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TELEPORTATION OF SCHRÖDINGER CAT STATES

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Abstract: In this work we propose a scheme in which a Schrödinger cat state generated in a cavity $C1$ can be teleported to another cavity $C3$ by using a three level atom $A2$ as the intermediate system. We have borrowed the teminology introduced by Bennett et al. (Phys. Rev. Lett. **29**, 1985 (1993)) and called this process as teleportation of Schrödinger cat states.

Key-words: Teleportation; Cat states.

Nowadays, the possibility of constructing electromagnetic cavities with large quality factors allows for the engineering of electromagnetic field states with very peculiar features. Among these states are Schrödinger cat states. Such states are coherent superpositions of two distinct macroscopic states and have interesting properties related to fundamental questions in quantum mechanics [1]. One may construct mesoscopic superpositions of coherent states which are known in the literature as even and odd coherent states which are superpositions of two different coherent states and have been studied extensively in the literature[2].

In what follows we make use of these peculiar states in connection with an also very peculiar process, i.e., the teleportation process[3][4]. We have borrowed the term teleportation from the original work by Benett et al [3]. More specifically, using a relatively simple and interesting scheme, here we show that it is possible to reproduce a Schrödinger cat state $|\Phi\rangle_1$, which is a superposition of an even and an odd coherent state prepared in cavity $C1$ to another cavity $C3$, i.e., $|\Phi\rangle_1$ is teleported from cavity $C1$ to cavity $C3$. Therefore, the proposition presented here involves the teleportation of a quasi macroscopic state of the electromagnetic field. Of course, we have to work with cavities of extremely high quality factors (Q) so that the decoherence process does not "kill the cat" before, during and after the teleportation process. We should remind that the progress in building up cavities of very high Q factors is relatively recent [5][6] and maybe future technological developments will permit the construction of cavities with Q factors higher than the ones available nowadays. This would impose less restrictive conditions in experiments involving quasi macroscopic quantum objects such as Schrödinger cat states. In spite of the restrictive conditions due to decoherence, it is important to develop studies involving Schrödinger cat states since they shed light on fundamental questions in quantum mechanics, and also envisaging future technological achievements that could yield to the possibility of realization of experiments which are considered as hard in the present days. The process described below could also serve as an starting point for future simple schemes of teleportation involving field states or atomic states as well.

In what follows we will show that the teleportation procedure works for a given set of states. Alice is given a Shrödinger cat, so that she cannot identify completely which state she has. However, she will be able to send this state to Bob. The set of allowed states which can be teleported are,

$$|\Phi\rangle = c_+|+\rangle + c_-|-\rangle, \tag{1}$$

where the states $|\pm\rangle$ are the normalized Schrödinger cat states in a cavity,

$$|\pm\rangle \cong \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle), \tag{2}$$

where we have assumed $|\alpha|^2 \gg 1$ and $2(1 \pm e^{-2|\alpha|^2}) \cong 2$. Notice that this is valid for say $|\alpha|^2 \approx 10$.

First we will discuss a possible way of building up states like (1) in cavity $C1$ and the restrictions on these states. Secondly we discuss the teleportation process, that is, given a cavity $C1$ prepared in an unknown state $|\Phi\rangle_1$ to the sender (Alice) she will be able to "teleport" it to the receiver's cavity $C3$ elsewhere (Bob).

We start with the Hamiltonian of a degenerate three-level lambda atom interacting with a field cavity mode

$$\begin{aligned} H &= \hbar\omega a^\dagger a + \hbar\omega_{aa}|a\rangle\langle a| \\ &\quad +\hbar\omega_{bb}|b\rangle\langle b| + \hbar\omega_{cc}|c\rangle\langle c| \\ &\quad +\hbar g a(|a\rangle\langle b| + |a\rangle\langle c|) + \hbar g a^\dagger(|b\rangle\langle a| + |c\rangle\langle a|), \end{aligned} \tag{3}$$

