

On the Consistency of Quantum Geometroynamics and Quantum Field Theories in the Bohm-de Broglie Interpretation

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We consider quantum geometrodynamics and parametrized quantum field theories in the framework of the Bohm-de Broglie interpretation. In the first case, and following the lines of our previous work [1], where a hamiltonian formalism for the bohmian trajectories was constructed, we show the consistency of the theory for any quantum potential, completing the scenarios for canonical quantum cosmology presented there. In the latter case, we prove the consistency of scalar field theory in Minkowski spacetime for any quantum potential, and we show, using this alternative hamiltonian method, a concrete example already known in the literature where Lorentz invariance of individual events is broken.

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I. INTRODUCTION

The Copenhagen interpretation of quantum mechanics assumes the existence of a classical domain outside the observed quantum system. The necessity of a classical domain comes, according with Von Neumann's approach, from the way it solves the measurement problem ([2]): in order to explain why a unique eigenvalue of some observable is measured out of all possible outcomes, it is postulated that the wave function suffers a real collapse, a process that cannot be described by the unitary Schrödinger evolution but must occur outside the quantum world, in a classical domain. In a quantum theory of the whole Universe, there is no place for a classical domain. Hence, the Copenhagen interpretation cannot be used in quantum cosmology. However, there are some alternative solutions to this quantum cosmological dilemma. One can say that the Schrödinger evolution is an approximation of a more fundamental non-linear theory which can accomplish the collapse of the wave function [3,4], or that the collapse is effective but not real, in the sense that the branches related to unmeasured eigenvalues of the observable which appear in the splitting of a wave function in an impulsive measurement disappear from the observer but do not disappear from existence. In this second category we can cite the Many-Worlds Interpretation [5] and the Bohm-de Broglie (BdB) Interpretation [6] [7] [8]. In the former, all the possibilities in the splitting are actually realized. In each branch there is an observer with the knowledge of the corresponding eigenvalue of this branch, but she or he is not aware of the other observers and the other possibilities because the branches do not interfere. In the

latter, a point-particle in configuration space describing the observed system and apparatus is supposed to exist, independently on any observations. It follows a trajectory in configuration space which is not the classical one due to the action of a quantum potential which arises naturally from the Schrödinger equation. In the splitting of the wave function in a measurement, this point particle will enter into one of the branches (which one depends on the initial position of the point particle before the measurement, which is unknown), and the other branches will be empty. It can be shown [8] that the empty waves can neither interact with other particles, nor with the point particle containing the apparatus. Hence, no observer can be aware of the other branches which are empty. Again we have an effective but not real collapse (the empty waves continue to exist), but now with no multiplication of observers. Of course these interpretations can be used in quantum cosmology. Schrödinger evolution is always valid, and there is no need of a classical domain outside the observed system.

The BdB interpretation has been successfully applied to quantum minisuperspace models [9–15] and even to full superspace models [1] [16]. In the first case it was discussed the classical limit, the singularity problem, the cosmological constant problem, and the time issue. It was shown in scalar and radiation models for the matter content of the early universe that quantum effects driven by the quantum potential can inhibit the formation of a singularity by producing a repulsive quantum force that counteract the gravitational field, yielding inflation. The quantum universe usually reach the classical limit for large scale factors. However, it is possible to have small classical universes and large quantum ones: it

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depends on the state functional and on initial conditions [14]. It was shown that the quantum evolution of homogeneous hypersurfaces form the same four-geometry independently on the choice of lapse function [10].

For the case of a general superspace model (i.e. the full theory) a BdB picture of quantum geometrodynamics was constructed [1]. As the BdB interpretation admits the notion of quantum trajectories (in this case, the evolution of spacelike geometries in superspace, the space of all spacelike geometries), called bohmian trajectories, we were able to construct a hamiltonian formalism with constraints which generates such orbits.¹ This hamiltonian formalism differs from the classical one only by the presence of the quantum potential in one of its terms which deviates the bohmian trajectory from the classical one. In this framework, it was possible to study the structures generated by the quantum evolution of spacelike 3-geometries using the theory of constrained hamiltonians developed by Dirac [17] for the case of geometrodynamics [18,19]. In this way it was shown that, irrespective of any regularization and factor ordering of the Wheeler-De Witt equation, the BdB interpretation of quantum cosmology yields basically two possible scenarios: In the first one, the constraints which generates the quantum bohmian evolution of the geometry of the spacelike hypersurfaces obey an specific algebra, the Dirac-Teitelboim's algebra [18], which assures a consistent evolution that form a non degenerate 4-geometry with two possibilities: the usual hyperbolic classical spacetime and an euclidean spacetime where the change of signature from Lorentzian to Euclidean is driven by the quantum potential. In the second scenario, the constraints are still conserved in time but they form an algebra different from the Dirac-Teitelboim's algebra yielding a consistent quantum evolution that form a degenerate 4-geometry. For example, in the case of real solutions of the Wheeler-DeWitt equation, we have a structure satisfying the Carroll group connected with the strong gravity limit. Another example with a non-local quantum potential was also studied.

In that same paper [1] we left open the possibility of an inconsistent evolution: a complicated non-local quantum potential would avoid the closure of the algebra, whatever it would be (Dirac-Teitelboim or another alternative algebra), implying that the constraints are not conserved

in time. As noted by Dirac, it would imply an inconsistent hamiltonian evolution of the spacelike geometries. Soon we realized that such possibility could also happen in quantum field theory. If this would be the case, then the subquantum reality of the BdB interpretation would be imposing selection rules on possible quantum states which are absent in other interpretations, either for quantum geometrodynamics as for quantum field theory. This could be used to find ways to distinguish experimentally between interpretations and/or impose new boundary conditions which could be used in quantum cosmology. In the case of quantum field theory, which is more accessible to experimental investigations, if one realize in nature quantum states which presents this pathological bohmian behaviour, it would be a serious drawback of the BdB interpretation. On the contrary, if one finds impossible to realize such states in practice, this would be a strong point in favour of the BdB interpretation because it would give a reason for this impossibility which is not present in other interpretations.

In the present work we show that this is not the case: the algebra of the constraints is closed for any quantum potential when restricted to the quantum bohmian trajectories and hence there is no inconsistency in the sense of Dirac, neither for quantum geometrodynamics, nor for quantum field theory. Hence, the subquantum world of the Bohm-de Broglie interpretation is not imposing any restriction at all to the possible quantum states of a field theory: the admissible quantum states are the same as in any other interpretation.

We begin the present paper with the application of the formalism developed in [1] to a parametrized quantum field theory in flat background. We obtain analogous results as in quantum geometrodynamics, namely, the break of Dirac-Teitelboim's algebra and the consistency of the theory, in the sense of Dirac, for any quantum potential. In this case, the break of Dirac-Teitelboim's algebra means a loss of Lorentz invariance of individual events, a result already known in the literature [8] [20] but which is presented here in a different way. Note that this break of Lorentz invariance appears only at the level of individual quantum trajectories, the bohmian trajectories or the subquantum world, a notion which is absent in other interpretations of quantum mechanics.²

¹It is important to stress that such hamiltonian formalism is relevant only for the subquantum world of the BdB interpretation, where trajectories are supposed to have objective reality. It has no meaning in interpretations where this subquantum world is absent, as in the Copenhagen interpretation.

²The study of individual events in quantum field theory was initiated already in an early paper by Bohm [7] and developed in much more detail in [20] and [8]. The loss of Lorentz invariance of individual events does not contradict relativity because all statistical predictions of the BdB interpretation are the same as the predictions of the Copenhagen interpretation, which are confirmed experimentally. The quantum trajectories which break Lorentz invariance in the BdB interpretation will not be accessible to observation as long as the quantum theory in its current form is valid. However, it is possible that quantum theory will fail in some new domain, unexplored until now. For instance, following Bohm [7] [20], in an extension of quantum theory to include stochastic processes there will exist some relaxation time along which the probability density is approaching, but is not

The present paper is organized as follows: in the next section we revisit the parametrized scalar field theory in Minkowski spacetime and we synthesize the Teitelboim's result about the spacetime structure reflected in the constraint's algebra. In section III we apply the Bohm-de Broglie interpretation to a parametrized scalar quantum field theory and we show that the bohmian evolution of the fields, irrespective to any regularization and factor ordering of the functional Schrödinger equation, can be obtained from a specific hamiltonian which is, of course, different from the classical one. We prove that this hamiltonian formalism is consistent for any quantum potential. We then use this approach to rederive, in a concrete example, the well known loss of Lorentz invariance of individual events in BdB theory in terms of the break of the Dirac-Teitelboim's algebra of the constraints. In section IV we consider quantum geometrodynamics in the BdB interpretation following the approach of our previous work, and we prove the consistency of the theory for any quantum potential, completing the possible scenarios for the BdB view of quantum cosmology presented in [1]. Section V is for discussion and conclusions. The appendix shows an alternative way to compute a relevant Poisson bracket (PB).

