Space-and Time-Like Superselection Rules in Conformal Quantum Field Theory

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Abstract

In conformally invariant quantum field theories one encounters besides the standard DHR superselection theory based on spacelike (Einstein-causal) commutation relations and their Haag duality another timelike ("Huygens") based superselection structure. Whereas the DHR theory based on spacelike causality of observables confirmed the Lagrangian internal symmetry picture on the level of the physical principles of local quantum physics, the attempts to understand the timelike based superselection charges associated with the center of the conformal covering group in terms of timelike localized charges lead to a more dynamical role of charges outside the DR theorem and even outside the Coleman-Mandula setting. The ensuing plektonic timelike structure of conformal theories explains the spectrum of the anomalous scale dimensions in terms of admissable braid group representations, similar to the explanation of the possible anomalous spin spectrum expected from the extension of the DHR theory to stringlike d=1+2 plektonic fields.

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1 Introduction

Among the oldest and most fruitful concepts in quantum mechanics and quantum field theory are the spin-statistics connection, the PCT-theorem and the factorization of the total symmetry into inner- and spacetime-symmetries [1]. Spin&Statistics and PCT were first seen in the formal Lagrangian quantization approach, whereas the internal symmetry entered particle physics initially via the phenomenologically motivated approximate isospin symmetry of nuclear physics and was easily incorporated into the Lagrangian framework in the form of field/particle multiplets. The DHR-theory [2], which started from the properly mathematically formulated causality and spectral principles for observables of (what become later known as) algebraic QFT [3] and aimed at the reconstruction of charge carrying (non-observable, superselected) field operators, finally culminated in the theorem of Doplicher and Roberts [4]. In this way it became clear that the local quantum physics generated by a physically admissible field multiplet with a specific internal symmetry group was already uniquely (after fixing some conventions) characterized by the observable structure. This de-mystified to a large degree the concept of internal symmetries by showing a new way to derive the representation category of compact groups (all compact groups arise in this way) from localization principles of quantum observables, a quite unexpected connection which has not yet been fully appreciated by the particle physics community.

Moreover the Spin&Statistics and TCP issue became inexorable linked with that of internal symmetry and the original Einstein-causal observable algebra was reattained as the fixed-point algebras under the compact global "gauge group". Although this picture about internal symmetries confirmed the formal observations in the Lagrangian quantization setting i.e. there were no completely unexpected new physical concepts (the innovative power especially of the DR theory remained on the mathematical side), the superselection analysis of observable algebras was able to relate hitherto seemingly unrelated structures and thus lead to a fresh and novel point of view with different perspectives besides contributing a new mathematical duality theory on group representations.

The only exceptions were low dimensional field theories ($D < 1 + 3$) where models were found by special non-Lagrangian methods and where the algebraic methods led to the more general braid group- instead

of permutation group- statistics [3] for which there are no natural Lagrangian realizations.

Since one of the main localization prerequisites of this theory is the possibility of compact spacetime localization and since this requirement in conformal quantum field theories is formally automatically met as a result of the conformal equivalence of noncompact regions (e.g. wedges) with compact ones (e.g. double cones or "diamonds"), the Doplicher Roberts theorem is in particular applicable to all conformal higher dimensional ($D \geq 1+3$) theories.

However it was realized rather early that conformal theories have additional superselection rules which have a somewhat different conceptual basis and are intimately related to anomalous scale dimensions. They result from the structure of the center Z of the conformal covering whose action describes a timelike rotational sweep and hence they are not accounted for by the DR theory. In this paper we look for arguments that the coherent subspaces associated with the conformal covering group are also of local origin i.e. associated with the representation theory of an algebra with timelike locality [9]. In fact it was noticed that the ensuing conformal decomposition theory is nonlocal at spacelike differences [5], but its timelike structure remained unexplored.