where $|a\rangle$, $|b\rangle$ and $|c\rangle$ are the upper and the two degenerated lower atomic levels respectively. We have assumed the same coupling constant g for the coupling of the field mode with the transitions $|a\rangle \rightleftharpoons |c\rangle$ and $|a\rangle \rightleftharpoons |b\rangle$. In the far off resonance limit we can eliminate level $|a\rangle$ adiabatically so that the dynamic evolution of the atom-field system is given by the time evolution operator [7] [8]

$$\begin{aligned} U(\tau) &= \frac{1}{2}(e^{i\varphi a^\dagger a} + 1)|b\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^\dagger a} - 1)|b\rangle\langle c| \\ &\quad +\frac{1}{2}(e^{i\varphi a^\dagger a} - 1)|c\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^\dagger a} + 1)|c\rangle\langle c|, \end{aligned} \tag{4}$$

where $\varphi = 2g^2\tau/\Delta$ with $\Delta = \omega_{ab} - \omega$, and τ is the atom-field interaction time.

In the first step someone prepares a cat state superposition in cavity $C1$. For instance, a possibility of building up state $|\Phi\rangle_1$ in cavity $C1$ can be realized as follows (see Fig.

1). Suppose we prepare cavity $C1$ initially in a coherent state $|\alpha'\rangle_1$. Then we prepare a two-level atom $A0$ in a coherent superposition passing through a first Ramsey zone $R0_1$

$$| \psi\rangle_0 = c_a \mid a\rangle_0 + c_b \mid b\rangle_0$$

and after that, it flies through cavity $C1$. The $\mid a\rangle_0 \rightleftharpoons \mid b\rangle_0$ transition is far off resonance with the cavity so that the interaction of the atom with the cavity mode in $C1$ is described by the time evolution operator

$$U(\tau) = e^{-i\theta(a^\dagger a+1)} \mid a\rangle_{00}\langle a \mid + e^{i\theta a^\dagger a} \mid b\rangle_{00}\langle b \mid,$$

where $\theta = \tau g^2/\Delta$. Let us write the amplitude of the coherent state $|\alpha'\rangle_1$ as $\alpha' = \alpha e^{i\pi/2}$ (α being any complex number), where $\mid \alpha \mid^2 \gg 1$. After the atom passes through $C1$ the state of the system $A0 + C1$ is given by

$$\mid \psi\rangle_{01} = -ic_a \mid a\rangle_0 \mid \alpha\rangle_1 + c_b \mid b\rangle_0 \mid -\alpha\rangle_1,$$

where we have chosen $\theta = \pi/2$. Then, the atom enters a second Ramsey zone $R0_2$. After the atom cross the Ramsey zone $R0_2$, the state of the system $A0 + C1$ is given by

$$\begin{aligned} |\psi\rangle_{01} &= i\frac{1}{2}\left[(c_a - e^{i\phi}c_b) \mid +\rangle_1 + i(c_a + e^{i\phi}c_b) \mid -\rangle_1\right] \mid a\rangle_0 \\ &\quad -\frac{1}{2}\left[(e^{-i\phi}c_a - c_b) \mid +\rangle_1) + (e^{-i\phi}c_a + c_b) \mid -\rangle_1\right] \mid b\rangle_0, \end{aligned}$$

where ϕ is the relative phase betwenn the fields in $R0_1$ and $R0_2$ and we have written the field states in terms of $\mid \pm\rangle_1 = (\mid \alpha\rangle_1 \pm \mid -\alpha\rangle_1)/\sqrt{2}$. Now, in order to obtain a state $|\Phi\rangle_1$(see Eq. (2)) in cavity $C1$, we just detect atom $A0$ in $\mid a\rangle_0$ or in $\mid b\rangle_0$.

