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Realization of Atomic GHZ States Via Cavity QED

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Abstract

In this work we propose a scheme in which it is possible to generate atomic GHZ states by letting three-level atoms in a lambda configuration to interact with a cavity field followed by a displacement of the cavity field and a selective measurements on two-level atoms which disentangle the atoms and field states. We also propose a GHZ test based on such states.

Key-words: GHZ-states; Bell's theorem.

I INTRODUCTION

Since the development of quantum mechanics, there has been several proposals to test the theory against theories based on local realism. The first formal tests which would decide between the two theories rely on Bell's theorem [1] based on an inequality which yields different predictions depending on if we consider a deterministic hidden-variable theory or quantum mechanics. There has been several attempts to implement experiments based on this theorem with results overwhelmingly supporting quantum mechanics [2]. Recently Greeberger, Horne and Zeilinger (GHZ) [3] have proposed a test in which a particular variable take on a specific value depending on which of the two theories is considered. Whereas experiments based on Bell's theorem involves a statistical experimental analysis, the GHZ proposal involves only one experimental run. For a very elegant and simple discussion about Bell's theorem without inequalities see the paper by Mermim [4]. However, the experimental implementation of such test is not simple. There has been several proposals of models in which the GHZ test could be realized. In a recent work, Gerry [5] has proposed a mesoscopic cavity QED realization of the GHZ test. The GHZ state, in this case, would be entangled coherent states built up from even and odd coherent states by means of a dispersive atom-cavity field interaction. In this work we propose the realization of an atomic GHZ state based also on dispersive atom-cavity field interaction. Here the atoms are three-level atoms in a lambda configuration which interact with the cavity field as described below. We also assume that the atoms used in the scheme discussed here are Rydberg atoms [6]

II PREPARATION OF GHZ STATES

We start with the Hamiltonian of a three-level lambda atom interacting with a field cavity mode

$$
\begin{aligned}
H &= \hbar\omega a^{\dagger}a + \hbar\omega_{aa}|a\rangle\langle a| \\
&\quad +\hbar\omega_{bb}|b\rangle\langle b| + \hbar\omega_{cc}|c\rangle\langle c| \\
&\quad +\hbar g a(|a\rangle\langle b| + |a\rangle\langle c|) + \hbar g a^{\dagger}(|b\rangle\langle a| + |c\rangle\langle a|), \qquad (1)
\end{aligned}
$$

where $|a\rangle$, $|b\rangle$ and $|c\rangle$ are the upper and the two quasi-degenerated lower atomic levels respectively. We have assumed the same coupling constant g for the coupling of the field mode with the transitions $|a\rangle \Rightarrow |c\rangle$ and $|a\rangle \Rightarrow |b\rangle$. In the far off resonance limit we can eliminate level $|a\rangle$ adiabatically so that the dynamic evolution is given by the evolution operator [7]

$$
\begin{aligned}
U &= \frac{1}{2}(e^{i\varphi a^{\dagger}a} + 1)|b\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^{\dagger}a} - 1)|b\rangle\langle c| \\
&\quad +\frac{1}{2}(e^{i\varphi a^{\dagger}a} - 1)|c\rangle\langle b| + \frac{1}{2}(e^{i\varphi a^{\dagger}a} + 1)|c\rangle\langle c| \qquad (2)
\end{aligned}
$$

where $\varphi = 2g^2\tau/\Delta$ with $\Delta = \omega_a - \omega_b - \omega = \omega_{ab} - \omega$, ω_a, $\omega_b$and ω are the frequencies associated to levels $|a\rangle$ and $|b\rangle$ and the cavity frequency respectively and τ is the atom-field interaction time.