Only in the very special and atypical D=1+1 conformal theories which permits a topology preserving interchange between the space- and time-like regions and which leads to a tensor decomposition into two "chiral" lightray theories, a sufficiently rich family of nontrivial ("minimal") models (abelian braid group illustrations with exponential Bose fields were already discussed in [5]) was later found by Belavin Polyakov and Zamolodchikov [6]; in fact the chiral version of the conformal central decomposition theory is part of their "block"-decomposition. The BPZ methods were based on special algebraic structures which had no counterpart in higher spacetime dimensions. By emphasizing the charge transport around the compactified Minkowski world (charge-monodromy) [7] and the related braid group statistics as expressed in terms of exchange algebras, it was possible to incorporate chiral quantum field theory into the algebraic setting of superselection sectors i. e. to place it under a common roof with higher dimensional QFT.

The suggestion to look for timelike braid group commutations in higher dimensional conformal theories is consistent with the analytic structure of the two-point function which for scalar fields is

$$\langle A(x)A(y)^* \rangle \simeq lim_{\varepsilon \to 0} \frac{1}{\left[-(x-y)^2_\varepsilon \right]^{\delta_A}} \qquad (1.1)$$

$$(x-y)^2_\varepsilon = (x-y)^2 + i\varepsilon(x_0 - y_0)$$

$$\langle A(x)A(y)^* \rangle = e^{2i\delta_A} \langle A(y)^* A(x) \rangle \ , \ x > y \qquad (1.2)$$

where the ε boundary prescription is just the spacetime version of the energy-momentum positivity and $\lessgtr$ denote $\mp$timelike separationes. One observes that for timelike distances the commutation relation can be at best plektonic[1] and certainly not bosonic/fermionic. But the two-point function does not reveal anything substantial concerning localization of fields and in particular 2- and 3-point functions cannot distingush anyonic (abelian) from general plektonic (nonabelian) timelike braid group structure. The consistency with higher point functions will be presented in section 3.

It is well known from chiral theories (where distances are lightlike) that the plektonic superselection structure is inexorably linked to the appearance of nontrivial central projectors which are the spectral projectors in the spectral resolution of the abelian generator Z of $center(\widetilde{SO(4,2)}) = \{Z^n; n \in \mathbb{Z}\}$. In chiral theories, which are based on the factorization $\widetilde{S(2,2)} \simeq \widetilde{SL(2,R)} \times \widetilde{SL(2,R)}$, a very good understanding about a one-to-one relation between algebraic nets of AQFT and conformal equivalence classes of generating "field coordinates" used in standard QFT has been achieved, and even the problem how to construct pointlike fields from nets of algebras has received successful attention [10]. There can be no reasonable doubts that these considerations can be generalized the higher dimensional conformal case, and in the present paper we will present some consistency arguments to this effect.

Even though conformal theories are somewhat outside of particle physics proper (since interactions, although consistent with all other properties of QFT, are inconsistent with a bona fide zero mass particle structure [8]), they still are expected to furnish useful illustrations of interacting local quantum physics.

In the next section we prepare the geometric prerequisites. This material is well known but we need to remind the reader and to set our notation. In the same section we also review the expansions with respect to the central projectors of Z. The core of this paper is section 4 where the consistency of timelike plektonic structures is discussed within the Wightman framework and were one can also find some remarks about some concepts which hopefully will turn out to be important in an algebraic setting.

2 Conformal Central Decomposition

According to Wigner the projective aspect of states in quantum theories requires the action of the universal covering of symmetry groups to act on state vectors in Hilbert space. Whereas for the Poincaré group the double covering explains the phenomenon of halfinteger spin and its relation to Fermi statistics, the larger conformal group has a much richer infinite covering symmetry i.e. the center of the conformal covering $\widetilde{SO(4,2)}$

[1] As it has become costumary in AQFT "plektonic" is used for the general (abelian and nonabelian) physically admissible braid group representation whereas anyonic refers to the abelian case (which is closer to the standard formulation of QFT).

