta_{\mathcal{W}} \psi_\mu = -D_\mu \phi \ , & \delta_{\mathcal{W}} \phi = 0 \ , \\ \delta_{\mathcal{W}} \chi_{\mu\nu} = F^+_{\mu\nu} \ , & \delta_{\mathcal{W}} \overline{\phi} = 2\eta \ , & \delta_{\mathcal{W}} \eta = \frac{1}{2} \left[\phi, \overline{\phi} \right] \ . \end{array} \tag{9}$$

The action of the vector generator on the set of fields is given by

$$\begin{aligned} \delta_\mu A_\nu &= \frac{1}{2} \chi_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \eta \ , \\ \delta_\mu \psi_\nu &= F_{\mu\nu} - \frac{1}{2} F^+_{\mu\nu} - \frac{1}{16} g_{\mu\nu} [\phi, \overline{\phi}] \ , \\ \delta_\mu \eta &= \frac{1}{2} D_\mu \overline{\phi} \ , \\ \delta_\mu \chi_{\sigma\tau} &= \frac{1}{8} (\varepsilon_{\mu\sigma\tau\nu} D^\nu \overline{\phi} + g_{\mu\sigma} D_\tau \overline{\phi} - g_{\mu\tau} D_\sigma \overline{\phi}) \ , \\ \delta_\mu \phi &= -\psi_\mu \ , \\ \delta_\mu \overline{\phi} &= 0 \ , \end{aligned} \tag{10}$$

and

$$\delta_{\mathcal{W}} \mathcal{S}_{TSYM} = \delta_\mu \mathcal{S}_{TSYM} = 0 \ . \tag{11}$$

We underline here that the form of the TSYM action (1) is not completely specified by the fermionic symmetry $\delta_{\mathcal{W}}$. In other words, (1) is not the most general gauge invariant action compatible with the $\delta_{\mathcal{W}}$-invariance. Nevertheless, it turns out to be uniquely characterized by δ_μ. The conditions (11) fix all the relative numerical coefficients of the Witten's action (1) , allowing, in particular, for a single coupling constant. This feature will be recovered in the renormalizability analysis of the model. The last generator, $\delta_{\mu\nu}$, will reproduce, together with the operators $\delta_{\mathcal{W}}, \delta_\mu$, the complete $N = 2$ susy algebra (5) , (6). The reasons why we do not actually take in further account the transformations $\delta_{\mu\nu}$ are due in part to the fact that, as previously remarked, the TSYM action is already uniquely fixed by the $(\delta_{\mathcal{W}},\ \delta_\mu)$−symmetries and in part to the fact that the generator $\delta_{\mu\nu}$ turns out to be trivially realized on the fields in terms of the $\delta_{\mathcal{W}}$-transformations *[12]*.

The second remark is related to the standard perturbative Feynman diagram computations. From the equivalence between $\mathcal{S}^{N=2}_{YM}$ and $\mathcal{S}_{TSYM}$ it is very tempting to argue that the values of quantities like the β-function should be the same when computed in the ordinary $N = 2$ Yang-Mills and in the twisted version. After all, at least at the perturbative level, the twisting procedure has the effect of a linear change of variables on the fields. The computation of the one loop β-function for the twisted theory has indeed been performed by R. Brooks et al. *[9]*. As expected, the result agrees with that of the untwisted $N = 2$ Yang-Mills.

III Quantizing the Twisted Theory

Following the procedure of *[7]*, we shall begin by looking for an extended BRST operator $\mathcal{Q}$ which turns out to be nilpotent on shell. To this purpose we first introduce the Faddeev-Popov ghost field c corresponding to the gauge transformations (2),

$$\begin{array}{lll} sA_\mu = -D_\mu c\ , & sc = c^2\ , & s\phi = [c,\phi]\ , \\ s\psi_\mu = \{c,\psi_\mu\}\ , & s\chi_{\mu\nu} = \{c,\chi_{\mu\nu}\}\ , & s\eta = \{c,\eta\}\ , \\ s\overline{\phi} = \left[c,\overline{\phi}\right]\ , & & \end{array} \tag{12}$$

$$s\mathcal{S}_{TSYM} = 0\,,\quad s^2 = 0\ . \tag{13}$$

We associate to each generator entering the algebra (5), namely $\delta_{\mathcal{W}}$, δ_μ and ∂_μ, the constant ghost parameters $(\omega,\varepsilon^\mu,v^\mu)$ respectively, defining, in this way, the extended BRST operator

$$\mathcal{Q} = s + \omega\delta_{\mathcal{W}} + \varepsilon^\mu\delta_\mu + v^\mu\partial_\mu - \omega\varepsilon^\mu\frac{\partial}{\partial v^\mu}\ . \tag{14}$$