In the first step we send atom $A1$ prepared in level $|b\rangle_1$ through cavity C. Cavity C is prepared initially in a coherent state $|\alpha\rangle$. If we choose the phase $\varphi = \pi$ we have

$$
\begin{aligned}
|\Phi\rangle_{C,1} &= \Pi_{+}|\alpha\rangle|b\rangle_1 + \Pi_{-}|\alpha\rangle|c\rangle_1 \\
&= \frac{1}{2}(|+\rangle|b\rangle_1 - |-\rangle|c\rangle_1), \qquad (3)
\end{aligned}
$$

where we have defined the non normalized states

$$
|\pm\rangle = |\alpha\rangle \pm |-\alpha\rangle \qquad (4)
$$

with $N^{\pm} = \langle \pm \mid \pm\rangle = 2\left(1 \pm e^{-2|\alpha|^2}\right)$[8] and

$$
\Pi_{+} = \frac{1}{2}(e^{i\pi a^{\dagger}a} + 1), \qquad (5)
$$

$$\Pi_- = \frac{1}{2}(e^{i\pi a^\dagger a} - 1). \tag{6}$$

Notice also that

$$\Pi_+|+\rangle = |+\rangle, \tag{7}$$
$$\Pi_+|-\rangle = 0, \tag{8}$$
$$\Pi_-|-\rangle = -|-\rangle, \tag{9}$$
$$\Pi_-|+\rangle = 0. \tag{10}$$

Now we let a three-level lambda atom $A2$ fly through cavity C. As above the levels of atom $A2$, $|a\rangle_2$, $|b\rangle_2$ and $|c\rangle_2$ are in a lambda configuration so that the $|a\rangle_2 \rightleftharpoons |c\rangle_2$ and $|a\rangle_2 \rightleftharpoons |b\rangle_2$ are the far off resonance interaction limit. Again the atom-field time evolution operator is given by Eq. (2). Assume now that $A2$ is prepared initially in the state $\mid b\rangle_2$. If $A2$ passes through the cavity and again we have $\varphi = \pi$, taking into account

$$U = \Pi_+|b\rangle_{22}\langle b| + \Pi_-|b\rangle_{22}\langle c| \ + \Pi_-|c\rangle_{22}\langle b| + \Pi_+|c\rangle_{22}\langle c| \tag{11}$$

the initial state $\mid b\rangle_2 \otimes \ |\Phi\rangle_{C,1}$ evolves to

$$|\Phi\rangle_{C,12} = \frac{1}{2}\left[|+\rangle|b\rangle_1|b\rangle_2 + |-\rangle|c\rangle_1|c\rangle_2\right] \tag{12}$$

Finally we let another three-level lambda $A3$ fly through cavity C. Following the above prescription the state (12) evolves to

$$|\Phi\rangle_{C,123} = |\Phi\rangle_{\mathrm{f-at;GHZ}} = \frac{1}{2}\left[|+\rangle|b\rangle_1|b\rangle_2|b\rangle_3 - |-\rangle|c\rangle_1|c\rangle_2|c\rangle_3\right]. \tag{13}$$

Now let us inject a coherent field $|\alpha\rangle$ in cavity C. Then we have

$$|\Phi;+\alpha\rangle_{C,123} = \frac{1}{2}\left[(|2\alpha\rangle + |0\rangle)|b\rangle_1|b\rangle_2|b\rangle_3 - (|2\alpha\rangle - |0\rangle)|c\rangle_1|c\rangle_2|c\rangle_3\right]. \tag{14}$$

Then, we send two-level atom that we call $A4$, with $|s\rangle_4$ and $|r\rangle_4$ being the lower and upper levels respectively, through C. If $A4$ is sent through C in the lower state, under the Jaynes Cummings dynamics we know that the state $|s\rangle_4|0\rangle$ does not evolve, however, the state $|s\rangle_4|2\alpha\rangle$ evolves to $|r\rangle_4|\chi_r\rangle + |s\rangle_4|\chi_s\rangle$, where $|\chi_s\rangle = \sum_n C_n \sin(gt\sqrt{n})|n\rangle$ and $|\chi_r\rangle = \sum_n C_n \cos(gt\sqrt{n})|n\rangle$ and $C_n = e^{-|2\alpha|^2}(2\alpha)^n/\sqrt{n!}$. Using this fact we can write the state of the system $C + A1 + A2 + A3 + A4$ as follows

$$|\Phi;+\alpha\rangle_{C,1234} = \frac{1}{2}[|r\rangle_4|\chi_r\rangle + |s\rangle_4|\chi_s\rangle + |s\rangle_4|0\rangle \quad]|b\rangle_1|b\rangle_2|b\rangle_3 - \frac{1}{2}[|r\rangle_4|\chi_r\rangle + |s\rangle_4|\chi_s\rangle - |s\rangle_4|0\rangle] \quad |c\rangle_1|c\rangle_2|c\rangle_3). \tag{15}$$