Let us therefore assume that the field in cavity $C1$ has been prepared in a state of the form

$$|\Phi\rangle_1 = c_+|+\rangle_1 + c_-|-\rangle_1, \tag{5}$$

where the states $|\pm\rangle_1$ are the normalized Schrödinger cat states in cavity $C1$

$$|\pm\rangle_1 = \frac{1}{\sqrt{2}}(|\alpha\rangle_1 \pm |-\alpha\rangle_1) \tag{6}$$

and this cavity is given to Alice. Notice that if $c_+ = c_-$, then the renormalized state in $C1$ is $|\Phi\rangle_1 = |\alpha\rangle_1$ and if $c_+ = -c_-$, this state is $|\Phi\rangle_1 = |-\alpha\rangle_1$. In the cases in which $c_+ = 0$

the renormalized state in $C1$ is $|\Phi\rangle_1 = |-\rangle_1$, and if $c_- = 0$ the renormalized state in $C1$ is $|\Phi\rangle_1 = |+\rangle_1$. which are not a coherent superposition of the states $|\pm\rangle_1$. We are not interested in these cases since the important information to be sent are the amplitudes c_+ and c_-. Finally notice that as the atom-field interaction is in the far off resonance limit, there is some flexibility in choosing the amplitude of the coherent field which builds up a state $|\pm\rangle$.

We will assume that the state $|\Phi\rangle_3 = c_+|+\rangle_3 + c_-|-\rangle_3$ produced in cavity $C3$ after the teleportation process has been completed, involves the coherent states $|\beta\rangle_3$ and $|-\beta\rangle_3$ where $|\beta|^2$ could or not be equal to $|\alpha|^2$. If $|\beta|^2 \neq |\alpha|^2$ we would have a process similar to the teleportation of the state of a two-level atom of a given specimen prepared in a coherent superposition to another two-level atom of a different specimen.

As we will see the teleportation process is based on the atom-field interaction in the far off resonance limit described by Eq. (4). Note, as it will be clear, that Alice and Bob need to share the information that in their teleportation machine they must use a three-level atom $A2$ (see Fig. 2) which interacts with the cavity fields in the far off resonance limit described by the time evolution operator (4). As we have seen, Alice is given a Shrödinger cat state with unknown parameters c_+ and c_-, so that she cannot identify completely which state she has. It will be clear that Alice needs to know the coherent state $|\alpha\rangle_1$ involved in the preparation of the $|\Phi\rangle_1$ since, as we will see, in the teleportation scheme to be discussed shortly below, Alice has to displace the cavity field state in $C1$ by α. Even though Alice does not know $|\Phi\rangle_1$completely, she will be able to send this state to Bob. The fact that Alice has to know $|\alpha\rangle_1$ imposes no restriction on the process, since in our teleportation machine, we could agree beforehand in always using a certain state $|\alpha\rangle_1$ known to everybody.

The teleportation process starts with Bob sending a three-level atom labeled as $A2$ and initially in the state $|b\rangle_2$, from cavity $C3$ to cavity $C1$ along the z-axis (see Fig. 2). Cavity $C3$ has been prepared previously in a coherent state $|\beta\rangle_3$. Making use of Eq. (4), the subsystem $A2 + C3$ evolves to the state

$$|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}(|+\rangle_3|b\rangle_2 - |-\rangle_3|c\rangle_2), \tag{7}$$

where $|c\rangle_2$ is the other lower level of $A2$ and we have assumed a coherent state such that

$|\beta|^2 \gg 1$. The states $|\pm\rangle_3$ are the Schrödinger cat states (defined in Eq. (2)) associated to cavity $C3$. Then the atom flies through cavity $C1$ which was prepared, as we have seen above, in the state $|\Phi\rangle_1$. Before the interaction of $A2$ with the field in $C1$ the global state of the system is given by $|\Phi\rangle_1 \otimes |\Psi\rangle_{23}$ and after the atom $A2$ has left cavity $C1$ the system evolves to the state

$$\begin{aligned} |\Psi\rangle_{123} &= \frac{1}{\sqrt{2}}|+\rangle_3(c_+|+\rangle_1|b\rangle_2 - c_-|-\rangle_1|c\rangle_2) \\ &+\frac{1}{\sqrt{2}}|-\rangle_3(c_-|-\rangle_1|b\rangle_2 - c_+|+\rangle_1|c\rangle_2). \end{aligned} \tag{8}$$