II. PARAMETRIZED FIELD THEORIES

An essential feature of geometrodynamics is the existence of the super-hamiltonian and super-momentum constraints which are present due to the invariance of General Relativity (GR) under general coordinate transformations [27]. We find an analogous situations in systems with a finite number of degrees of freedom and in field theory in flat spacetime when they are expressed as *parametrized* theories (see [27] and [28] cap VI). To parametrize a system with a finite number of degrees of freedom, with canonical coordinates X^i , ($i = 1 \dots n$) depending on the physical time T and with lagrangean \mathcal{L}_o , we simply promote $T \equiv X^0$ as being one of the canonical coordinates $X^\alpha = (T, X^i)$ (with $\alpha = 0 \dots n$) and depending on an arbitrary label time t : $T = T(t)$. In defining the conjugate momenta π_T of T a constraint appears: $\mathcal{H} \equiv \pi_T + H_o = 0$ (where $H_o \equiv \pi_i \frac{dX^i}{dT} - \mathcal{L}_o$ is the hamiltonian in the old coordinates and π_i is the canonical momentum conjugate to X^i). The original action functional

$S = \int dT \mathcal{L}_o \left(T, X^i, \frac{dX^i}{dT} \right) = \int dT \left(\pi_i \frac{dX^i}{dT} - H_o \right)$ will be expressed in the new canonical variables and after incorporating the constraint by mean a lagrange multiplier N , as

$$S = \int dt \left(\pi_\beta \dot{X}^\beta - N \mathcal{H} \right), \quad (2.1)$$

where $X^0 = T$. This is the *parametrized* action and the variables X^α , π_α , N are varied freely.

For the case of a relativistic field theory we have four parameters X^α , instead of one T . We will introduce them as canonical coordinates, together with the fields $\phi(X^\alpha)$, by expressing them in terms of an arbitrary set of curvilinear coordinates x^β as $X^\alpha = X^\alpha(x^\beta)$. In this manner (see below), we can build the field theory with the states defined on a general spacelike hypersurface, which plays the role of time. We have a manifest relativistic invariant hamiltonian formalism. The parametrized form of the action of a field in flat spacetime will help us in the implementation of the BdB interpretation to quantum gravity where the action is, from the beginning, parametrized. In fact, up to now, it is imposible to deparametrize GR in general by separating the dynamical (i.e relevant) degrees of freedom from the kinematical (i.e. redundant) ones. In GR we are forced to use redundant variables as canonical coordinates, and then constraints appears.

Specifying, let $\phi(X^\alpha)$ be a scalar field propagating in a 4-dimensional flat spacetime with minkowskian coordinates $X^\alpha \equiv (T, X^i)$. Greek indices run from 0 to 3 and latin indices from 1 to 3. Let us consider curvilinear coordinates $x^\beta = (t, x^i)$ and a transformation

$$X^\alpha = X^\alpha(x^\beta) \quad (2.2)$$

For t fixed, this equation defines a hypersurface with a spatial coordinate system x^i defined on it. We will have a family of hypersurfaces for different values of the parameter (label) t . The action in minkowskian coordinates is

$$S = \int d^4 X \mathcal{L}_o \left(\phi, \frac{\partial \phi}{\partial X^\alpha} \right), \quad (2.3)$$

where \mathcal{L}_o stands for the lagrangean density in minkowskian coordinates. Writing the action in curvilinear coordinates we have:

$$S = \int d^4 x J \mathcal{L}_o \left(\phi, \frac{\partial \phi}{\partial x^\beta} \frac{\partial x^\beta}{\partial X^\alpha} \right) = \int d^4 x \mathcal{L} \left(\phi, \phi_{,i}, \dot{\phi}, X_{,i}^\alpha, \dot{X}^\alpha \right), \quad (2.4)$$

yet equal, to $|\psi|^2$. An experiment in times shorter than this relaxation time might reveal this discrepancy, yielding in general results which are not Lorentz invariant. In such situation, relativity would hold only as a statistical approximation valid for a distribution close to equilibrium in the stochastic process of the subquantum world that underly quantum mechanics.

where $\dot{\phi} \equiv \frac{\partial \phi}{\partial x^0}$, $\phi_{,k} \equiv \frac{\partial \phi}{\partial x^k}$, and

$$J \equiv \frac{\partial(X^0 \dots X^3)}{\partial(x^0 \dots x^3)} \quad (2.5)$$

is the jacobian of the transformation. Here \mathcal{L} represents the lagrangean density in curvilinear coordinates. Defining the momentum π_ϕ , conjugate to ϕ , as usual,

$$\pi_\phi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}}, \quad (2.6)$$

we have the hamiltonian density

$$\mathbf{h} = \pi_\phi \dot{\phi} - \mathcal{L}, \quad (2.7)$$

yielding

$$\mathbf{h} = \frac{\partial x^0}{\partial X^\alpha} J T_\beta^\alpha \dot{X}^\beta \equiv K_\beta \dot{X}^\beta, \quad (2.8)$$

where T_β^α is the energy-momentum tensor of the field in minkowskian coordinates, given by

$$T_\beta^\alpha = \frac{\partial \mathcal{L}_o}{\partial \frac{\partial \phi}{\partial X^\alpha}} \frac{\partial \phi}{\partial X^\beta} - \eta_\beta^\alpha \mathcal{L}_o, \quad (2.9)$$

and K_β is defined as

$$K_\beta \equiv \frac{\partial x^0}{\partial X^\alpha} J T_\beta^\alpha \quad (2.10)$$

The hamiltonian density \mathbf{h} has a linear dependence in the ‘kinematical velocities’ X^β because K_β does not depend on them. The lagrangean density is given by

$$\mathcal{L} = \pi_\phi \dot{\phi} - K_\beta \dot{X}^\beta, \quad (2.11)$$

We can define the ‘kinematical’ momentum as

$$\Pi_\alpha \equiv \frac{\partial \mathcal{L}}{\partial \dot{X}^\alpha} = -K_\alpha, \quad (2.12)$$

which yields the constraint

$$\Pi_\alpha + K_\alpha = 0, \quad (2.13)$$

i.e.

$$\mathcal{H}_\alpha \equiv \Pi_\alpha + \frac{\partial x^0}{\partial X^\beta} J T_\alpha^\beta = 0. \quad (2.14)$$

In such a way, it is possible to write the action linearly in the dynamical velocities $\dot{\phi}$ and in the kinematical velocities X^β

$$S = \int d^4 x (\pi_\phi \dot{\phi} + \Pi_\beta \dot{X}^\beta). \quad (2.15)$$

In order to vary freely the action without worrying about the constraint (2.14), we must add the term $N^\alpha \mathcal{H}_\alpha$, where N^α are lagrange multipliers, yielding

$$S = \int d^4 x (\pi_\phi \dot{\phi} + \Pi_\beta \dot{X}^\beta - N^\alpha \mathcal{H}_\alpha). \quad (2.16)$$

It is possible to make a deeper analysis by projecting the constraints (2.14) onto the normal direction and onto the parallel (or tangential) directions to the hypersurfaces $t = \text{constant}$:

$$\mathcal{H} \equiv \mathcal{H}_\alpha n^\alpha, \quad (2.17)$$

$$\mathcal{H}_i \equiv \mathcal{H}_\alpha X_i^\alpha, \quad (2.18)$$

where X_i^α are the components of the tangent vectors in the basis $\frac{\partial}{\partial X^\alpha}$, $\frac{\partial}{\partial x^i} = \frac{\partial X^\alpha}{\partial x^i} \frac{\partial}{\partial X^\alpha}$ and the normal vector is defined by

$$\eta_{\alpha\beta} n^\alpha n^\beta = \epsilon = \mp 1, \quad (2.19)$$

$$n_\alpha X_i^\alpha = 0, \quad (2.20)$$

(– for hyperbolic signature and + for euclidean signature). Hence, the general form of the constraints is given by the sum of a kinematical part plus a dynamical or field part:

$$\mathcal{H} \equiv \Pi_\alpha n^\alpha + \frac{\partial x^0}{\partial X^\beta} J T_\alpha^\beta n^\alpha = 0, \quad (2.21)$$

$$\mathcal{H}_i \equiv \Pi_\alpha X_i^\alpha + \frac{\partial x^0}{\partial X^\beta} J T_\alpha^\beta X_i^\alpha = 0. \quad (2.22)$$

The constraint \mathcal{H} is known as super-hamiltonian and the constraint \mathcal{H}_i as super-momentum. Expanding N^α in the basis (n^α, X_i^α) , $N^\alpha = N n^\alpha + N^i X_i^\alpha$, we get the action in parametrized form:

$$S = \int d^4 x (\pi_\phi \dot{\phi} + \Pi_\beta \dot{X}^\beta - N \mathcal{H} - N^i \mathcal{H}_i) \quad (2.23)$$