is generated by one abelian element Z of infinite order[2]. Corresponding to spacelike 2π rotation as compared to the timelike sweep through $\bar{M}$, the related physical phenomena are somewhat different. Whereas the spatial spin-statistics connection and associated univalence superselection rule appeared quite early in the famous work of Wick Wightman and Wigner at the beginning of the 50s [3] and marked the beginning of the discussion about limitations of the quantum mechanical superposition principle due to superselection sectors, the conformal superselection rule which required the setting of local QFT and led to the temporal conformal decomposition theory, was discovered only twenty years later [5]. There is a formal similarity between both since whereas in the case of spin a 2π rotation in space results in a $e^{2\pi is}$ phase factor on vectors of spin s is related to the spacelike commutation structure for localized operators, the conformal case permits in addition a timelike rotational sweep which is associated via the eigenvalues $e^{2\pi i\delta}$ of Z to the spectrum of anomalous dimensions. Whereas the connections between the anomalous dimensions with the central phases in full timelike sweeps and the associated timelike decomposition theory into superselected sectors of local operators was obtained already in the 70s [5], the possible connection (there are as yet no controllable models) with a timelike braid group structure of charge-carrying fields associated with timelike commuting observables fulfilling Huygens principle is of a fairly recent vintage [9] and constitute the main subject of this report.

One reason for this delayed attention to such a fundamental problem is of course that the required methods have neither a natural place in the Lagrangian approach, nor are they in reach of the BPZ [6] representation theoretical methods (e.g. no immediate analog of locally acting diffeomorphisms beyond the finite parametric conformal group exists) whose algebraic structure is restricted to chiral theories. As will be seen in the sequel they are even somewhat outside the formalism of DHR since the issue of global causality in the presence of a covering of spacetime tends to be more "dynamical" than the basically kinematical DHR superselection analysis. The step to re-derive or incorporate the chiral results into the general setting of locally generated superselected charges by liberating them from the rather special diffeomorphism- and loop-group algebras has been achieved in a series of papers, for the most recent (with references to prior ones) see [11].

Although the similarities with D=1+1 in the covering and causality aspects are helpful, one must also appreciate the differences. The most important difference is already visible on the classical level when one studies the characteristic value problem. It is well known that for D>1+1 that data on that part of the light front which constitutes the upper causal horizon of a wedge region already fully determines the data in the wedge region, whereas for D=1+1 one needs the data on both lower and upper horizon to determine the data inside the wedge. The latter fact is of course intimately related to the D=1+1 decomposition into chiral right and left movers which for the quantum observables prevails even the presence of interactions. These differences have their counterpart in local quantum physics [9].

The content of the present paper aims at showing consistency between the Boson/Fermion statistics structure of the spacelike based DHR theory and the appearance of new central decomposition superselection sectors which require an autonomous role for the timelike region. Whereas the timelike region is the arena of interactions which in massive interactions has remained impenetrable to direct investigation, conformal symmetry opens this region to a full analysis for interacting charge-carrying fields. This phenomenon has no chiral counterpart.

Before we present the timelike braid group structure and the resulting classification theory for anomalous scale dimensions in the next section, we will review briefly the known facts about the conformal covering structure and the decomposition theory of local fields in the remainder of this section.

It is customary to compactify D-dimensional Minkowski space M within a $(D+2)$-dimensional linear formalism [15] with signature $(D,2)$ corresponding to the $SO(D,2)$ group with signature (+—-+) where + means timelike. The surface of the forward light cone is a D+1 dimensional submanifold $LC^{(d+1)} = \{\xi,\ \xi_\mu\xi^\mu = 0\}$ and the D-dimensional manifold of directions on this surface is the model for the compactified Minkowski space $\bar{M}$. The following parametrization which is also useful for the infinite sheeted covering $\widetilde{\bar{M}}$ of $\bar{M}$ is well known ($\tau =$ "conformal time")

$$\bar{M} = (sin\tau, \mathbf{e}, cos\tau),\ -\pi < \tau < \pi,\ \mathbf{e}^2 = 1 \qquad (2.1)$$
$$\bar{M} \simeq S^3 \times S^1$$