Now, we have to define the action of the four generators $s,\delta_{\mathcal{W}},\delta_\mu$ and ∂_μ on the ghosts $(c,\omega,\varepsilon^\mu,v^\mu)$. Let us analyse in detail the case of the two operators s and $\delta_{\mathcal{W}}$. Working out eq.(5) explicitly, one looks then for an operator $(s+\omega\delta_{\mathcal{W}})$ nilpotent on the set of fields $(A_\mu,\psi_\mu,\eta,\phi,\overline{\phi},c,\omega)$ and nilpotent on shell on the field $\chi_{\mu\nu}$ *[12]*. After a little experiment, it is not difficult to convince oneself that these conditions are indeed verified by defining the action of s and $\delta_{\mathcal{W}}$ on the ghost (c,ω) as

$$s\omega = 0\ ,\quad \delta_{\mathcal{W}}\omega = 0\ ,\quad \delta_{\mathcal{W}}c = -\omega\phi\ . \tag{15}$$

The above procedure can be easily repeated in order to include in the game also the operators δ_μ and ∂_μ. The final result is that the extension of the operator $\mathcal{Q}$ on the ghosts $(c, \omega, \varepsilon^\mu, v^\mu)$ is found to be

$$\begin{aligned} \mathcal{Q}c &= c^2 - \omega^2\phi - \omega\varepsilon^\mu A_\mu + \frac{\varepsilon^2}{16}\overline{\phi} + v^\mu\partial_\mu c \,, \\ \mathcal{Q}\omega &= 0 \,, \quad \mathcal{Q}\varepsilon^\mu = 0 \,, \quad \mathcal{Q}v^\mu = -\omega\varepsilon^\mu \,, \end{aligned} \tag{16}$$

with

$$\mathcal{Q}^2 = 0 \quad \text{on} \quad \left(A, \phi, \overline{\phi}, \eta, c, \omega, \varepsilon, v\right) \,, \tag{17}$$

and

$$\begin{aligned} \mathcal{Q}^2\psi_\sigma &= \frac{g^2}{4}\omega\varepsilon^\mu \frac{\delta\mathcal{S}_{TYM}}{\delta\chi^{\mu\sigma}} \\ &+ \frac{g^2}{32}\varepsilon^\mu\varepsilon^\nu \left(g_{\mu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\nu} + g_{\nu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\mu} - 2g_{\mu\nu}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\sigma} \right) \,, \end{aligned} \tag{18}$$

$$\begin{aligned} \mathcal{Q}^2\chi_{\sigma\tau} &= -\frac{g^2}{2}\omega^2 \frac{\delta\mathcal{S}_{TYM}}{\delta\chi^{\sigma\tau}} \\ &+ \frac{g^2}{8}\omega\varepsilon^\mu \left(\varepsilon_{\mu\sigma\tau\nu}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi_\nu} + g_{\mu\sigma}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\tau} - g_{\mu\tau}\frac{\delta\mathcal{S}_{TYM}}{\delta\psi^\sigma} \right) \,. \end{aligned} \tag{19}$$

For the usefulness of the reader, we give in the tables 1 and 2 the quantum numbers and the Grassmanian characters of all the fields and constant ghosts. We observe that the grading is chosen to be the sum of the ghost number and of the $\mathcal{R}$-charge.

	A_μ	$\chi_{\mu\nu}$	ψ_μ	η	ϕ	$\overline{\phi}$
dim.	1	3/2	3/2	3/2	1	1
$\mathcal{R}$-charge	0	-1	1	-1	2	-2
gh-number	0	0	0	0	0	0
nature	*comm.*	*ant.*	*ant.*	*ant.*	*comm.*	*comm.*

Table 1: Quantum numbers

	c	ω	ε_μ	v_μ
dim.	0	$-1/2$	$-1/2$	-1
$\mathcal{R}$-charge	0	-1	1	0
gh-number	1	1	1	1
nature	*ant.*	*comm.*	*comm.*	*ant.*

Table 2: Quantum numbers

The construction of the gauge fixing term is now straightforward. We introduce an antighost $\overline{c}$ and a Lagrangian multiplier b transforming as *[7, 8]*

$$\mathcal{Q}\overline{c} \; = b + v^\mu \partial_\mu \overline{c} \; , \quad \mathcal{Q}b = \omega \varepsilon^\mu \partial_\mu \overline{c} + v^\mu \partial_\mu b \; . \tag{20}$$

Thus, for the gauge fixing action we get

$$\begin{aligned} \mathcal{S}_{gf} &= Q \int d^4x \; tr(\overline{c}\partial A) \\ &= tr \int d^4x \; \left(b\partial A + \overline{c}\partial Dc - \omega \overline{c}\partial\psi - \frac{\varepsilon^\nu}{2}\overline{c}\partial^\mu \chi_{\nu\mu} - \frac{\varepsilon^\mu}{8}\overline{c}\partial_\mu \eta \right) \; , \end{aligned} \tag{21}$$

so that the gauge fixed action $(\mathcal{S}_{TSYM} + S_{gf})$ is $\mathcal{Q}$-invariant. The above equation means that the gauge fixing procedure has been worked out by taking into account not only the pure local gauge symmetry but also the additional nonlinear invariances $\delta_{\mathcal{W}}$ and δ_μ.