The canonical variables $\phi, \pi_\phi, X^\alpha, \Pi_\alpha$ are varied independently. The hamiltonian equations resulting from this variation will determine the evolution in time t of the canonical variables. Varying with respect to the lagrange multipliers N and N^i , we obtain the constraints:

$$\mathcal{H} \approx 0, \quad \mathcal{H}_i \approx 0 \quad (2.24)$$

We write the last equations in Dirac’s notation and terminology: the constraints are *weakly zero*, which means that the Poisson brackets between a functional of the canonical variables $A(\phi, \pi_\phi, X^\alpha, \Pi_\alpha)$ and a weakly zero

constraint are not necessarily zero [17]. For consistency of the theory, the constraints must be preserved in time, which means that their PB with the hamiltonian must be weakly zero. The hamiltonian is

$$H = \int d^3x (N\mathcal{H} + N^i\mathcal{H}_i) \quad (2.25)$$

and the constraints \mathcal{H} and \mathcal{H}_i will be conserved in time only if all the PB between them, evaluated on two points x e y of the hypersurface, are weakly zero. Dirac made this computation (with $\epsilon = -1$), and he showed that these brackets can be written as a linear combination of the original constraints (i.e. new constraints do not arise) satisfying the following algebra (known as ‘Dirac-Teitelboim’s algebra’)³ [27] [17]:

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = \mathcal{H}^i(x)\partial_i\delta^3(x, y) - \mathcal{H}^i(y)\partial_i\delta^3(y, x) \quad (2.26)$$

$$\{\mathcal{H}_i(x), \mathcal{H}(y)\} = \mathcal{H}(x)\partial_i\delta^3(x, y) \quad (2.27)$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = \mathcal{H}_i(x)\partial_j\delta^3(x, y) - \mathcal{H}_j(y)\partial_i\delta^3(y, x) \quad (2.28)$$

where upper indices of the supermomentum are raised by the metric tensor h_{ij} induced on the hypersurface $t = \text{constant}$, given by $h_{ij} = \eta_{\alpha\beta}X_i^\alpha X_j^\alpha$. (We take an usual convention in which we write the derivatives of the delta function with respect to the first argument every time: $\delta_i(x, y) \equiv \frac{\partial}{\partial x^i}\delta(x, y)$ and $\delta_i(y, x) \equiv \frac{\partial}{\partial y^i}\delta(y, x)$). Dirac found this result with the constraints given in the form (2.14).

A. An important result by Teitelboim

It is appropriate at this point to remind a result that will be of fundamental importance to our approach, which was obtained by Claudio Teitelboim [18]. He obtained this algebra (but with the signature of spacetime appearing explicitly) in a general form that is independent of the form of the constraints and without assuming a Minkowski spacetime. He studied the deformations of spacelike hypersurfaces embedded in a riemannian spacetime. Intuitively, a labeled hypersurface can be deformed in general according to two operations: leaving it fixed in the embedding spacetime and relabeling its points or keep fixed its labels and deform it into another hypersurface. The first operation represents a deformation

$\delta N^i \equiv \delta t N^i$ tangential to the hypersurface, generated by some $\bar{\mathcal{H}}_i$. The second operation represent a deformation $\delta N \equiv \delta t N$ ortogonal to the hypersurface, generated by some $\bar{\mathcal{H}}$. (For the previous case we have $\bar{\mathcal{H}}_i \equiv \mathcal{H}_i$ and $\bar{\mathcal{H}} \equiv \mathcal{H}$). Any functional F of canonical variables (fields and kinematical variables) defined on the hypersurface will change, according to the hamiltonian given by

$$\bar{H} = \int d^3x (N\bar{\mathcal{H}} + N^i\bar{\mathcal{H}}_i), \quad (2.29)$$

in such a way that

$$\delta F = \int d^3x \{F, \delta N\bar{\mathcal{H}} + \delta N^i\bar{\mathcal{H}}_i\}, \quad (2.30)$$

which we write as

$$\delta F = \int d^3x \{F, \delta N^\alpha \bar{\mathcal{H}}_\alpha\}, \quad (2.31)$$

where $\bar{\mathcal{H}}_0 \equiv \bar{\mathcal{H}}$ and $\delta N^0 \equiv \delta N$. Teitelboim follows a purely geometrical argument founded in the ‘path independence’ of the dynamical evolution: the change in the canonical variables during the evolution from a given initial hypersurface to a given final hypersurface must be independent of the particular sequence of intermediary hypersurfaces along which the change is actually evaluated. Then, assuming that the 3-geometries are embedded in a 4-dimensional non-degenerate manifold and consistency of the theory, he showed that the constraints $\bar{\mathcal{H}} \approx 0$ and $\bar{\mathcal{H}}_i \approx 0$ obey the following algebra (‘Dirac-Teitelboim’s algebra’)

$$\{\bar{\mathcal{H}}(x), \bar{\mathcal{H}}(x')\} = -\epsilon[\bar{\mathcal{H}}^i(x)\partial_i\delta^3(x', x) - \bar{\mathcal{H}}^i(x')\partial_i\delta^3(x', x)] \quad (2.32)$$

$$\{\bar{\mathcal{H}}_i(x), \bar{\mathcal{H}}(x')\} = \bar{\mathcal{H}}(x)\partial_i\delta^3(x, x'), \quad (2.33)$$

$$\{\bar{\mathcal{H}}_i(x), \bar{\mathcal{H}}_j(x')\} = \bar{\mathcal{H}}_i(x)\partial_j\delta^3(x, x') - \bar{\mathcal{H}}_j(x')\partial_i\delta^3(x', x), \quad (2.34)$$

where indices of supermomentum are raised with the metric h_{ij} induced on the hypersurface, $h_{ij} = g_{\alpha\beta}X_i^\alpha X_j^\alpha$, and $g_{\alpha\beta}$ is the metric of the embedding spacetime. The constant ϵ in Eq.(2.32) can be ± 1 depending if the 4-geometry where the hypersurfaces are embedded is euclidean ($\epsilon = 1$) or hyperbolic ($\epsilon = -1$). This analysis can be applied to a field evolving in a prescribed riemannian background or when the embedding spacetime is generated by the evolution, as in GR. In the first situation the algebra imposes conditions for preserving the

³It is not strictly an algebra because the structure constants depend on the metric [19]

local Lorentz invariance. In the case of GR the algebra provides the conditions for the existence of spacetime: the evolution of a 3-geometry can be viewed as the ‘motion’ of a 3-dimensional cut in a 4-dimensional spacetime (embeddability conditions). This result, when applied to the case of a parametrized field theory in a flat spacetime, means that the constraints obey the algebra given in (2.26) (2.27) (2.28).

B. A simple model

We will consider a scalar field in a flat spacetime, with a lagrangean given by

$$\mathcal{L}_o = -\frac{1}{2} \left(\eta^{\alpha\beta} \frac{\partial\phi}{\partial X^\alpha} \frac{\partial\phi}{\partial X^\beta} + U(\phi) \right), \quad (2.35)$$

where $\eta^{\alpha\beta} = \eta_{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$. Computing the energy-momentum tensor given by Eq.(2.9), and substituting it in (2.21) and (2.22) we obtain the super-hamiltonian and super-momentum constraints as

$$\mathcal{H} = \frac{1}{\nu} (\Pi_\alpha \nu^\alpha + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \nu^2 (h^{ij} \phi_{,i} \phi_{,j} + U(\phi))) = 0, \quad (2.36)$$

$$\mathcal{H}_i = \Pi_\alpha X_i^\alpha + \pi_\phi \phi_{,i} = 0, \quad (2.37)$$

where the vector normal to the hypersurface was written in the form (see [29] chap.7) $n^\alpha = \frac{\nu^\alpha}{\nu}$, where

$$\nu_\alpha \equiv -\frac{1}{3!} \epsilon_{\alpha\alpha_1\alpha_2\alpha_3} \frac{\partial(X^{\alpha_1} X^{\alpha_2} X^{\alpha_3})}{\partial(x^1 x^2 x^3)}, \quad (2.38)$$

and ν is the norm of ν^α

$$\nu = \sqrt{-\nu^\alpha \nu_\alpha}. \quad (2.39)$$

It can be shown that $-\nu^\alpha \nu_\alpha = h$, where $h \equiv \det(h_{ij})$ is the determinant of the metric induced on the hypersurface.

We know that the constraints obey the Dirac’s algebra [27] (see the appendix for a prove of (2.26) in a somewhat different way). In the next section we will quantize this model and we will interpret it within the Bohm-de Broglie picture.