In terms of the D-dimensional standard coordinates it reads

$$x^0 = \frac{sin\tau}{cos\tau + e^d},\ \ \vec{x} = \frac{\vec{\mathbf{e}}}{cos\tau + e^d} \qquad (2.2)$$

where the boundary of $\bar{M}$ correspond to infinite remote points in the Cartesian coordinates (the usual covering model which one associates with the standard coordinates is $\bar{M}/Z_2$ together with the corresponding group $\widetilde{SO(D,2)}/Z_2$). Since the covering space has the topology

$$\widetilde{\bar{M}} = (\mathbf{e}, \tau) \simeq S^{d-1} \times \mathbb{R}, \qquad (2.3)$$

the causal dependence region in the global sense of the covering space is the noncompact complement of the compact spacelike region. In terms of differences between events $(\mathbf{e}, \tau)$ and $(\mathbf{e}', \tau')$ in $\widetilde{\bar{M}}$ we have for globally causal relations

$$|\tau - \tau'| > |Arcos(\mathbf{e}\cdot\mathbf{e}')|\ \pm timelike \qquad (2.4)$$
$$|\tau - \tau'| < |Arcos(\mathbf{e}\cdot\mathbf{e}')|\ \ spacelike \qquad (2.5)$$

[2] Actually the physical conformal group is $SO(d,2)/Z_2$, but for our purpose its double covering is more suitable.

and in a graphical representation[3] in terms of the surface of a D+1 dimensional cylinder one has a tiling in terms of infinitely many repeated diamond-shaped Minkowski spacetimes with a d-1 dimensional "infinity" $\bar{M}\backslash M$ which is spanned by the backward light cone with apex at $m_{+\infty} = (0,0,0,0,1,\tau = \pi)$ intersected with the forward cone with apex $m_{-\infty}(\tau = -\pi)$ [9]. As on S^1, there is no genuine causality concept on $\bar{M}$; algebras commute whenever the light rays emanated from the localization region of one do not intersect the other. A glocal notion of causality is however restored on $\widetilde{M}$ [16][15].

There exists an economical way to organize the conformal transformations relative to the Poincaré subgroup which consists in defining the analogue of the chiral rotation with the help of the conformal inversion acting on translations

$$R_\mu = P_\mu + IP_\mu I \qquad (2.6)$$
$$I : x \to \frac{-x}{x^2}$$

The inversion itself is not part of the connected conformal group (except in free theories), but the product $IP_\mu I$ is the generator of the fractionally acting abelian subgroup corresponding to $x \to \frac{x-bx^2}{1-2bx+b^2x^2}$ whereas R_μ generates a kind of "translation" analogue which acts as a timelike rotation through the compact $\bar{M}$. In fact if one looks at

$$U_e(\tau) = e^{i\tau e\cdot R} \qquad (2.7)$$
$$eR = e^\mu R_\mu,\ e^2 = 1,\ e_0 > 0$$

one realizes that $U_e(\tau)$ in the rest frame is precisely the so called conformal-time transformation which plays the crucial role in the compactification and which for $\tau = 2\pi$ defines the generator of the center of $\widetilde{S(D,2)}$. The advantage of the above formalism is that it presents the full conformal group by starting from the Poincaré group extended by scale transformations and associating only one additional one parametric subgroup namely the conformal "time" rotations; the rest follows from Lorentz transformations. This makes the topological similarity of $S^3 \times S^1$ with the well known chiral case analytically very explicit. In particular the well-known statement that observable chiral fields on S^1 have meromorphic analytically continued correlation functions paases to higher dimensional conformal observables on $\bar{M} \simeq S^3 \times S^1$. In this analytic language the cuts of correlations of charge-carrying fields on the complex extension of $\bar{M}$ disappear in the transition to the complexification on $\widetilde{M}$.