Now, if we detect state $|r\rangle_4$, we get

$$\begin{aligned}|\Psi;+\alpha\rangle_{C,123} &= \frac{1}{\mathcal{N}}|\chi_r\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3) \\ &= |\Phi\rangle_C \otimes |\Phi;-\rangle_{\text{at}}\end{aligned} \tag{16}$$

where $1/\mathcal{N}$ is a normalization factor. That is we have disentangled the cavity-atoms state, where

$$|\Phi;-\rangle_{\text{at}} = |\Phi\rangle_{\text{at;GHZ}} = \frac{1}{\sqrt{2}}[|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3) \tag{17}$$

If again we start from

$$|\Phi\rangle_{C,123} = \frac{1}{2}\left[|+\rangle|b\rangle_1|b\rangle_2|b\rangle_3 - |-\rangle|c\rangle_1|c\rangle_2|c\rangle_3\right]. \tag{18}$$

but now we inject a coherent field $|-\alpha\rangle$ in cavity C, then we have

$$|\Phi;-\alpha\rangle_{C,123} = \frac{1}{2}\left[(|0\rangle + |-2\alpha\rangle)|b\rangle_1|b\rangle_2|b\rangle_3 - (|0\rangle - |-2\alpha\rangle)|c\rangle_1|c\rangle_2|c\rangle_3\right]. \tag{19}$$

As above we send two-level atoms that we call $A4$, with $|s\rangle_4$ and $|r\rangle_4$ being the lower and upper levels respectively, through C. Then we can write the state of the system $C + A1 + A2 + A3 + A4$ as follows

$$|\Phi;-\alpha\rangle_{C,1234} = \frac{1}{2}[|s\rangle_4|0\rangle + |r\rangle_4|\chi_r\rangle + |s\rangle_4|\chi_s\rangle \quad]|b\rangle_1|b\rangle_2|b\rangle_3 - \frac{1}{2}[|s\rangle_4|0\rangle - |r\rangle_4|\chi_r\rangle - |s\rangle_4|\chi_s\rangle] \quad |c\rangle_1|c\rangle_2|c\rangle_3). \tag{20}$$

and if we detect state $|r\rangle_4$ and we get

$$\begin{aligned}|\Psi;-\alpha\rangle_{C,123} &= \frac{1}{\mathcal{N}}|\chi_r\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 + |c\rangle_1|c\rangle_2|c\rangle_3) \\ &= |\Phi\rangle_C \otimes |\Phi;+\rangle_{\text{at}}\end{aligned} \tag{21}$$

Now let us go back to the displaced states and assume that $|\,|\chi_r\rangle\,|^2 \gg |\,|\chi_s\rangle\,|^2$

$$|\Phi;+\alpha\rangle_{C,1234} \approx \frac{1}{2}[|r\rangle_4|\chi_r\rangle + |s\rangle_4|0\rangle \quad]|b\rangle_1|b\rangle_2|b\rangle_3 - \frac{1}{2}[|r\rangle_4|\chi_r\rangle - |s\rangle_4|0\rangle] \quad |c\rangle_1|c\rangle_2|c\rangle_3). \tag{22}$$

and

$$|\Phi;-\alpha\rangle_{C,1234} \approx \frac{1}{2}[|s\rangle_4|0\rangle + |r\rangle_4|\chi_r\rangle \quad]|b\rangle_1|b\rangle_2|b\rangle_3 - \frac{1}{2}[|s\rangle_4|0\rangle - |r\rangle_4|\chi_r\rangle] \quad |c\rangle_1|c\rangle_2|c\rangle_3). \tag{23}$$

and if we detect atom $A4$ in state $|s\rangle_4$, we get (injecting $|\alpha\rangle$ in C)