III. PARAMETRIZED FIELD THEORY IN THE BOHM-DE BROGLIE INTERPRETATION

In this section we will study the Bohm-de Broglie interpretation of the parametrized field theory for the model presented in the last section. In first place, we quantize following Dirac prescription. Coordinates $\phi^A \equiv (X^0, X^1, X^2, X^3, \phi)$ and momentum $\pi_A \equiv (\Pi_0, \Pi_1, \Pi_2, \Pi_3, \pi_\phi)$ become operators, obeying commutation relations

$$[\phi^A(x), \phi^B(y)] = 0, [\pi_A(x), \pi_B(y)] = 0, \quad (3.1)$$

$$[\phi^A(x), \pi_B(y)] = i\hbar \delta_B^A \delta(x, y), \quad (3.2)$$

where x, y are two points of the hypersurface. Constraints acts annihilating the state, giving conditions for the possible states:

$$\hat{\mathcal{H}}_i | \Psi \rangle = 0 \quad (3.3)$$

$$\hat{\mathcal{H}} | \Psi \rangle = 0 \quad (3.4)$$

In the representation of ‘coordinates’ $\phi^A(x)$, the state of the scalar field is given by the functional $\Psi[\phi^A(x)]$ and the momentum operator is a functional derivative: $\pi_A(x) = -i\hbar \frac{\delta}{\delta\phi^A(x)}$. Substituting it into Eq. (3.3), and taking into account the super-momentum Eq.(2.37) we have:

$$-i\hbar X_i^\alpha \frac{\delta\Psi}{\delta X^\alpha(x)} - i\hbar \phi_{,i} \frac{\delta\Psi}{\delta\phi(x)} = 0, \quad (3.5)$$

It follows from this equation that Ψ is invariant under spatial coordinate transformations on the hypersurface.

Using the super-hamiltonian given in Eq. (2.36), Eq. (3.4) yields

$$\mathcal{H}(x)\Psi = \frac{1}{\nu} \left(-i\hbar \nu^\alpha \frac{\delta\Psi}{\delta X^\alpha(x)} - (\hbar)^2 \frac{1}{2} \frac{\delta^2\Psi}{\delta\phi(x)^2} + \frac{1}{2} \nu^2 \left(h^{ij}(x) \phi_{,i}(x) \phi_{,j}(x) + U(\phi(x)) \right) \right) \Psi = 0. \quad (3.6)$$

To interpret the above functional equation according to the BdB view, we follow the usual procedure. First we write the wave functional in polar form $\Psi = A e^{\frac{i}{\hbar} S}$. Substituting it into Eq. (3.5) yields two equations saying that S and A are invariants under general space coordi-

nate transformations:

$$X_i^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \phi_{,i} \frac{\delta S}{\delta\phi(x)} = 0. \quad (3.7)$$

$$X_i^\alpha \frac{\delta A}{\delta X^\alpha(x)} + \phi_{,i} \frac{\delta A}{\delta \phi(x)} = 0. \quad (3.8)$$

Substituting the polar form of Ψ into Eq.(3.6) we obtain two equations that depend on the factor ordering we choose. However, in any case, one of the equations will have the form, after dividing by A ,

$$\frac{1}{\nu} \left(\nu^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \frac{1}{2} \left(\frac{\delta S}{\delta \phi} \right)^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q} = 0. \quad (3.9)$$

where, for simplicity of notation, we define $W \equiv h^{ij}(x)\phi_{,i}\phi_{,j} + U(\phi(x))$. This is like a Hamilton-Jacobi equation modified by the quantum potential, given by the last term. Only the quantum potential depends on factor ordering and regularization; the other terms are well defined. All subsequent results depend only on the fact that \mathcal{Q} is a scalar density, which is true for any factor ordering, as it will be shown shortly. According to the non-regulated version given in Eq (3.6), \mathcal{Q} is:

$$\mathcal{Q} = -\frac{1}{\nu} \frac{\hbar^2}{2A} \frac{\delta^2 A}{\delta \phi(x)^2}. \quad (3.10)$$

The other equation is:

$$\nu^\alpha \frac{\delta A^2}{\delta X^\alpha} + \frac{\delta(A^2 \frac{\delta S}{\delta \phi})}{\delta \phi} = 0. \quad (3.11)$$

In the BdB interpretation the canonical variables exist independently on measurements and the evolution of canonical coordinates ϕ and X^α can be obtained from Bohm's guidance relations given by

$$\Pi_\alpha = \frac{\delta S(\phi, X^\alpha)}{\delta X^\alpha}, \quad (3.12)$$

$$\pi_\phi = \frac{\delta S(\phi, X^\alpha)}{\delta \phi}. \quad (3.13)$$

Given the initial values of the field $\phi(t_0, x^i)$ and kinematical variables $X^\alpha(t_0, x^i)$ on a initial hypersurface $x^0 = t = \text{const.}$, we can integrate these first order differential equations and compute the bohmian trajectories, i.e., the values of the field $\phi(t, x^i)$ and $X^\alpha(t, x^i)$ for any value of the parameter t . The evolution of those fields will be different from the classical one because of the presence of the quantum potential in Eq. (3.9). The classical limit is obtained by imposing the conditions for $\mathcal{Q} = 0$. In this case, the functional S obeys the classical Hamilton-Jacobi equation and we know that integrating equations (3.12) and (3.13) the solutions obtained represent a classical field evolving in a Minkowski spacetime. This follows from the fact that the constraints of the classical theory satisfy the Dirac-Teitelboim's algebra (2.32) (2.33) (2.34) with $\epsilon = -1$. However, if the quantum potential is different from zero, then S is a solution

of the *modified* Hamilton-Jacobi equation (3.9). Hence, we cannot assert that the obtained solution for ϕ^A still represent a field in a Minkowski spacetime. The quantum effects may break Lorentz invariance and modify the einsteinian causality of special relativity. Which type of structure corresponds to this case? To answer this question we rewrite the BdB theory, originally formulated in a Hamilton-Jacobi picture, in a hamiltonian picture.

Using the Bohm's guidance relations (3.12) (3.13) we can write (3.9) as:

$$\frac{1}{\nu} \left(\nu^\alpha \Pi_\alpha + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q} = 0. \quad (3.14)$$

Note that, whatever is the form of the quantum potential \mathcal{Q} , it must be a scalar density of weight one. This comes from the Hamilton-Jacobi equation Eq.(3.9). From this equation we can express \mathcal{Q} as

$$\mathcal{Q} = -\frac{1}{\nu} \left(\nu^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \frac{1}{2} \left(\frac{\delta S}{\delta \phi} \right)^2 + \frac{1}{2} \nu^2 W \right) \quad (3.15)$$

We remember that $\nu = \sqrt{\hbar}$ is a scalar density of weight 1 and that S is an invariant under general coordinate transformations on the hypersurface (this follows from the supermomentum constraint applied to Ψ , Eq.(3.7)). Thus, $\frac{\delta S}{\delta X^\alpha}$ is a vector density which, when contracted with the normal vector, produces a scalar density of weight 1. For the second term we use the same reasoning and the third term is obviously a scalar density of weight 1. Hence \mathcal{Q} is a sum of scalar densities of weight 1. We can write Eq.(3.14) as

$$\mathcal{H} + \mathcal{Q} = 0 \quad (3.16)$$

where \mathcal{H} is the classical superhamiltonian given by (2.36). The Bohm's quantum superhamiltonian reads:

$$\mathcal{H}_Q \equiv \mathcal{H} + \mathcal{Q}. \quad (3.17)$$

The hamiltonian that generates the bohmian trajectories, once the guidance relations (3.12) e (3.13) are imposed initially, is

$$H_Q = \int d^3x \left[N \mathcal{H}_Q + N^i \mathcal{H}_i \right]. \quad (3.18)$$

This can be shown by noting that the guidance relations are consistent with the constraints $\mathcal{H}_Q \approx 0$ and $\mathcal{H}_i \approx 0$, because S satisfy (3.7) and (3.9). Furthermore the guidance relations are conserved in the evolution given by the hamiltonian (3.18). This can be shown first by writing the guidance relations (3.12) (3.13) in a form adapted for the hamiltonian formalism as:

$$\Phi_\alpha \equiv \Pi_\alpha - \frac{\delta S}{\delta X^\alpha} \approx 0, \quad (3.19)$$

$$\Phi_\phi \equiv \pi_\phi - \frac{\delta S}{\delta \phi} \approx 0. \quad (3.20)$$

Conservation in time of the guidance relations means that $\dot{\Phi}_\phi \equiv \{\Phi_\phi, H_Q\} = 0$ e $\dot{\Phi}_\alpha \equiv \{\Phi_\alpha, H_Q\} = 0$. This is equivalent to prove that their Poisson brackets with the constraints \mathcal{H}_Q and \mathcal{H}_i are zero. Let us compute then

$$\begin{aligned} \{\Phi_\phi(y), \mathcal{H}_Q(x)\} &= \left\{ \Pi_\alpha - \frac{\delta S}{\delta X^\alpha}, \frac{1}{\nu} \left(\nu^\alpha \Pi_\alpha + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q} \right\} = \\ &= -\frac{\delta}{\delta \phi(y)} \left\{ \frac{1}{\nu} \left(\nu^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \frac{1}{2} \left(\frac{\delta S}{\delta \phi} \right)^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q} \right\} - \frac{1}{\nu} \frac{\delta^2 S}{\delta \phi^2} \Phi_\phi, \end{aligned} \quad (3.22)$$

where the first term from the RHS of this equation stands for the functional derivative with respect to $\phi(y)$ of the LHS of the modified Hamilton-Jacobi equation, Eq (3.9). Hence, it is identically zero. The second term from RHS is weakly zero because of the guidance relation (3.20).