In order to obtain the Slavnov-Taylor identity we first couple the nonlinear $\mathcal{Q}$-transformations of the fields $(c, \phi, A, \psi, \overline{\phi}, \eta, \chi)$ to a set of antifields $(c^*, \phi^*, A^*, \psi^*, \overline{\phi}^*, \eta^*, \chi^*)$,

$$\mathcal{S}_{ext} = tr \int d^4x \; (\; \Phi^{*i} \mathcal{Q} \Phi_i) \; , \tag{22}$$

where Φ^i, Φ^{*i} represent all fields and respective antifields. Moreover, taking into account that the extended operator $\mathcal{Q}$ is nilpotent only modulo the equations of motion of the fields ψ_μ and $\chi_{\mu\nu}$, we also introduce a term quadratic in the corresponding antifields $\psi^{*\mu}, \chi^{*\mu\nu}$, *i.e.*

$$\mathcal{S}_{quad} = tr \int d^4x \left(\frac{g^2}{8}\omega^2 \chi^{*\mu\nu}\chi^*_{\mu\nu} - \frac{g^2}{4}\omega \chi^{*\mu\nu}\varepsilon_\mu \psi^*_v - \frac{g^2}{32}\varepsilon^\mu \varepsilon^\nu \psi^*_\mu \psi^*_v + \frac{g^2}{32}\varepsilon^2 \psi^{*\mu}\psi^*_\mu \right) \; . \tag{23}$$

The complete action

$$\Sigma = \mathcal{S}_{TSYM} + \mathcal{S}_{gf} + \mathcal{S}_{ext} + \mathcal{S}_{quad} \ , \tag{24}$$

obeys the classical Slavnov-Taylor identity

$$\begin{aligned} \mathcal{S}(\Sigma) = tr \int d^4x \left(\frac{\delta\Sigma}{\delta\Phi^{*i}} \frac{\delta\Sigma}{\delta\Phi_i} + (b + v^\mu \partial_\mu \overline{c}) \frac{\delta\Sigma}{\delta\overline{c}} \right. \\ \left. + (\omega\varepsilon^\mu \partial_\mu \overline{c} + v^\mu \partial_\mu b) \frac{\delta\Sigma}{\delta b} \right) - \omega\varepsilon^\mu \frac{\partial\Sigma}{\partial v^\mu} = 0 \ . \end{aligned} \tag{25}$$

This equation will be the starting point for the analysis of the renormalizability of the model.

At this point, it is worthwhile drawing the attention to a particular feature of the complete action Σ given by eq.(24). One should notice that, in this action, the global ghost parameter ω only appears analytically. Accordingly, we expect that the physical sectors of the theory should be characterized by field polynomials which are analytic in ω. This information will be of great relevance when we come to the characterization of the possible counterterms and anomalies of the theory.

The Slavnov-Taylor identity (25) can be simplified by using the fact that the complete action Σ is invariant under space-time translations. Indeed, the dependence of Σ on the corresponding translation constant ghost v^μ turns out to be fixed by the following linearly broken Ward identity

$$\begin{aligned} \frac{\partial\Sigma}{\partial v^\mu} = \Delta^{cl}_\mu = tr \int d^4x (c^* \partial_\mu c - \phi^* \partial_\mu \phi - A^{*\nu} \partial_\mu A_\nu + \psi^{*\nu} \partial_\mu \psi_\nu - \overline{\phi}^* \partial_\mu \overline{\phi} \\ + \eta^* \partial_\mu \eta + \frac{1}{2} \chi^{*\nu\sigma} \partial_\mu \chi_{\nu\sigma}) \ . \end{aligned} \tag{26}$$

This means that we can completely eliminate the global constant ghost v^μ without any further consequence. Introducing the action $\widehat{\Sigma}$ through

$$\Sigma = \widehat{\Sigma} + v^\mu \Delta^{cl}_\mu \ , \qquad \frac{\partial\widehat{\Sigma}}{\partial v^\mu} = 0 \ , \tag{27}$$

it is easily verified from (25) that $\widehat{\Sigma}$ obeys the modified Slavnov-Taylor identity

$$\mathcal{S}(\widehat{\Sigma}) = \omega\varepsilon^\mu \Delta^{cl}_\mu \ . \tag{28}$$

Besides (28), the classical action $\widehat{\Sigma}$ turns out to be characterized by further additional constraints *[13]*, namely the Landau gauge fixing condition, the antighost equation, and the linearly broken ghost Ward identity (typical of the Landau gauge), respectively

$$\frac{\delta\widehat{\Sigma}}{\delta b} = \partial A \ , \qquad \frac{\delta\widehat{\Sigma}}{\delta\overline{c}} + \partial_\mu \frac{\delta\widehat{\Sigma}}{\delta A^*_\mu} = 0 \ ,$$
$$\int d^4x \left(\frac{\delta\widehat{\Sigma}}{\delta c} + \left[\overline{c}, \frac{\delta\widehat{\Sigma}}{\delta b} \right] \right) = \Delta^{cl}_c \ , \tag{29}$$

with Δ^{cl}_c a linear classical breaking

$$\Delta^{cl}_c = \int d^4x \ \left([c, c^*] - [A, A^*] - [\phi, \phi^*] + [\psi, \psi^*] - [\overline{\phi}, \overline{\phi}^*] + [\eta, \eta^*] + \frac{1}{2}[\chi, \chi^*] \right) \ . \tag{30}$$