$$\begin{aligned}|\Psi;+\alpha\rangle_{C,123} &\approx \frac{1}{\sqrt{2}}|0\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 + |c\rangle_1|c\rangle_2|c\rangle_3] = \\ &|\Phi\rangle_C \otimes |\Phi;+\rangle_{\text{at}}\end{aligned} \tag{24}$$

and (injecting $|-\alpha\rangle$ in C)

$$\begin{aligned}|\Psi;-\alpha\rangle_{C,123} &\approx \frac{1}{\sqrt{2}}|0\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3] = \\ &|\Phi\rangle_C \otimes |\Phi;-\rangle_{\text{at}}.\end{aligned} \tag{25}$$

Notice that we can probe the cavity field in C, which is left approximately in the vacuum state $|0\rangle$ by sending an auxiliary atom $A5$ posteriorly. If we send another two level atom $A5$ in the lower state through C, as $|s\rangle_5|0\rangle$ does not change, after atom $A5$ leaves the cavity it will be detected in state $|s\rangle_5$ with large probability (since we have assumed $|\ |\chi_r\rangle\ |^2 \gg |\ |\chi_s\rangle\ |^2$).

On the other hand, if we detect atom $A4$ in state $|r\rangle_4$, we get (injecting $|\alpha\rangle$ in C)

$$\begin{aligned}|\Psi;+\alpha\rangle_{C,123} &\approx \frac{1}{\sqrt{2}}|\chi_r\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3] = \\ &|\Phi\rangle_C \otimes |\Phi;-\rangle_{\text{at}}\end{aligned} \tag{26}$$

and (injecting $|-\alpha\rangle$ in C)

$$\begin{aligned}|\Psi;-\alpha\rangle_{C,123} &\approx \frac{1}{\sqrt{2}}|\chi_r\rangle[|b\rangle_1|b\rangle_2|b\rangle_3 + |c\rangle_1|c\rangle_2|c\rangle_3] = \\ &|\Phi\rangle_C \otimes |\Phi;+\rangle_{\text{at}}\end{aligned} \tag{27}$$

Again, the displacement of the cavity filed followed by the selective detection of a state of a two-level atom plays the role of a Stern-Gerlach apparatus for measurement of $\sigma_{x1}\sigma_{x2}\sigma_{x3}$.

III GHZ TEST

As we know the GHZ test is based on the measurement of the three operator product $\sigma_{x1}\sigma_{x2}\sigma_{x3}$, where in our case σ_{xi}, belongs to the algebra defined by the operators

$$\begin{aligned}\sigma_{zk} &= |b\rangle_{kk}\langle b| - |c\rangle_{kk}\langle c| \\ \sigma_{xk} &= |b\rangle_{kk}\langle c| + |c\rangle_{kk}\langle b| \\ \sigma_{yk} &= i(|b\rangle_{kk}\langle c| - |c\rangle_{kk}\langle b|)\end{aligned} \tag{28}$$

and $k = 1, 2$ and 3. Namely we have

$$\sigma_{x1}\sigma_{x2}\sigma_{x3}|\Phi; -\rangle_{\rm at} = -\tfrac{1}{\sqrt{2}}(|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3), \tag{29}$$

Therefore

$$_{\rm at}\langle\Phi; -|\sigma_{x1}\sigma_{x2}\sigma_{x3}|\Phi; -\rangle_{\rm at} = -1 \tag{30}$$

Then a single set of measurements of σ_{xi} is sufficient to demonstrate that local theories can be discarded since such theories would predict as result $+1$ for the expectation value of $\sigma_{x1}\sigma_{x2}\sigma_{x3}$ contrasting with the prediction based on quantum theory, that is, -1.