Now we perform a detection on atom $A2$ by the detector $D2$. If $A2$ is detected in level $|b\rangle_2$ the renormalized correlated state of the subsystem $C1 + C3$ is

$$|\Psi\rangle_{13,b_2} = (c_+|+\rangle_1|+\rangle_3 + c_-|-\rangle_1|-\rangle_3). \tag{9}$$

If the atom is detected in level $|c\rangle_2$ the renormalized state is

$$|\Psi\rangle_{13,c_2} = -(c_-|-\rangle_1|+\rangle_3 + c_+|+\rangle_1|-\rangle_3). \tag{10}$$

Alice wishes to teleport state $|\Phi\rangle_1$ to cavity $C3$. The state she wants to teleport to cavity $C3$ is already present in the correlated state $|\Psi\rangle_{13,b_2}$ and not in $|\Psi\rangle_{13,c_2}$. She can achieve this goal in the following way. First she displaces the field in cavity $C1$, applying the displacement operator $D(\alpha)$ to the state (9). Making use of Eq. (6) the displacement of the field in cavity $C1$ transform the state $|\Psi\rangle_{13,b_2}$ to

$$\begin{aligned} D(\alpha)|\Psi\rangle_{13,b_2} &= \frac{1}{\sqrt{2}}|0\rangle_1 \otimes (c_+|+\rangle_3 - c_-|-\rangle_3) \\ &+\frac{1}{\sqrt{2}}|2\alpha\rangle_1 \otimes (c_+|+\rangle_3 + c_-|-\rangle_3). \end{aligned} \tag{11}$$

Now she lets a two level atom $A4$, with excited state $|e\rangle_4$ and lower state $|f\rangle_4$, to interact resonantly with cavity $C1$. If $A4$ is sent in the lower state, under the Jaynes Cumming dynamics we know that the state $|f\rangle_4|0\rangle_1$ does not change, however, the state $|f\rangle_4|2\alpha\rangle_1$ evolves to $|e\rangle_4|\chi_e\rangle_1 + |f\rangle_4|\chi_f\rangle_1$, where $|\chi_e\rangle_1 = \sum_n C_n \sin(gt\sqrt{n})|n\rangle$ and $|\chi_f\rangle_1 = \sum_n C_n \cos(gt\sqrt{n})|n\rangle$ and $C_n = e^{-|2\alpha|^2/2}(2\alpha)^n/\sqrt{n!}$. By using this fact we can write the state of the system $C1 + C3 + A4$ as follows

$$\begin{aligned} |\Psi\rangle_{134} &= \frac{1}{\sqrt{2}}|f\rangle_4|0\rangle_1(c_+|+\rangle_3 - c_-|-\rangle_3) \\ &+\frac{1}{\sqrt{2}}(|e\rangle_4|\chi_e\rangle_1 + |f\rangle_4|\chi_f\rangle_1)(c_+|+\rangle_3 + c_-|-\rangle_3). \end{aligned} \tag{12}$$

From the above expression we see that in order to complete the teleportation of $|\Phi\rangle_1$ to cavity $C3$ Alice must now perform a measurement of the atomic level $|e\rangle_4$ by the detector $D4$, which gives us the desired result

$$|\Psi\rangle_{13} = |\chi_e\rangle_1|\Phi\rangle_3 = |\chi_e\rangle_1(c_+|+\rangle_3 + c_-|-\rangle_3). \tag{13}$$

In this way Alice has "teleported" the state $|\Phi\rangle_1$ (generated in $C1$) to a distant cavity $C3$. Notice that the state generated in cavity $C1$ has to be destroyed ($|\Phi\rangle_1 \rightarrow |\chi_e\rangle_1$) in order to teleport this state to cavity $C3$ ($|\Phi\rangle_3$).