Following the standard procedure, let us introduce the so called reduced action *[13]* $\widetilde{\mathcal{S}}$ defined through the gauge fixing condition (29) as

$$\widehat{\Sigma} = \widetilde{\mathcal{S}} \ + \ tr \int d^4x \ b\partial A \ , \tag{31}$$

so that $\widetilde{\mathcal{S}}$ is independent from the Lagrangian multiplier b. Moreover, from the antighost equation (29) it follows that $\widetilde{\mathcal{S}}$ depends from the antighost $\overline{c}$ only through the combination $A^*_\mu + \partial_\mu \overline{c}$. From now on A^*_μ will stand for this combination. Accordingly, for the Slavnov-Taylor identity we get

$$\mathcal{S}(\widetilde{\mathcal{S}}) = tr \int d^4x \left(\frac{\delta\widetilde{\mathcal{S}}}{\delta\Phi^{*i}} \frac{\delta\widetilde{\mathcal{S}}}{\delta\Phi_i} \right) = \omega\varepsilon^\mu \Delta^{cl}_\mu \ . \tag{32}$$

As a consequence the linearized Slavnov-Taylor operator $\mathcal{B}_{\widetilde{\mathcal{S}}}$ defined as

$$\mathcal{B}_{\widetilde{\mathcal{S}}} = tr \int d^4x \left(\frac{\delta\widetilde{\mathcal{S}}}{\delta\Phi^i} \frac{\delta}{\delta\Phi^*_i} + \frac{\delta\widetilde{\mathcal{S}}}{\delta\Phi^*_i} \frac{\delta}{\delta\Phi^i} \right) \tag{33}$$

is not nilpotent. Instead, we have

$$\mathcal{B}_{\widetilde{\mathcal{S}}}\mathcal{B}_{\widetilde{\mathcal{S}}} = \omega\varepsilon^\mu \mathcal{P}_\mu \ , \tag{34}$$

meaning that $\mathcal{B}_{\widetilde{\mathcal{S}}}$ is nilpotent only modulo a total derivative. It follows then that $\mathcal{B}_{\widetilde{\mathcal{S}}}$ becomes a nilpotent operator when acting on the space of the integrated local polynomials in the fields and antifields. This is the case, for instance, of the invariant counterterms and of the anomalies.

IV Renormalization of the Twisted Theory

We are now ready to discuss the renormalization of the twisted $N = 2$ Yang-Mills theory. The first task is that of characterizing the cohomology classes of the linearized Slavnov-Taylor operator which turn out to be relevant for the anomalies and the invariant counterterms. Let us recall that both anomalies and invariant counterterms are integrated local polynomials Δ^G in the fields, antifields, and in the global ghosts (ω, ε), with dimension four, vanishing $\mathcal{R}$-charge and ghost number G respectively one and zero. In addition, they are constrained by the consistency condition

$$\mathcal{B}_{\widetilde{\mathcal{S}}}\Delta^G = 0 \ , \qquad G = 0,1 \ . \tag{35}$$

In order to characterize the integrated cohomology of $\mathcal{B}_{\widetilde{\mathcal{S}}}$ we introduce the operator $\mathcal{N}_\varepsilon = \varepsilon^\mu \partial/\partial\varepsilon^\mu$, which counts the number of global ghosts ε^μ contained in a given field polynomial. Accordingly, the functional operator $\mathcal{B}_{\widetilde{\mathcal{S}}}$ displays the following ε-expansion

$$\mathcal{B}_{\widetilde{\mathcal{S}}} = b_{\widetilde{\mathcal{S}}} + \varepsilon^\mu \mathcal{W}_\mu + \frac{1}{2}\varepsilon^\mu\varepsilon^\nu \mathcal{W}_{\mu\nu} \ , \tag{36}$$

where, from eq.(34) the operators $b_{\widetilde{\mathcal{S}}}$, $\mathcal{W}_\mu$, $\mathcal{W}_{\mu\nu}$ are easily seen to obey the following algebraic relations

$$b_{\widetilde{\mathcal{S}}} b_{\widetilde{\mathcal{S}}} = 0 \ , \qquad \left\{ b_{\widetilde{\mathcal{S}}}, \mathcal{W}_\mu \right\} = \omega \mathcal{P}_\mu \ , \tag{37}$$

$$\begin{aligned} &\{\mathcal{W}_\mu, \mathcal{W}_\nu\} + \left\{ b_{\widetilde{\mathcal{S}}}, \mathcal{W}_{\mu\nu} \right\} = 0 \ , \\ &\{\mathcal{W}_\mu, \mathcal{W}_{\nu\rho}\} + \{\mathcal{W}_\nu, \mathcal{W}_{\rho\mu}\} + \{\mathcal{W}_\rho, \mathcal{W}_{\mu\nu}\} = 0 \ , \\ &\{\mathcal{W}_{\mu\nu}, \mathcal{W}_{\rho\sigma}\} + \{\mathcal{W}_{\mu\rho}, \mathcal{W}_{\nu\sigma}\} + \{\mathcal{W}_{\mu\sigma}, \mathcal{W}_{\nu\rho}\} = 0 \ . \end{aligned} \tag{38}$$