$$\begin{aligned} \{\Phi_\alpha(y), \mathcal{H}_Q(x)\} &= -\frac{\delta}{\delta X^\alpha(y)} \left\{ \frac{1}{\nu} \left(\nu^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \frac{1}{2} \left(\frac{\delta S}{\delta \phi(x)} \right)^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q} \right\} \\ &= -\frac{1}{\nu} \frac{\delta \nu^\beta}{\delta X^\alpha(y)} \Phi_\beta - \frac{\delta \nu^{-1}}{\delta X^\alpha(y)} \nu^\beta \Phi_\beta - \frac{1}{2} \frac{\delta \nu^{-1}}{\delta X^\alpha(y)} \left(\Phi_\phi^2 + 2 \frac{\delta S}{\delta \phi} \Phi_\phi \right) - \frac{1}{\nu} \frac{\delta^2 S}{\delta \phi(x) \delta X^\alpha(y)} \Phi_\phi \approx 0, \end{aligned} \quad (3.24)$$

where the first term from the RHS of this equation stands for the functional derivative with respect to $X^\alpha(y)$ of the LHS of the modified Hamilton-Jacobi equation, Eq (3.9). It is identically zero. The other terms are weakly zero because of the guidance relations (3.19) (3.20). To compute the Poisson brackets involving the supermomentum constraint, as S is an invariant, then Φ_α is a vector density and Φ_ϕ is a scalar density, both of weight one. As \mathcal{H}_i is the generator of space coordinate transformations, we get

$$\{\Phi_\phi(y), \mathcal{H}_i(x)\} = -\Phi_\phi(x) \partial_i \delta(y, x) \approx 0, \quad (3.25)$$

$$\{\Phi_\alpha(y), \mathcal{H}_i(x)\} = \Phi_i(x) \partial_\alpha \delta(y, x) - \Phi_\alpha(y) \partial_i \delta(y, x) \approx 0. \quad (3.26)$$

Combining these results we obtain

$$\dot{\Phi}_\phi = \{\Phi_\phi, H_Q\} \approx 0, \quad (3.27)$$

$\{\Phi_\phi, \mathcal{H}_Q\}$, $\{\Phi_\alpha, \mathcal{H}_Q\}$, $\{\Phi_\phi, \mathcal{H}_i\}$ and $\{\Phi_\alpha, \mathcal{H}_i\}$. The quantum superhamiltonian is given by

$$\mathcal{H}_Q \equiv \mathcal{H} + \mathcal{Q} = \frac{1}{\nu} \left(\nu^\alpha \Pi_\alpha + \frac{1}{2} \pi_\phi^2 + \frac{1}{2} \nu^2 W \right) + \mathcal{Q}. \quad (3.21)$$

Computing, we have

Then, we have that

$$\{\Phi_\phi(y), \mathcal{H}_Q(x)\} = -\frac{1}{\nu} \frac{\delta^2 S}{\delta \phi(y)^2} \Phi_\phi(x) \approx 0. \quad (3.23)$$

For the bracket $\{\Phi_\alpha(y), \mathcal{H}_Q(x)\}$ we have

$$\dot{\Phi}_\alpha = \{\Phi_\alpha, H_Q\} \approx 0. \quad (3.28)$$

Then, the Bohm's guidance relations are conserved.

Knowing that the quantum potential does not depend on the momenta, we have that the definitions of the momenta in terms of the velocities are the same as in the classical case:

$$\dot{\phi} = \{\phi, H_Q\} = \{\phi, H\}, \quad (3.29)$$

$$\dot{X}^\alpha = \{X^\alpha, H_Q\} = \{X^\alpha, H\}. \quad (3.30)$$

We now have the BdB theory written in hamiltonian form and we want to know which type of structure corresponds to the Bohmian evolution generated by the hamiltonian (3.18)⁴. The constraints $\mathcal{H}_i \approx 0$ and $\mathcal{H}_Q \approx 0$

⁴We would like to emphasize that the BdB interpretation is only a different interpretation: its relevant hamiltonian operator

must be conserved in time for the consistency of the theory. This will be true only if all the PB between two of them, evaluated on two points x and y on the hypersurface, are weakly zero. In the context of the Teitelboim's work described in section II, let us analyze the algebra satisfied by the constraints $\mathcal{H}_i \approx 0$ and $\mathcal{H}_Q \approx 0$. The PB $\{\mathcal{H}_i(x), \mathcal{H}_j(y)\}$ satisfies Eq. (2.28) because \mathcal{H}_i in H_Q defined by Eq. (3.18) is the same as in the classical theory. In the same way, $\{\mathcal{H}_i(x), \mathcal{H}_Q(y)\}$ satisfies Eq. (2.27) since \mathcal{H}_i is the generator of space coordinate transformations, and because \mathcal{H}_Q is a scalar density of weight 1. What remains to be verified is if the PB $\{\mathcal{H}_Q(x), \mathcal{H}_Q(y)\}$ closes in the same way as in (2.26). We will show that this bracket is weakly zero for any quantum potential (note that it does not mean that it closes necessarily as

in Dirac-Teitelboim's algebra Eq. 2.26). This means that the theory is consistent for any Q and thus for any state. We have

$$\{\mathcal{H}_Q(x), \mathcal{H}_Q(y)\} = \{\mathcal{H}(x), \mathcal{H}(y)\} + \{\mathcal{H}(x), Q(y)\} + \{Q(x), \mathcal{H}(y)\}. \quad (3.31)$$

From equation (3.9) we can write the quantum potential as:

$$Q = -\frac{1}{\nu} \left(\nu^\alpha \frac{\delta S}{\delta X^\alpha(x)} + \frac{1}{2} \left(\frac{\delta S}{\delta \phi} \right)^2 + \frac{1}{2} \nu^2 W \right). \quad (3.32)$$

Replacing the last equation into Eq.(3.31) and taking into account the Bohm's guidance relations given by (3.19) and (3.20) we find that

$$\begin{aligned} \{\mathcal{H}_Q(x), \mathcal{H}_Q(y)\} = & \frac{1}{\nu(x)\nu(y)} \left(\left(\frac{\delta S}{\delta \phi(y)} \frac{\delta^2 S}{\delta \phi(x) \delta \phi(y)} + \right. \right. \\ & \left. \left. \nu^\alpha(y) \frac{\delta^2 S}{\delta X^\alpha(y) \delta \phi(x)} \right) \Phi_\phi(x) - \left(\frac{\delta S}{\delta \phi(x)} \frac{\delta^2 S}{\delta \phi(y) \delta \phi(x)} + \right. \right. \\ & \left. \left. \nu^\alpha(x) \frac{\delta^2 S}{\delta X^\alpha(x) \delta \phi(y)} \right) \Phi_\phi(y) + \nu^\alpha(y) \frac{\delta \nu^\beta(x)}{\delta X^\alpha(y)} \Phi_\beta(y) - \nu^\alpha(x) \frac{\delta \nu^\beta(y)}{\delta X^\alpha(x)} \Phi_\beta(y) \right) \approx 0, \end{aligned} \quad (3.33)$$

The RHS of this equation is weakly zero in view of Bohm's guidance relations (3.19) and (3.20).

Hence, we see that the Bohm-de Broglie interpretation of a field theory in Minkowski spacetime is a consistent theory. However, the constraint's algebra may not close as Dirac-Teitelboim's algebra. It will depend on the quantum potential. If Q breaks the Dirac-Teitelboim's algebra then, following Teitelboim's result, the structure of the background spacetime will be modified. This means that Lorentz invariance is broken. A similar situation is shown in quantum geometrodynamics where the quantum potential determine the quantum evolution of the Universe [1] [16]. We will now show that already the ground state of the free scalar field produces a quantum potential that breaks the Dirac-Teitelboim's algebra and, consequently, Lorentz invariance.