The basic observation which led the present author et. al. [5] to the decomposition theory for covariant local Boson/Fermi charge-carrying fields F was that one obtains quasiperiodic fields on M which remain irreducible even under global conformal transformations including those involving the action of the center of the group which has one abelian generator Z

$$center(\widetilde{S(D,2)}) = \{Z^n, n \in \mathbb{Z}\}$$

$$F(x) = \sum_{\alpha,\beta} F_{\alpha,\beta}(x),\ F_{\alpha,\beta}(x) \equiv P_\alpha F(x) P_\beta \qquad (2.8)$$
$$Z = \sum_\alpha e^{2\pi i\theta_\alpha} P_\alpha$$

in terms of central projectors. In a way the existence of this decomposition facilitates the use of the standard parametrization of Minkowski space augmented by the quasiperiodic central transformation

$$ZF_{\alpha,\beta}(x)Z^* = e^{2\pi i(\theta_\alpha - \theta_\beta)} F_{\alpha,\beta}(x) \qquad (2.9)$$

and hence one may to a large part avoid the use of the complicated covering parametrization and its $\widetilde{SO(D,2)}$ transformations which the unprojected fields F would require. For the latter fields on $\widetilde{M}$ the notation would be insufficient; one also has to give an equivalence class of path (the number $n \gtrless 0$ of the heaven/hell one is in) with respect to our copy of M embedded in $\widetilde{M}$. The projected fields on the other hand are analogous to sections in a trivialized vectorbundle.

Whereas spacelike causality on M and $\bar{M}$ is not conformally invariant (only lightlike separations are invariant), the global distinction in $\widetilde{M}$ between positive/negative timelike and spacelike is invariant and corresponds to the two sign in (2.4). The attachment of an index n to the projection in Minkowski spacetime prevents that a pair of points in $\widetilde{M}$ which was spacelike can become timelike under a transformation since e.g. it prevents the passing through lightlike infinity via special conformal transformations $x \to \frac{x-bx^2}{1-2bx+b^2x^2}$. In this way we solved the causality paradox [17] since it only came about by forgetting the path dependence which linked the Minkowski "heavens and hells" to our Minkowski space [5]. Instead of the $(\mathbf{e},\tau)$ parametrization (2.4), we used a function of a pair of conformal transformations $\sigma(C_1,C_2)$ which can be obtained from the quadratic expression $\sigma(b,x) = 1-2bx+b^2x^2$ [5][3].

It was shown [5] via the conformal properties of 3-point functions that $\mathrm{spec}\{Z^n, n \in \mathbb{Z}\} = e^{2\pi i\delta}, \delta\ an.dim.\}$. Strictly speaking it is not the dimension but rather the so called "twist" $t = \delta + s$ where s is the spin [5], but here we restrict ourselves to bosonic theories.

All above formula in fact remain true in the case D=1+1 if one takes care of the chiral tensor product structure which leads to a bigger tensor product center $\widetilde{SO(2,2)} = \{Z_+^{n_+} \times Z_-^{n_-}\}$. In that case the D=1+1 fields can be projected by factorizing double-indexed projectors $P_{\alpha_+\alpha_-} = P_{\alpha_+}^{(+)} \times P_{\alpha_-}^{(-)}$ onto charge sectors which refine the central projectors i.e. a central projector is a sum over charge projectors. Restricting to one chiral factor, one finds a lightlike plektonic exchange

[3] M looks then like a Penrose world, except that Penrose does not make the $\bar{M}$ identifications because his matter content is not invariant in the sense of Huygens priniple.

algebra for double indexed charge-carrying fields or operators (removing the $\pm$ notation)

$$F_{\alpha,\beta}(x)G_{\beta,\gamma}(y) = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'} G_{\alpha,\beta'}(y)F_{\beta',\gamma}(x),\ x > y \tag{2.10}$$

$$F_{a,\beta}G_{\beta,\gamma} = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'} G_{\alpha,\beta'}F_{\beta',\gamma},\ locF > locG \tag{2.11}$$

i.e. a commutation relation with R-matrices which form a representation of the infinite braid group. The more general algebraic form (2.11) of the exchange algebra in terms of operator algebras instead of fields was derived in [7].