From (37) we observe that the operator $b_{\widetilde{\mathcal{S}}}$ is strictly nilpotent and that the vector operator $\mathcal{W}_\mu$ allows to decompose the space-time translations $\mathcal{P}_\mu$ as a $b_{\widetilde{\mathcal{S}}}-$anticommutator, providing thus an off-shell realization of the algebra (5). The operator $b_{\widetilde{\mathcal{S}}}$ is just given by

$$b_{\widetilde{\mathcal{S}}} = s + \omega\delta_{\mathcal{W}} \ . \tag{39}$$

According to the general results of *[14]*, the integrated cohomology of $\mathcal{B}_{\widetilde{\mathcal{S}}}$ is isomorphic to a subspace of the integrated cohomology of $b_{\widetilde{\mathcal{S}}}$ *[14]*. Since $b_{\widetilde{\mathcal{S}}}$ is exactly nilpotent, one can pass from the integrated version of the Wess-Zumino consistency condition (35) to its local version, which leads to the following set of descent equations,

$$
\begin{aligned}
b_{\widetilde{S}}\Omega^G_4 \;+\; \omega\partial^\mu\Omega^G_{\frac{7}{2}\mu} &= 0\;, \\
b_{\widetilde{S}}\Omega^G_{\frac{7}{2}\mu} \;+\; \omega\partial^\nu\Omega^G_{3[\mu\nu]} &= 0\;, \\
b_{\widetilde{S}}\Omega^G_{3[\mu\nu]} \;+\; \omega\partial^\rho\Omega^G_{\frac{5}{2}[\mu\nu\rho]} &= 0\;, \\
b_{\widetilde{S}}\Omega^G_{\frac{5}{2}[\mu\nu\rho]} \;+\; \omega\partial^\sigma\Omega^G_{2[\mu\nu\rho\sigma]} &= 0\;, \\
b_{\widetilde{S}}\Omega^G_{2[\mu\nu\rho\sigma]} &= 0\;,
\end{aligned}
\tag{40}
$$

where the cocycle Ω^G_D has ghost number G and dimension D.

The presence of the parameter ω in front of all the derivatives in the above set of equations is a feature of the algebra given by (37). In fact, as we have commented before in the preceding section, we are interested in the characterization of those cohomologically nontrivial cocycles which are given by local field polynomials depending analytically on the parameter ω. In other words, a cocycle will be nontrivial if it is analytic in ω and if it cannot be written as a $b_{\widetilde{S}}$-variation of any local field polynomial analytic in ω. Now, using the algebra (37), one can see that the solutions Ω^G_D of the descent equations (40) can be obtained by suitably applying the operator $\mathcal{W}_\mu$ on the nontrivial solutions of the local cohomology of $b_{\widetilde{S}}$ in each level of the descent equations *[15]*. Also, it is not difficult to show that the operator $\mathcal{W}_\mu$ preserves the analyticity in ω of the space where it acts upon, *i.e* it transforms local polynomials analytic in ω into local polynomials analytic in ω. Then, as the nontrivial solutions of the local cohomology of $b_{\widetilde{S}}$ belong to this analytic space, it is assured that $\mathcal{W}_\mu$ will map such solutions into nontrivial solutions of the cohomology of $b_{\widetilde{S}}$ modulo total derivatives. As a consequence, this latter cohomology will also be restricted to the space of field polynomials analytic in ω.

We are interested in the solutions of the descent equations in the case of the invariant counterterms and of the gauge anomalies, corresponding respectively to the sectors of ghost number $G = 0, 1$. In the case of the gauge anomalies, one can show that there is no possible nontrivial solution for the local cohomology of $b_{\widetilde{S}}$ with the correct quantum numbers at any level of the descent equations (40). It is important to mention that this result, already obtained in *[8]* in the analysis of the $N = 2$ untwisted gauge theories, means that there is no possible extension of the nonabelian Adler-Bardeen gauge anomaly compatible with $N = 2$ supersymmetry.

In the case of the invariant counterterms, the analysis of the local cohomology of $b_{\widetilde{S}}$ shows the existence of two nontrivial solutions in different levels of the descent equations. The first one has dimension 2 and is given by

$$\Delta = \frac{1}{2} tr\phi^2 \ . \tag{41}$$

It is a solution of the local cohomology of $b_{\widetilde{S}}$ for the last of the eqs.(40). The second term has dimension 3,

$$\Delta_{\mu\nu} = a \left(F^+_{\mu\nu}\phi + \frac{g^2\omega}{2} B_{\mu\nu}\phi \right) \ , \tag{42}$$

and it is a solution of the local cohomology of $b_{\widetilde{S}}$ at the intermediate level of $\Omega^0_{3[\mu\nu]}$.