The wave functional for the ground state of the free scalar field is given by ([30] chap. 10):

$$\Psi_0[\phi, T] = e^{-\frac{iE_0 T}{\hbar}} \eta e^{-\int d^3 X d^3 Y \phi(X) g(X, Y) \phi(Y)}, \quad (3.34)$$

where E_0 is the renormalized energy,

$$g(X, Y) = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \omega_k e^{ik \cdot (X-Y)}, \quad (3.35)$$

and $\omega_k = \hbar \sqrt{k^2 + m^2}$. The quantity η is a normalization factor which does not depend on ϕ , and hence it is of no importance in the following calculations. We denote $X \equiv \vec{X}$ and $k \equiv \vec{k}$ for the 3-vectors. Computing the amplitude from (3.34) and using that

$$Q(\phi) = -\hbar^2 \frac{1}{2A} \int d^3 X \frac{\delta^2 A}{\delta \phi^2}, \quad (3.36)$$

the quantum potential reads

as well as its functional Schrödinger's equation are the same as in any other interpretation. Hence, the eigenstates, including the ground state, and statistical predictions are the same, the difference residing in the interpretation, where trajectories independent of any observation (an underlying objective reality) are supposed to exist. Hence, the *classical like* hamiltonian (not strictly classical because of the presence of the quantum potential, which has no parallel in classical physics) presented above is an extra structure meaningful only within the BdB interpretation. Such *classical like* hamiltonian formalism has no place in the Copenhagen interpretation, where only probabilities of measurement results have an objective reality: the subquantum world does not exist.

$$Q = -\frac{1}{2} \int d^3X \left(\int d^3Y \frac{d^3k}{(2\pi)^3} \omega_k \cos\{k \cdot (X - Y)\} \phi(Y) \right)^2 + \frac{1}{2} \int d^3X \int \frac{d^3k}{(2\pi)^3} \omega_k. \quad (3.37)$$

The last term is the zero-point energy which can be renormalized to E_0 by choosing, e.g., a normal factor ordering in Eq. (3.36). Anyway, it is not important for our calculations because it is independent of the field ϕ and hence its functional derivative with respect to ϕ vanishes.

We write the last equation, after an adequate renor-

malization of the zero point energy, as

$$Q = \int d^3X f(X^i, \phi), \quad (3.38)$$

where f depends on X^i as a function and depends on ϕ as a functional, and is given by

$$f \equiv -\frac{1}{2} \left(\int d^3Y \frac{d^3k}{(2\pi)^3} \omega_k \cos\{k \cdot (X - Y)\} \phi(Y) \right)^2. \quad (3.39)$$

Writing it in hypersurface coordinates x^i we have

$$Q = \int d^3x J f(X^i(x^j), \phi), \quad (3.40)$$

where J is the jacobian $J = \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \frac{\partial X^i}{\partial x^a} \frac{\partial X^j}{\partial x^b} \frac{\partial X^k}{\partial x^c}$. The quantum potential density Q that enters into the quan-

tum superhamiltonian of Eq. (3.17) will be:

$$Q = J f(X^i(x^j), \phi) \quad (3.41)$$

Let us calculate the Poisson bracket $\{\mathcal{H}_Q(x), \mathcal{H}_Q(y)\}$. We have

$$\begin{aligned} \{\mathcal{H}_Q(x), \mathcal{H}_Q(y)\} &= \{\mathcal{H}(x), \mathcal{H}(y)\} + \{\mathcal{H}(x), Q(y)\} + \{Q(x), \mathcal{H}(y)\} = \\ &= \mathcal{H}^i(x) \partial_i \delta^3(x, y) - \mathcal{H}^i(y) \partial_i \delta^3(y, x) + \{\mathcal{H}(x), Q(y)\} + \{Q(x), \mathcal{H}(y)\}, \end{aligned} \quad (3.42)$$

where the first two terms on the RHS are exactly those appearing in the Dirac's algebra Eq.(2.26). Hence, to maintain the Dirac's algebra, it is necessary that

$\{\mathcal{H}(x), Q(y)\} + \{Q(x), \mathcal{H}(y)\} = 0$ (strongly zero). Meanwhile,

$$\begin{aligned} \{\mathcal{H}(x), Q(y)\} + \{Q(x), \mathcal{H}(y)\} &= +2 \frac{\nu^\alpha(y)}{\nu(y)} f(y) \epsilon_{\alpha j k} \epsilon^{abc} \frac{\partial X^j}{\partial y_b} \frac{\partial X^k}{\partial y_c} \frac{\partial \delta(y, x)}{\partial y_a} + \\ &+ 2 \frac{J(y)}{\nu(x)} \pi_\phi B(y) \int \frac{d^3k}{(2\pi)^3} \omega_k \cos k \cdot (X(y) - x) - 2 \frac{\nu^\alpha(x)}{\nu(x)} f(x) \epsilon_{\alpha j k} \epsilon^{abc} \frac{\partial X^j}{\partial x_b} \frac{\partial X^k}{\partial x_c} \frac{\partial \delta(x, y)}{\partial x_a} + \\ &+ 2 \frac{J(x)}{\nu(y)} \pi_\phi(y) B(x) \int \frac{d^3k}{(2\pi)^3} \omega_k \cos k \cdot (X(x) - y), \end{aligned} \quad (3.43)$$

and the RHS of this equation is evidently strongly different from zero. Hence the Dirac-Teitelboim's algebra is not satisfied in this particular example. According to Ref. [18], the bohmian trajectories are generating a new structure which does not correspond to a relativistic field propagating in Minkowski spacetime. In other words, we have shown the breaking of Lorentz invariance in terms of the breaking of the Dirac-Teitelboim's algebra of the constraints. This constitutes an alternative derivation of a known result of the BdB interpretation, which is explained in Refs. [8] [20]⁵.

This method will be useful for quantum geometrodynamics. We point out that relativistic invariance is lost only at the level of individual events. However, the field properties, as we know, are basically statistical and are contained in the expectation values of the operators

$$\langle \Psi | \hat{A} | \Psi \rangle = \int \Psi^* [\phi] (\hat{A}\Psi) [\phi] D\phi \quad (3.46)$$

whose invariant character is maintained. As long as the invariance of the individual events can be broken in general, as we saw explicitly in the last example, special relativity is verified in the laboratory only statistically. Lorentz invariance is a statistical effect [8] [20]. We point out that the conclusions obtained in this work are easily extended to any Minkowski spacetime with dimension $n \geq 2$.

IV. QUANTUM GEOMETRODYNAMICS

In Ref. [1] we applied the BdB interpretation to canonical quantum cosmology. Let's briefly summarize what was done there and the results obtained. The steps taken

here are analogous to what was done in the previous section.

We quantized General Relativity with a minimally coupled scalar field in an arbitrary potential. All results remain essentially the same for any matter field which couples uniquely with the metric, not with their derivatives. The classical hamiltonian of GR with a scalar field is given by:

$$H = \int d^3x (N\mathcal{H} + N^j\mathcal{H}_j) \quad (4.1)$$

The lapse function N and the shift function N_j are the Lagrange multipliers of the *super-hamiltonian constraint* $\mathcal{H} \approx 0$ and the *super-momentum constraint* $\mathcal{H}^j \approx 0$, respectively. The constraints are present due to the invariance of GR under spacetime coordinate transformations. They are given by

$$\mathcal{H} = \kappa G_{ij}{}_{kl} \Pi^{ij} \Pi^{kl} + \frac{1}{2} h^{-1/2} \Pi_\phi^2 + V \quad (4.2)$$

$$\mathcal{H}_j = -2D_i \Pi_j^i + \Pi_\phi \partial_j \phi. \quad (4.3)$$

being V the classical potential given by

$$V = h^{1/2} \left[-\kappa^{-1} (R^{(3)} - 2\Lambda) + \frac{1}{2} h^{ij} \partial_i \phi \partial_j \phi + U(\phi) \right]. \quad (4.4)$$

In these equations, h_{ij} is the metric of closed 3-dimensional spacelike hypersurfaces, and Π^{ij} is its canonical momentum given by

⁵It goes succinctly in the following way: the equation for the field ϕ can be found by taking the functional derivative of the modified Hamilton-Jacobi equation of the field in the original Minkowski coordinates (see [16] [8]). Using the Bohm guidance relations, it is possible to show that ϕ satisfies

$$-\nabla^2 \phi(X, T) + \frac{\partial^2}{\partial T^2} \phi(X, T) + m^2 \phi(X, T) = -\frac{\delta Q[\phi(X), T]}{\delta \phi(X)} \Big|_{\phi(X)=\phi(X, T)} \quad (3.44)$$

This is the quantum version of the classical wave equation:

$$-\nabla^2 \phi(X, T) + \frac{\partial^2}{\partial T^2} \phi(X, T) + m^2 \phi(X, T) = 0. \quad (3.45)$$

The 'quantum force' that appears in the RHS of (3.44) is responsible for all the quantum effects. For the ground state we have

$$-\nabla^2 \phi(X, T) + \frac{\partial^2}{\partial T^2} \phi(X, T) + m^2 \phi(X, T) = (-\nabla^2 + m^2) \phi(X) \Big|_{\phi(X)=\phi(X, T)},$$

which is not a Lorentz invariant equation. We emphasize that $\phi(X, T)$ is a c-number at each spacetime point. It is the eigenvalue of the Schrödinger field operator: $\hat{\phi}(x) | \Psi \rangle = \phi(x) | \Psi \rangle$, evaluated along a system "trajectory" (the bohmian trajectory), a notion that has no meaning in the conventional interpretation. Equation (3.44) belongs only to the subquantum world of the BdB interpretation. It is important to the reader to not confuse $\phi(x, t)$ with the Heisenberg field operator $\hat{\phi}(x, t)$ which satisfies the usual Lorentz invariant Klein-Gordon equation [8].