The teleportation process described above can be summarized as follows. First, as we have mentioned above an information that Alice and Bob must share is that the coherent field states prepared in their cavities should interact with three-level atoms in a lambda scheme in the far off resonance limit. Then, say that Alice is going to teleport state $|\Phi\rangle_1$ to Bob. Notice that it is not necessary that she knows where Bob is. When Bob receives a classical information from Alice informing him that she wishes to start the teleportation process, he sends an atom $A2$ through $C3$ towards cavity $C1$, preparing $|\Psi\rangle_{23}$. After this atom has interacted with the field in $C1$ and has been detected in level $|b\rangle_2$, Alice completes the teleportation process displacing the state in $C1$ and sending an atom $A4$ through cavity $C1$ to be detected in $|e\rangle_4$ after leaving cavity $C1$.

A limitation of the above scheme is related to the detection of $|b\rangle_2$ and $|e\rangle_4$. If Alice detects $|b\rangle_2$ and $|e\rangle_4$ successfully she calls back Bob and informs him that the teleportation was successful. If states $|b\rangle_2$ and $|e\rangle_4$ are not detected successfully, the state $|\Phi\rangle_1$ has to be prepared again and given to Alice. Then she calls back Bob informing him that they should start the process again. The cavities must also have a high quality factor. However nowadays it is possible to build up niobium superconducting cavities with high quality factors Q. The atoms to be used in the teleportation scheme described above could also be Rydberg atoms of long radiative lifetimes [9]. It is possible to construct cavities with quality factors $Q \sim 10^8$ [5]. Even cavities with quality factors as high as $Q \sim 10^{12}$ have been reported [6], which, for frequencies $\nu \sim 50$ GHz gives us a cavity field lifetime of the order of a few seconds.

The process of injecting a coherent field to displace the field in cavity $C1$ can be considered also as a realization of a quantum switch. We have two choices to displace the field in cavity $C1$

$$D(\alpha)|\Psi\rangle_{13,b_2} = \frac{1}{\sqrt{2}}|0\rangle_1 \otimes (c_+|+\rangle_3 - c_-|-\rangle_3) + \frac{1}{\sqrt{2}}|2\alpha\rangle_1 \otimes (c_+|+\rangle_3 + c_-|-\rangle_3), \tag{14}$$

and

$$D(-\alpha)|\Psi\rangle_{13,b_2} = \frac{1}{\sqrt{2}}|-2\alpha\rangle_1 \otimes (c_+|+\rangle_3 - c_-|-\rangle_3) + \frac{1}{\sqrt{2}}|0\rangle_1 \otimes (c_+|+\rangle_3 + c_-|-\rangle_3). \tag{15}$$

Then after sending atom $A4$ through $C1$ and detecting it in the excited state, as we discussed above, the state of the system $C1 + C3$ can be found in one of the states

$$|\Psi\rangle_{13\pm} = |\chi_{e\pm}\rangle_1 (c_+|+\rangle_3 \pm c_-|-\rangle_3). \tag{16}$$

Where $|\chi_{e\pm}\rangle_1$ are the states of the field in $C1$ after the detection of $|e\rangle_4$. Therefore, changing the field in cavity $C1$ properly, we can switch cavity $C3$ to $|\psi\rangle_3 = c_+|+\rangle_3 + c_-|-\rangle_3$ or $|\psi\rangle_3 = c_+|+\rangle_3 - c_-|-\rangle_3$.

Concluding, we have presented a scheme which permits teleportation of superpositions of quasi macroscopic states of the electromagnetic field. In addition the scheme allows for the realization of a quantum switch. The case involving two-mode cavities is also interesting to be investigated. This study is under development and will be the subject of a future work.

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[8] In the expressions involving the field operators we could have used indices in these operators, i.e., a_k and $a_k^\dagger$ ($k = 1$ or 3) depending if they refer to the field in $C1$ or $C3$. However we have dropped these indices in the operators and prefered to use indices in the field states $\mid \alpha\rangle_k$, $\mid \beta\rangle_k$ and $\mid \pm\rangle_k$ ($k = 1$ or 3).