Before analyzing the consequences which follow from the above results on the cohomology of the complete operator $\mathcal{B}_{\widetilde{S}}$, let us discuss here the important issue of the analyticity in the constant ghosts. In fact, the requirement of analyticity in the ghosts $(\varepsilon_\mu, \omega)$, stemming from pure perturbative considerations, is one of the most important ingredients in the cohomological analysis that we are doing. It is an almost trivial exercise to show that both cocycles (41) and (42) can be expressed indeed as a pure $b_{\widetilde{S}}-$variation, namely

$$\Delta = \frac{1}{2} b_{\widetilde{S}} tr \left(-\frac{1}{\omega^2} c\phi + \frac{1}{3\omega^4} c^3 \right) \ , \tag{43}$$

$$\Delta_{\mu\nu} = a b_{\widetilde{S}} tr \left(\frac{1}{\omega} \phi \chi_{\mu\nu} \right) \ . \tag{44}$$

These expressions illustrate in a very clear way the relevance of the analyticity requirement. It is apparent from the eq.(43) that the price to be payed in order to write $tr\phi^2$ as a pure $b_{\widetilde{S}}-$variation is in fact the loss of analyticity in the ghost ω.

In other words, as long as one works in a functional space whose elements are power series in the constant ghosts, the cohomology of $b_{\widetilde{S}}$ is not empty. On the other hand, if the analyticity requirement is given up, the cohomology of $b_{\widetilde{S}}$, and therefore that of the complete operator $\mathcal{B}_{\widetilde{S}}$, becomes trivial, leading thus to the cohomological interpretation of Baulieu-Singer *[4]* and Labastida-Pernici *[3]*. One goes from the standard field theory point of view, of $N = 2$ super Yang-Mills, to the cohomological one, of the topological Yang-Mills theory, by simply setting $\omega = 1$, which of course implies that analyticity is lost. In addition, it is rather simple to convince oneself that setting $\omega = 1$ has the meaning of identifying the $\mathcal{R}$-charge with the ghost number, so that the fields $(\chi, \psi, \eta, \phi, \overline{\phi})$ acquire a nonvanishing ghost number given respectively by $(-1, 1, -1, 2, -2)$. They correspond now to the so called topological ghosts of the cohomological interpretation.

It is also interesting to point out that there is some relationship between the analyticity in the global ghosts and the so called equivariant cohomology proposed by *[5, 6]* in order to recover the Witten's observables *[16, 17]*. Roughly speaking, the equivariant cohomology can be defined as the restriction of the BRST cohomology to the space of the gauge invariant polynomials which cannot be written as the BRST variation of local quantities which are independent from the Faddeev-Popov ghost c. Considering now the polynomial $tr\phi^2$, we see that it yields a nontrivial equivariant cocycle in the cohomological interpretation (*i.e.* $\omega = 1$), due to the unavoidable presence of the Faddeev-Popov ghost c on the right hand side of eq.(43). However, the nontriviality of the second cocycle (42) relies exclusively on the analyticity requirement.

In fact, it is not surprising at all to have found two independent solutions of the local cohomology of $b_{\widetilde{S}}$. One should remember that the operator $\delta_{\mathcal{W}}$, which builds with s the operator $b_{\widetilde{S}}$ of eq.(39) is not sufficient to completely fix the coefficients of the TSYM action (1). One needs to impose the invariance under δ_μ in order to completely specify (1). The reflection of this point is the existence of a second cocycle in the descent equations for the operator $b_{\widetilde{S}}$. At this level, this does not mean the existence of a second β-function, beyond that associated to the gauge coupling g. Our interest, at the end, is in the integrated cocycles invariant under $\mathcal{B}_{\widetilde{S}}$ (36). Then, our approach is to climb the descent equations (40), which will give us the final solution on the integrated cohomology of $b_{\widetilde{S}}$, and afterwards demand invariance under $\mathcal{B}_{\widetilde{S}}$.

In order to climb the descent equations, one can apply the operator $\mathcal{W}_\mu$, and reach the solution at the upper level for Ω^0_4. This solution, when integrated, can be written in the form

$$\Omega^0 \;=\; \varepsilon^{\mu\nu\rho\tau}\mathcal{W}_\mu\mathcal{W}_\nu\mathcal{W}_\rho\mathcal{W}_\tau \int d^4x\ \Delta\ + \mathcal{W}^\mu\mathcal{W}^\nu \int d^4x\ \Delta_{\mu\nu} + b_{\widetilde{S}} - variation$$

$$= \mathcal{S}_{TSYM} + a\Xi + b_{\widetilde{\mathcal{S}}}\widetilde{\Omega}^{-1} \ , \tag{45}$$

where $\widetilde{\Omega}^{-1}$ is an arbitrary integrated polynomial in the fields analytic in ω with ghost number -1, and

$$\begin{aligned}\Xi &= \int d^4x \left(\frac{g^2\omega}{4} F^{+\mu\nu} B_{\mu\nu} + \frac{g^4\omega^2}{8} B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\chi^{\mu\nu}(D_\mu\psi_\nu - D_\nu\psi_\mu)^+ \right. \\ &\quad -\frac{1}{4}\phi\left\{\chi^{\mu\nu},\chi_{\mu\nu}\right\} - \frac{3}{4}\,\psi^\mu D_\mu\eta - \frac{3}{4}\omega g^2\phi D^\mu\xi_\mu - \frac{3}{4}\omega g^2\psi^\mu\gamma_\mu \\ &\quad \left. +\frac{3}{16}\phi\left\{\eta,\eta\right\} - \frac{3}{2}\omega^2 g^2\phi L + \frac{3}{4}\omega g^2\phi\left[\overline{\phi},\tau\right]\right) \ .\end{aligned} \tag{46}$$