$$\Pi^{ij} = -h^{1/2}(K^{ij} - h^{ij}K) = G^{ijkl}(\dot{h}_{kl} - D_k N_l - D_l N_k), \quad (4.5)$$

where

$$K_{ij} = -\frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i), \quad (4.6)$$

is the extrinsic curvature of the hypersurfaces (indices are raised and lowered by the 3-metric h_{ij} and its inverse h^{ij}). The canonical momentum of the scalar field is now

$$\Pi_\phi = \frac{h^{1/2}}{N} \left(\dot{\phi} - N^i \partial_i \phi \right). \quad (4.7)$$

The quantity $R^{(3)}$ is the intrinsic curvature of the hypersurfaces and h is the determinant of h_{ij} . The quantities G_{ijkl} and its inverse G^{ijkl} ($G_{ijkl}G^{ijab} = \delta_{kl}^{ab}$) are given by

$$G^{ijkl} = \frac{1}{2}h^{1/2}(h^{ik}h^{jl} + h^{il}h^{jk} - 2h^{ij}h^{kl}), \quad (4.8)$$

$$G_{ijkl} = \frac{1}{2}h^{-1/2}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}), \quad (4.9)$$

which is called the DeWitt metric. The quantity D_i is the i -component of the covariant derivative operator on the hypersurface, and $\kappa = 16\pi G/c^4$. The algebra of the constraints close in the following form [19] [31]:

$$\begin{aligned} \{\mathcal{H}(x), \mathcal{H}(x')\} &= \mathcal{H}^i(x) \partial_i \delta^3(x, x') - \mathcal{H}^i(x') \partial_i \delta^3(x', x) \\ \{\mathcal{H}_i(x), \mathcal{H}(x')\} &= \mathcal{H}(x) \partial_i \delta^3(x, x') \\ \{\mathcal{H}_i(x), \mathcal{H}_j(x')\} &= \mathcal{H}_i(x) \partial_j \delta^3(x, x') + \mathcal{H}_j(x') \partial_i \delta^3(x, x') \end{aligned} \quad (4.10)$$

It is a feature of the hamiltonian of GR that the 4-metrics $ds^2 = -(N^2 - N^i N_i) dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j$ constructed by integrating the Hamilton equations, with the same initial conditions, describe the same four-geometry for any choice of N and N^i .

After quantizing this theory following the Dirac procedure, we obtain the Wheeler-DeWitt equation and, in order to construct the BdB interpretation, the wave functional is written in polar form $\Psi = A \exp(iS/\hbar)$ yielding two equations for A and S which of course depend on the factor ordering we choose. However, in any case, one of the equations will have the form

$$\kappa G_{ijkl} \frac{\delta S}{\delta h_{ij}} \frac{\delta S}{\delta h_{kl}} + \frac{1}{2} h^{-1/2} \left(\frac{\delta S}{\delta \phi} \right)^2 + V + Q = 0, \quad (4.11)$$

where V is the classical potential given in Eq. (4.4). Contrary to the other terms in Eq. (4.11), which are already well defined because they do not need any regularization, only the precise form of Q depends on the regularization and factor ordering procedures which are prescribed for the Wheeler-DeWitt equation. Eq. (4.11) is like the

Hamilton-Jacobi equation for GR, supplemented by an extra term Q , the quantum potential. The trajectories of the 3-metric and scalar field (which in this interpretation always exist by assumption) can be obtained from the guidance relations

$$\Pi^{ij} = \frac{\delta S(h_{ab}, \phi)}{\delta h_{ij}}, \quad (4.12)$$

$$\Pi_\phi = \frac{\delta S(h_{ij}, \phi)}{\delta \phi}, \quad (4.13)$$

These are first order differential equations which can be integrated to yield the 3-metric and scalar field for all values of the t parameter. These solutions depend on the initial values of the 3-metric and scalar field at some initial hypersurface. The evolution of these fields will of course be different from the classical one due to the presence of the quantum potential term Q in Eq. (4.11). What kind of structure do we obtain when we integrate this equations in the parameter t ? Does this structure form a 4-dimensional geometry with a scalar field for any choice of the lapse and shift functions? Note that if the functional S were a solution of the classical Hamilton-Jacobi equation, which does not contain the quantum potential term, then the answer would be in the affirmative because we would be in the scope of GR.

In order to answer the questions formulated above, we moved from this Hamilton-Jacobi picture of quantum geometrodynamics to a hamiltonian picture, where strong results concerning geometrodynamics exist [18] [19]. In this way we have constructed a quantum geometrodynamical picture of the Bohm-de Broglie interpretation of canonical quantum gravity and we found that once the Bohm's guidance relations, given by Eqs. (4.12) and (4.13), are imposed initially, the bohmian trajectories will be generated by the hamiltonian, H_Q , given by

$$H_Q = \int d^3x \left[N(\mathcal{H} + Q) + N^i \mathcal{H}_i \right] \quad (4.14)$$

where we define

$$\mathcal{H}_Q \equiv \mathcal{H} + Q. \quad (4.15)$$

We have found that the bohmian evolution of the 3-geometries can yield, depending on the quantum potential (i.e. on the wave functional), basically two possible types of structures which are determined by the algebra satisfied by the constraints:

A. A consistent non-degenerate four geometry. In this scenario the quantum potential does not break the Dirac-Teitelboim's algebra, and the most important quantum effect is the change of signature yielding an euclidean spacetime.

B. A consistent but degenerate four-geometry indicating the presence of special vector fields and the

breaking of the spacetime structure as a single entity. The constraints satisfy an algebra different from Dirac-Teitelboim's algebra.

We left open the possibility of a third structure that would correspond to an inconsistent bohmian evolution in which the algebra of the constraints does not close i.e. one of the PB would not be zero (the constraints would not be conserved in time). The relevant PB is $\{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\}$ (the other two PB are weakly zero for

any quantum potential as was shown in Ref. [1]). Some complicated non-local quantum potential could make this PB weakly different from zero. In the present section we show that the PB $\{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\}$, when restricted to the bohmian trajectories, is weakly zero for any quantum potential. Hence there is no inconsistency: quantum geometrodynamics in the Bohm-de Broglie interpretation is always a consistent theory for any quantum state. In order to show this fact, we calculate

$$\begin{aligned} \{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\} &= \{\mathcal{H}(x), \mathcal{H}(x')\} - 2\kappa G_{abcd}(x)\Pi^{cd}(x)\frac{\delta\mathcal{Q}(x')}{\delta h_{ab}(x)} \\ &+ 2\kappa G_{abcd}(x')\Pi^{cd}(x')\frac{\delta\mathcal{Q}(x)}{\delta h_{ab}(x')} - h^{-1/2}(x)\Pi_\phi(x)\frac{\delta\mathcal{Q}(x')}{\delta\phi(x)} + h^{-1/2}(x')\Pi_\phi(x')\frac{\delta\mathcal{Q}(x)}{\delta\phi(x')}. \end{aligned} \quad (4.16)$$

Our steps are similar to the ones followed in the previous section. We write the quantum potential as follows from the modified Hamilton-Jacobi equation (4.11)

$$\mathcal{Q} = -\kappa G_{ijkl}\frac{\delta S}{\delta h_{ij}}\frac{\delta S}{\delta h_{kl}} - \frac{1}{2}h^{-1/2}\left(\frac{\delta S}{\delta\phi}\right)^2 - V, \quad (4.17)$$

and replace it into (4.16) yielding

$$\begin{aligned} \{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\} &= \{\mathcal{H}(x), \mathcal{H}(x')\} + 2\kappa G_{abcd}(x)\Pi^{cd}(x)\frac{\delta V(x')}{\delta h_{ab}(x)} + h^{-1/2}(x)\Pi\frac{\delta V(x')}{\delta\phi(x)} \\ &- \kappa G_{abcd}(x')\Pi^{cd}(x')\frac{\delta V(x)}{\delta h_{ab}(x')} - h^{-1/2}(x')\Pi\frac{\delta V(x)}{\delta\phi(x')} + 4\kappa^2 G_{abcd}(x)\Pi^{cd}(x)\frac{\delta^2 S}{\delta h_{ab}(x)h_{ij}(x')}\frac{\delta S}{\delta h_{kl}(x')}G_{ijkl}(x') \\ &+ 2\kappa G_{abcd}(x)\Pi^{cd}(x)h^{-1/2}(x')\frac{\delta S}{\delta\phi(x')}\frac{\delta^2 S}{\delta h_{ab}(x)\delta\phi(x')} + 2\kappa h^{-1/2}(x)\Pi_\phi(x)G_{ijkl}(x')\frac{\delta^2 S}{\delta\phi(x)\delta h_{ij}(x')}\frac{\delta S}{\delta h_{kl}(x')} \\ &+ h^{-1/2}(x)h^{-1/2}(x')\Pi_\phi(x)\frac{\delta S}{\delta\phi(x')}\frac{\delta^2 S}{\delta\phi(x)\phi(x')} - (x \longleftrightarrow x'), \end{aligned} \quad (4.18)$$