Let us assume that we have prepared the state

$$|\Phi; -\rangle_{\rm at} = |\Phi\rangle_{\rm at;GHZ} = \frac{1}{\sqrt{2}}[|b\rangle_1|b\rangle_2|b\rangle_3 - |c\rangle_1|c\rangle_2|c\rangle_3] \tag{31}$$

and let us denote the eigenstates of $\sigma_{xk}$by

$$|Ak; \pm\rangle = \frac{1}{\sqrt{2}}[|b\rangle_k \pm |c\rangle_k]$$

where $k = 1, 2$ and 3. Then we can write

$$|\Phi; -\rangle_{\rm at} = \frac{1}{2}[(|A1; +\rangle + |A1; -\rangle)|b\rangle_2|b\rangle_3 - (|A1; +\rangle - |A1; -\rangle)|c\rangle_2|c\rangle_3] \tag{32}$$

$$= \frac{1}{2}[|A1; +\rangle(|b\rangle_2|b\rangle_3 - |c\rangle_2|c\rangle_3) + |A1; -\rangle(|c\rangle_2|c\rangle_3 + |b\rangle_2|b\rangle_3)]. \tag{33}$$

Consider the rotation matrix (see Appendix A)

$$K = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

or

$$K = \frac{1}{\sqrt{2}}\left[|b\rangle\langle b| + |c\rangle\langle c| + |b\rangle\langle c| - |c\rangle\langle b|\right]$$

where we have omitted the atomic subindexes. Applying the rotation on $A1$

$$K|\Phi;-\rangle_{\mathrm{at}} = \frac{1}{2}[|c\rangle_1(|b\rangle_2|b\rangle_3 - |c\rangle_2|c\rangle_3) + |b\rangle_1(|c\rangle_2|c\rangle_3 + |b\rangle_2|b\rangle_3)] \tag{34}$$

Detection $A1$ in $|c\rangle_1$ is equivalent to detection of $|A1;+\rangle$. The renormalized state we obtain is

$$|\Phi\rangle_{23} = \frac{1}{\sqrt{2}}(|b\rangle_2|b\rangle_3 - |c\rangle_2|c\rangle_3) \tag{35}$$

$$= \frac{1}{2}((|A2;+\rangle + |A2;-\rangle)|b\rangle_3 - (|A2;+\rangle - |A2;-\rangle)|c\rangle_3) \tag{36}$$

$$= |A2;+\rangle(|b\rangle_3 - |c\rangle_3) + |A2;-\rangle(|c\rangle_3 + |b\rangle_3) \tag{37}$$

$$K|\Phi\rangle_{23} = \frac{1}{2}(|c\rangle_2(|b\rangle_3 - |c\rangle_3) + |b\rangle_2(|c\rangle_3 + |b\rangle_3)) \tag{38}$$

Detection $A2$ in $|c\rangle_2$ is equivalent to the detection of $|A2;+\rangle$. So the final state and after rotation lead us to $|A3;-\rangle_3$. Now we can apply the rotation K again and detect $A3$ in $|b\rangle_3$. Therefore, the rotations by K and the measurement sequence $|c\rangle_1|c\rangle_2|b\rangle_3$ is equivalent to a measurement of the three product operator $\sigma_{x1}\sigma_{x2}\sigma_{x3}$.

But we realize that any of the sequence of measurements presented in the bellow table lead us to the eigenvalue -1 for the expectation value of the three operator product $\sigma_{x1}\sigma_{x2}\sigma_{x3}$, that is

$\lvert c\rangle_1$	$\lvert c\rangle_2$	$\lvert b\rangle_3$
$\lvert c\rangle_1$	$\lvert b\rangle_2$	$\lvert c\rangle_3$
$\lvert b\rangle_1$	$\lvert c\rangle_2$	$\lvert c\rangle_3$
$\lvert b\rangle_1$	$\lvert b\rangle_2$	$\lvert b\rangle_3$

IV DISCUSSION

In the above schemes, as we have pointed, the injection of $|\pm\alpha\rangle$ in C followed by the detection of $A4$ in one of its levels play the role of a Stern-Gerlach apparatus. For instance, in the case we assume $|\ |\chi_r\rangle\ |^2 \gg |\ |\chi_s\rangle\ |^2$, the injection of $|\alpha\rangle$ in C followed by the detection of $A4$ in state $|r\rangle_4$ and the detection of $A5$ in state $|r\rangle_5$ is associated with the atomic state $|\Phi\rangle_{GHZ}$ and therefore with eigenvalue -1. The same happens if we inject $|-\alpha\rangle$ in C followed by the detection of $A4$ in state $|s\rangle_4$ and assuming $|\ |\chi_r\rangle\ |^2 \gg |\ |\chi_s\rangle\ |^2$, we detect $A5$ in state $|s\rangle_5$ with a large probability.