Now, we have to impose the invariance of Ω^0 under $\mathcal{B}_{\widetilde{\mathcal{S}}}$ (36), which, in particular, means invariance under $\mathcal{W}_\mu$

$$\mathcal{W}_\mu\Omega^0 = \mathcal{W}_\mu\,\left(\mathcal{S}_{TSYM} + a\Xi + b_{\widetilde{\mathcal{S}}}\widetilde{\Omega}^{-1}\right) = 0 \ . \tag{47}$$

Obviously, the action $\mathcal{S}_{TSYM}$ of eq.(1) is already invariant under $\mathcal{W}_\mu$. Then, using the algebra (38), the equation (47) gives the consistency condition

$$a\mathcal{W}_\mu\Xi = b_{\widetilde{\mathcal{S}}}\Lambda_\mu^{-1} \ , \tag{48}$$

where Λ_μ^{-1} is an arbitrary integrated polynomial in the fields analytic in ω with ghost number -1. This implies that either $\mathcal{W}_\mu\Xi$ is a trivial cocycle analytic in ω, or the coefficient a has to vanish. One can show, by a straightforward calculation, that the only way to write $\mathcal{W}_\mu\Xi$ as an exact cocycle is to loose the analyticity in ω

$$\begin{aligned}\mathcal{W}_\mu\Xi &= -\frac{1}{\omega}b_{\widetilde{\mathcal{S}}}\left(\frac{3}{8}F^-_{\mu\nu}D^\nu\overline{\phi} + \frac{3}{64}\left[\phi,\overline{\phi}\right]D_\mu\overline{\phi} - \frac{3}{8}\psi^\nu\left[\chi_{\mu\nu},\overline{\phi}\right]\right. \\ &\quad +\frac{3}{32}\psi_\mu\left[\eta,\overline{\phi}\right] - \frac{1}{16}\omega g^2 B_{\mu\nu}D^\nu\overline{\phi} + \frac{1}{4}\omega g^2\chi_{\mu\nu}\gamma^\nu \\ &\quad \left. -\frac{3}{8}\omega g^2 F^-_{\mu\nu}\xi^\nu - \frac{3}{64}\omega g^2\xi_\mu\left[\phi,\overline{\phi}\right] - \frac{3}{4}\omega g^2\psi_\mu D + \frac{1}{16}\omega^2 g^4 B_{\mu\nu}\xi^\nu\right) \ ,\end{aligned} \tag{49}$$

i.e., the only allowed solution for the equation (48) is given by $a = 0$. Then, going back to eq.(45), we can see that our cohomological analysis finally leads us to the conclusion that the nontrivial part of the counterterm Ω^0 can be written as the starting $N = 2$ super-Yang-Mills action modulo a trivial $b_{\widetilde{\mathcal{S}}}$-term

$$\Omega^0 = \mathcal{S}_{TSYM} + b_{\widetilde{\mathcal{S}}}\widetilde{\Omega}^{-1}. \tag{50}$$

The fact that we were left with only one arbitrary coefficient in the nontrivial part of the counterterm (which is the global coefficient of $\mathcal{S}_{TSYM}$) means the presence of only one coupling in the theory, and consequently, of only one β-function for the twisted $N = 2$ Yang-Mills theory.

V Conclusion

We have shown how the quantization of the twisted $N = 2$ super-Yang-Mills action (which is just Witten's action for TYM theory) can be done by taking into account the full $N = 2$ twisted supersymmetric algebra. Then, the analysis of the renormalizability was performed along the standard lines. Nontrivial cohomology classes were characterized by demanding analyticity in the twisted constant global ghosts of $N = 2$ supersymmetry in the quantized theory. The analyticity requirement, following from perturbation theory, plays a crucial rôle as it defines a criterium to select the nontrivial physical space of the $N = 2$ theory.

We have seen that the operator $b_{\widetilde{S}}$ has a nonvanishing integrated analytic cohomology only in the sector of the invariant counterterms, coming from the nontrivial local elements given by (41) and (42). Finally, we were able to show that the cohomology of $\mathcal{B}_{\widetilde{S}}$ in the sector of the invariant counterterms contains a unique element given in eq.(50). This result is in complete agreement with that found in the case of untwisted $N = 2$ YM *[8]*.

Moreover, as a conclusion of eq.(50), we stress that the origin of the twisted $N = 2$ super-Yang-Mills action (1) can be traced back, modulo an irrelevant exact cocycle, to the invariant polynomial $tr\phi^2$, eq.(41). Let us also recall here that the explicit Feynman diagrams computation yields a nonvanishing value for the renormalization of the gauge coupling, meaning that the twisted version of $N = 2$ YM possesses a nonvanishing β-function for g. The latter agrees with that of the pure $N = 2$ untwisted Yang-Mills *[9]*. Moreover, it is well known that the β-function of $N = 2$ Yang-Mills theory receives only one loop order contributions *[18]*. On the other hand it is known since several years that in the $N = 2$ susy gauge theories the Green's functions with the insertion of composite operators of the kind of the invariant polynomials of the form $tr\phi^n$ display remarkable finiteness properties and can be computed exactly, even when nonperturbative effects are taken into account *[19]*. It is natural therefore to conjecture that the finiteness properties of $tr\phi^2$ are at the origin of the absence of higher order corrections for the gauge β-function of both twisted and untwisted $N = 2$ gauge theories. This can give us a deeper understanding of the nonrenormalization theorem for the $N = 2$ gauge β-function. A purely algebraic proof of this important nonrenormalization theorem is under investigation.