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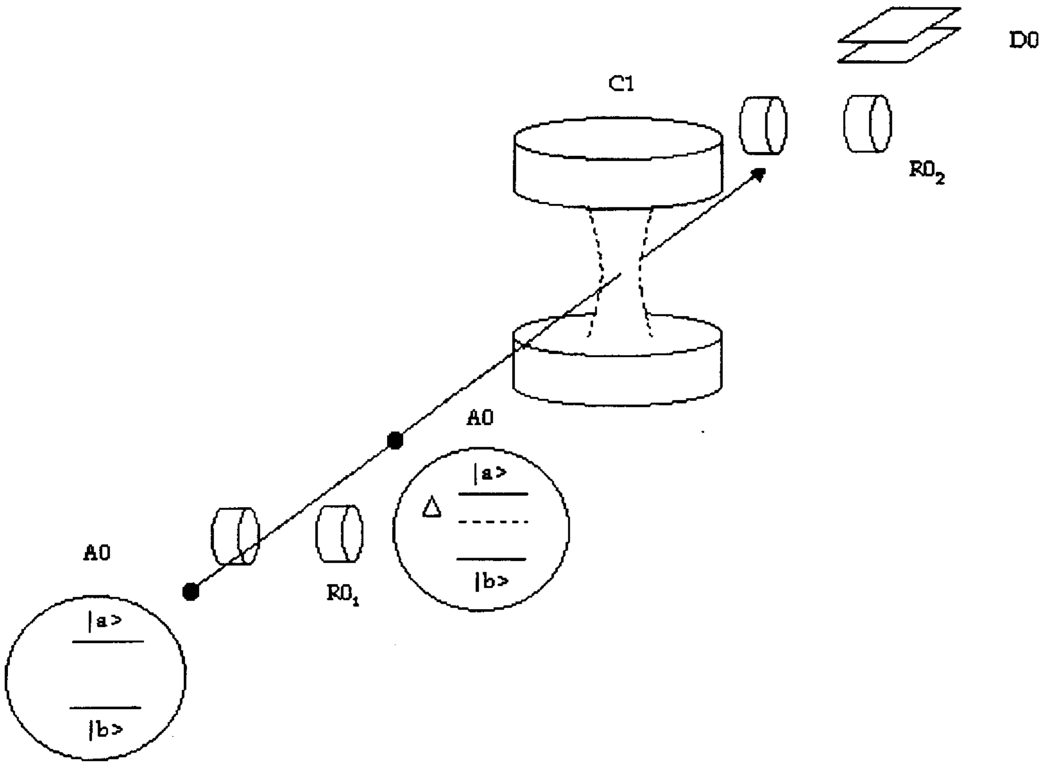
FIGURE CAPTION

Figure 1. The figure shows a scheme to prepare the initial state $|\Phi\rangle_1$ in cavity $C1$. The dashed lines represent the cavity field. The state $|\Phi\rangle_1$is prepared by the far off resonance interaction of the two-level atom $A0$ with a coherent field $|\alpha\rangle$ in $C1$. The atom $A0$ is prepared initially in state $\mid \psi\rangle_0 = c_a \mid a\rangle_0 + c_b \mid b\rangle_0$ in the first Ramsey zone $R0_1$ (bottom left). Then it flies through $C1$ and after leaving $C1$, $A0$ is rotated in the second Ramsey zone $R0_2$ and detected in $D0$ (top right).

Figure 2. The figure shows the cavities and the atoms used to build up our teleportation machine. The dashed lines represent the cavity fields. On the right we show cavity $C1$ where the state $|\Phi\rangle_1$ is generated. On the left we show cavity $C3$ where $|\Phi\rangle_3$ is produced. Atom $A2$ on the left travels along z axis through $C3$ and $C1$ entangling the fields, and is detected in $D2$. After the field in $C1$ has been displaced by α, atom $A4$, on the bottom right, travels through $C1$ and is detected in $D4$ (top left of $C1$) completing the teleportation process.

GUERRA - FIG. 1



GUERRA - FIG. 2