where $(x \longleftrightarrow x')$ means the same expression with x and x' interchanged. In the RHS, the terms proportionals to $\delta^3(x', x)$ (these terms come from the functional derivatives $\frac{\delta G_{ijkl}(x')}{\delta h_{ab}(x)}$ and $\frac{\delta h^{-1/2}(x')}{\delta h_{ab}(x)}$) were cancelled with the terms that come from the term $-(x \longleftrightarrow x')$. The four terms that follows after $\{\mathcal{H}(x), \mathcal{H}(x')\}$ will produce exactly $-\{\mathcal{H}(x), \mathcal{H}(x')\}$ and they will be cancelled out. Substituting the momenta expressed according Bohm's guidance relations

$$\Pi^{ij}(x) = \Phi^{ij}(x) + \frac{\delta S}{\delta h_{ij}(x)}, \quad (4.19)$$

$$\Pi_\phi(x) = \Phi_\phi(x) + \frac{\delta S}{\delta\phi(x)}, \quad (4.20)$$

it is easy to see, using the symmetry properties of G_{ijkl} , that all terms that are not weakly zero will be cancelled by pairs. At last we have

$$\begin{aligned} \{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\} &= +4\kappa^2 G_{abcd}(x)\Phi^{cd}(x)\frac{\delta^2 S}{\delta h_{ab}(x)h_{ij}(x')}\frac{\delta S}{\delta h_{kl}(x')}G_{ijkl}(x') \\ &+ 2\kappa G_{abcd}(x)\Phi^{cd}(x)h^{-1/2}(x')\frac{\delta S}{\delta\phi(x')}\frac{\delta^2 S}{\delta h_{ab}(x)\delta\phi(x')} + 2\kappa h^{-1/2}(x)\Phi_\phi(x)G_{ijkl}(x')\frac{\delta^2 S}{\delta\phi(x)\delta h_{ij}(x')}\frac{\delta S}{\delta h_{kl}(x')} \\ &+ h^{-1/2}(x)h^{-1/2}(x')\Phi_\phi(x)\frac{\delta S}{\delta\phi(x')}\frac{\delta^2 S}{\delta\phi(x)\phi(x')} - (x \longleftrightarrow x'). \end{aligned} \quad (4.21)$$

The RHS of this equation is weakly zero because of the Bohm's guidance relations and, then

$$\{\mathcal{H}_Q(x), \mathcal{H}_Q(x')\} \approx 0 \quad (4.22)$$

This prove the consistency. Note that it was very important to use the guidance relations to close the algebra. It means that the hamiltonian evolution with the quantum potential (4.17) is consistent only when restricted to the bohmian trajectories. For other trajectories, it may be inconsitent. This is an important remark on the BdB interpretation of canonical quantum cosmology, which sometimes is not noticed.

V. CONCLUSIONS

We have studied a hamiltonian description of the BdB interpretation of quantum field theory and canonical quantum gravity.

In the first case, we have examined certain state functionals where the quantum potential have such a form that the Dirac-Teitelboim's algebra of the classical constraints of the hamiltonian picture is broken, and the structure of the background spacetime becomes different from the Minkowski one. This implies the break of Lorentz invariance of individual events. We exhibited a concrete example given by the ground state of a free scalar field presenting this property. This is a well known result (see [8] [20]), but we have shown it in terms of the break of Dirac-Teitelboim's algebra. In this manner, the BdB view of quantum field theory expressed in a hamiltonian approach can give a nice picture of the loss of Lorentz invariance of the bohmian subquantum world: when Dirac-Teitelboim's algebra is broken, then Lorentz invariance of individual events is lost. We should remember that the statistical properties of the quantum field, which are the same in the BdB interpretation as in the conventional one, continue to be Lorentz invariant.

We have shown that both quantum field theory and canonical quantum gravity have a consistent bohmian hamiltonian formulation, in the sense of Dirac, for any quantum potential, i.e., any state functional, when restricted to the bohmian trajectories. In the case of canonical quantum gravity, this complete the quantum geometrodynamical picture of quantum cosmology in the BdB view which was constructed in our previous paper [1].

The importance of this result can be seen as follows: suppose there were states which had inconsistent quantum evolution in the BdB interpretation, in the sense we have described. Then, the subquantum reality of the BdB interpretation would be imposing selection rules on possible quantum states which are absent in other interpretations. This could be used to find ways to distinguish experimentally between interpretations and/or impose new boundary conditions which could be used in quantum cosmology. However, we have shown that this subquantum world is not imposing any restriction to the possible quantum states of a field theory: the admissible quantum states are the same as in any other interpretation.

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APPENDIX: COMPUTING $\{\mathcal{H}(X), \mathcal{H}(Y)\}$ FOR THE PARAMETRIZED FIELD THEORY

We present here the computation of $\{\mathcal{H}(x), \mathcal{H}(y)\}$ for the parametrized field theory. Even though it is a well known result, we compute it in a new way. The superhamiltonian is given by (Eq. 2.36)

$$\mathcal{H} = \frac{1}{\nu}(\Pi_\alpha \nu^\alpha + \frac{1}{2}\pi_\phi^2 + \frac{1}{2}\nu^2(h^{ij}\phi_{,i}\phi_{,j} + U(\phi))),$$

which we write, for simplicity, as

$$\mathcal{H} = \nu^{-1}\mathbb{H}, \quad (A1)$$

where we define

$$\mathbb{H} \equiv \Pi_\alpha \nu^\alpha + \frac{1}{2}\pi_\phi^2 + \frac{1}{2}\nu^2(h^{ij}\phi_{,i}\phi_{,j} + U(\phi)). \quad (A2)$$

Then, we have

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = \frac{1}{\nu(x)}\frac{1}{\nu(y)}\{\mathbb{H}(x), \mathbb{H}(y)\} + \frac{1}{\nu(x)}\mathbb{H}(y)\{\mathbb{H}(x), \frac{1}{\nu(y)}\} + \frac{1}{\nu(y)}\mathbb{H}(x)\{\frac{1}{\nu(x)}, \mathbb{H}(y)\}. \quad (1.3)$$

Using the fact that $\frac{\delta\nu_\alpha(x)}{\delta X^\beta(y)} = -\frac{\delta\nu_\beta(x)}{\delta X^\alpha(y)}$, which comes from (2.38), and the basic properties of the $\delta(x, y)$, it is possible to show that each of the last two brackets of the

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = -\frac{1}{\nu^2(y)}\Pi_\beta(y)\nu^\alpha(y)\frac{\partial\nu^\beta(y)}{\partial X_i^\alpha(x)}\frac{\partial}{\partial y^i}\delta(y, x) + \frac{1}{\nu^2(x)}\Pi_\alpha(x)\nu^\beta(x)\frac{\partial\nu^\alpha(x)}{\partial X_i^\beta(y)}\frac{\partial}{\partial x^i}\delta(x, y) - h^{ij}(y)\Pi_\phi(y)\frac{\partial\phi}{\partial y^j}\frac{\partial}{\partial y^i}\delta(y, x) + h^{ij}(x)\Pi_\phi(x)\frac{\partial\phi}{\partial x^j}\frac{\partial}{\partial x^i}\delta(x, y). \quad (1.4)$$

Finally, is possible to show that the first term of the RHS is equal to

$$-h^{ij}(y)\Pi_\alpha(y)\frac{\partial X^\alpha}{\partial y^j}\frac{\partial}{\partial y^i}\delta(y, x), \quad (1.5)$$

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = h^{ij}(x)\left(\Pi_\alpha(x)\frac{\partial X^\alpha}{\partial x^j} + \Pi_\phi(x)\frac{\partial\phi}{\partial x^j}\right)\frac{\partial}{\partial x^i}\delta(x, y) - h^{ij}(y)\left(\Pi_\alpha(y)\frac{\partial X^\alpha}{\partial y^j} + \Pi_\phi(y)\frac{\partial\phi}{\partial y^j}\right)\frac{\partial}{\partial y^i}\delta(y, x), \quad (1.7)$$

which means that

$$\{\mathcal{H}(x), \mathcal{H}(y)\} = \mathcal{H}^i(x)\frac{\partial}{\partial x^i}\delta(x, y) - \mathcal{H}^i(y)\frac{\partial}{\partial y^i}\delta(y, x). \quad (1.8)$$

which is Eq. (2.26).

RHS of this equation are identically zero. Next, using the same argument and the fact that the potential $U(\phi)$ does not contain derivatives of the metric, the last equation becomes:

and the second is equal to

$$+h^{ij}(x)\Pi_\alpha(x)\frac{\partial X^\alpha}{\partial x^j}\frac{\partial}{\partial x^i}\delta(x, y), \quad (1.6)$$

where Eq. (2.38) was used again. Thus

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