Acknowledgements The financial supports from CNPq - Brazil, SR-2 UERJ and from FAPERJ-Rio are gratefully acknowledged.

References

[1] E. Witten, **Comm. Math. Phys. 117 (1988) 353;**

[2] M. Mariño, "*The Geometry of Supersymmetric Gauge Theories in Four Dimensions*", **US-Ft-2/97, hep-th/9701128;**

[3] J.M.F. Labastida and M. Pernici, **Phys. Lett. B212 (1988) 56;**

[4] L. Baulieu and I.M. Singer, **Nucl. Phys. B15 (1988) 12;**
L. Baulieu and I.M. Singer, **Comm. Math. Phys. 135 (1991) 253;**

[5] S. Ouvry, R. Stora and P. Van Baal, **Phys. Lett. B220 (1989) 159;**

[6] J. Kalkman, **Comm. Math. Phys. 153 (1993) 447;**
R. Stora, "*Equivariant Cohomology and Topological Theories*", in BRS Symmetry, M. Abe, N. Nakanishi, Iojima eds., **Universal Academy Press, Tokyo, Japan, 1996;**
R. Stora, F. Thuillier and J.C. Wallet, "*Algebraic Structure of Cohomological Field Theory Models and Equivariant Cohomology*", lectures given at the First Caribbean Spring School of Mathematics and Theoretical Physics, R. Coquereaux, M. Dubois-Violette and P. Flad Eds., **World Scientific Publ., 1995;**

R. Stora, *"Exercises in Equivariant Cohomology"*, **ENSLAPP-A-619/96, hep-th/9611114**;

R. Stora, *"De la fixation de jauge consideree comme un des beaux arts et de la symetrie de Slavnov qui s 'ensuit"*, **ENSLAPP-A-620/96, hep-th/9611115;**

R. Stora, *"Exercises in Equivariant Cohomology and Topological Theories "*, **hep-th/9611116;**

[7] P. L. White, **Class. Quantum Grav. 9 (1992) 413;**
P. L. White, **Class. Quantum Grav. 9 (1992) 1663;**

[8] N. Maggiore, **Int. J. Mod. Phys. A10 (1995) 3781;**
N. Maggiore, **Int. J. Mod. Phys. A10 (1995) 3937;**

[9] R. Brooks, D. Montano and J. Sonnenschein, **Phys. Lett. B214 (1988) 91;**

[10] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, **Nucl. Phys. B346 (1990) 313;**
E. Guadagnini, N. Maggiore and S.P. Sorella, **Phys. Lett. B255 (1991) 65;**
N. Maggiore and S.P. Sorella, **Int. Journ. Mod. Phys. A8 (1993) 929;**
C. Lucchesi, O. Piguet and S.P. Sorella, **Nucl. Phys. B395 (1993) 325;**
L.C.Q. Vilar, O. S. Ventura, C.A.G. Sasaki and S.P. Sorella, *"Algebraic Characterization of Vector Supersymmetry in Topological Field Theories"*, **CBPF-NF-007/97, to appear in Journ. Math. Phys.;**

[11] D. Birmingham, M. Blau, M. Rakowski and G. Thompson, **Phys. Rep. 209 (1991) 129;**

[12] F. Fucito, A. Tanzini, L.C.Q. Vilar, O.S. Ventura, C.A.G. Sasaki and S.P. Sorella, *"Perturbative Twisted Considerations on Topological Yang-Mills Theory and on N=2 Supersymmetric Gauge Theories"*, lectures given at the First School on Field Theory and Gravitation of Vitoria, **CBPF-NF-043/97, hep-th/9707209;**

[13] O. Piguet and S.P. Sorella, *Algebraic Renormalization,* **Monographs Series, m 28, Springer Verlag, Berlin, 1995;**

[14] J. A. Dixon, **Comm. Math. Phys. 139 (1991) 495;**

[15] S.P. Sorella, **Comm. Math. Phys. 157 (1993) 231;**
S.P. Sorella and L. Tataru, **Phys. Lett. B324 (1994) 351;**

[16] F. Delduc, N. Maggiore, O. Piguet and S. Wolf, **Phys. Lett. B385 (96) 132;**

[17] A. Blasi and R. Collina, **Phys. Lett. B222 (1989) 159;**

[18] M. Sohnius and P. West, **Phys. Lett. B100 (1981) 45;**
P. Howe, K. Stelle and P. West, **Phys. Lett. B124 (1983) 55;**

[19] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi and G. Veneziano, **Phys. Rep. 162 (4) (88) 169.**