The magnitude of the coherent state $|\alpha\rangle$ can be small, and therefore, for a very high quality factor, decoherence could be negligible. Therefore, the atomic decoherence would be a more important factor of limitation in our scheme. However, Rydberg atoms present a relatively large lifetime and should be used in the experiments. Of course as in any other scheme involving atoms, the atomic state detectors are another limitation due to their efficiency. However, with tecnological developments of good cavities and atomic state detectors we believe that the above scheme could be implemented to produce atomic GHZ states and, most important, we have shown that applying a sequence of rotations and detections on $|\Phi;-\rangle_{\rm at}$ it would be possible to perform the GHZ test, in other words, to measure the eigenvalue of $\sigma_{x1}\sigma_{x2}\sigma_{x3}$.

V Appendix A

In this appendix we show how the operator K we consider in the measurement processes on the observable $\sigma_{x1}\sigma_{x2}\sigma_{x3}$, can be physically implemented. After each atom has passed through the main cavity, it enters an additional cavity before it is detected. Considering the classical limit in the three level lambda atom interacting with two dephased modes with the same frequency it is not difficult to show that the matrix elements of the evolution operator read as:

$$u_{bb} \;=\; 1+\epsilon_1^*\epsilon_1\frac{1}{2\mid\epsilon\mid^2}[e^{-i\Delta t/2}(\cos\sqrt{2\mid\epsilon\mid^2+\Delta^2/4}t$$

$$
\begin{aligned}
&+i\frac{\Delta}{2\sqrt{2\mid\epsilon\mid^2+\Delta^2/4}}\sin\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t})-1]\\
u_{cc} &= 1+\epsilon_2^*\epsilon_2\frac{1}{2\mid\epsilon\mid^2}[e^{-i\Delta t/2}(\cos\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t}\\
&+i\frac{\Delta}{2\sqrt{2\mid\epsilon\mid^2+\Delta^2/4}}\sin\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t})-1]\\
u_{cb} &= \epsilon_1^*\epsilon_2\frac{1}{2\mid\epsilon\mid^2}[e^{-i\Delta t/2}(\cos\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t}\\
&+i\frac{\Delta}{2\sqrt{2\mid\epsilon\mid^2+\Delta^2/4}}\sin\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t})-1]\\
u_{bc} &= \epsilon_1\epsilon_2^*\frac{1}{2\mid\epsilon\mid^2}[e^{-i\Delta t/2}(\cos\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t}\\
&+i\frac{\Delta}{2\sqrt{2\mid\epsilon\mid^2+\Delta^2/4}}\sin\sqrt{2\mid\epsilon\mid^2+\Delta^2/4t})-1]
\end{aligned}
$$

where we have used

$$
\begin{aligned}
g_1a_1 &\rightarrow \epsilon_1=\epsilon e^{i\theta_1}\\
g_2a_2 &\rightarrow \epsilon_2=\epsilon e^{i\theta_2}
\end{aligned}
$$

In the high detuning limit we obtain the expression

$$
\begin{aligned}
u_{bb} &= \frac{1}{2}(e^{i\varphi}+1)\\
u_{cc} &= \frac{1}{2}(e^{i\varphi}+1)\\
u_{cb} &= \frac{1}{2}e^{-i\phi}(e^{i\varphi}-1)\\
u_{bc} &= \frac{1}{2}e^{i\phi}(e^{i\varphi}-1)
\end{aligned}
$$

where $\phi=\theta_1-\theta_2$ and $\varphi=2\mid\epsilon\mid^2 t/\Delta$. Choosing $\varphi=\pi/2$ and $\phi=\pi/2$ we get

$$
U=\frac{1}{\sqrt{2}}e^{i\pi/4}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}
$$

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