Since in higher dimensions only the timelike region has an ordering structure which is maintained by positive respectively negative central transformations $Z^{\pm 1}$, the exchange relations are geometrically consistent for the timelike region in any dimension, with $\lessgtr$ meaning positive/negative timelike. In the next section we will test the consistency of this plektonic structure with the spacelike bosonic/fermionic commutation relation.

3 Timelike Decomposition Structure and the Braid Group

For chiral theories the structural investigations by the methods of algebraic quantum field theory were proceeded by a good understanding of exchange algebras in the more standard setting [12] of Wightman fields and their correlation functions. It is reasonable to proceed in the same way for higher dimensional conformal QFT.

The most powerful tool of Wightman's formulation is provided by the analytic properties of correlation functions. It is well known that the complexified Lorentz group may be used to extend the tube analyticity associated with the physical positive energy-momentum spectrum. The famous BHW theorem [1] insures that this extension remains univalued in the new complex domain and the Jost theorem characterizes its real points. Finally spacelike locality links the various permutations of the position field operators within the correlation function to one permutation (anti)symmetric analytic master function which is still univalued. The various correlation functions on the physical boundary with different operator ordering can be obtained by different temporal $i\varepsilon$ prescriptions.

Complexifying the scale transformations, the conformal correlations can be extended into a still bigger analyticity region which even incorporates "timelike Jost points" but trying to find a master function which links the various orders together fails in the presence of fields with anomalous dimensions and remains restricted to fields which live on the compactification $\bar{M}$. The latter are the analogs of chiral observables, except that apart from (composite) free fields one does not have algebraic examples since Virasoro- and Kac-Moody algebras do not exist in higher dimensions.

The analytic timelike structure of 3-point functions suggest that the permutation group should be replaced by the more general braid group. The global timelike ordering structure on the covering $\widetilde{M}$ is the prerequisite; without this ordering one can only have the more special permutation group commutations since the exchange and its inverse can then be continuously deformed into each other.

A plektonic (general braid-type) charge structure which is only visible in the timelike region would immediately explain the appearance of a nontrivial timelike center and the spectrum of anomalous dimension. It would kinematize conformal interactions and reveal conformal QFT as basically free theories if it would not be for that part of interaction which sustains the timelike plektonic structure. Of course the situation trivializes if the theory has no anomalous dimensions and nontrivial components. Analogous to [3] (remarks at end of section V.4) we conjecture that this characterizes interaction free conformal theories which are generated by free fields[4]. What makes this issue somewhat complicated is the fact that contrary to chiral theories we do not have a single nontrivial example because this issue is neither approachable from the representation theory of known infinite dimensional Lie-algebras nor from the formal euclidean functional integral method. The remaining strategy is to show structural consistency of the spacelike local- with the conjectured timelike plektonic- structure and to find a new construction method (non energy-momentum tensor- or current- algebra based, non-Lagrangian). Here we are mainly concerned with consistency arguments and in the following we will comment how local/plektonic on-vacuum relations between two fields can be commuted through to a generic position.

Assume for simplicity as before that we are in a "minimalistic" situation where the field theory has no internal symmetry group[5], but that the fields can be given "timelike" charge indices α, β, γ.. and their conjugates $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$.... resulting from projectors on charge spaces so that the decomposition is as in the chiral case where the charge projectors with the same phase factors $e^{2\pi i\delta}$ constitute a refinement of the central projectors. Clearly α and its conjugate $\bar{\alpha}$ contribute to the same central projector. In fact we may take over a substantial part of the formalism and concepts of [12] if one replaces the chiral translation+dilation augmented with the circular rotation generator L_0 by the spacetime symmetry group which leaves the timelike infinite point

[4] Note that this conjecture would be wrong in D=1+1 since from selfdual lattice construction on current algebras one obtains models without nontrivial sectors which are different from free fields.

[5] The general exchange algebra relations with group algebra valued R-matrices have been elaborated by K-H Rehren (private communication).

fixed (Poincaré+dilations) extended by the generator of conformal time R_0 instead of the chiral L_0. One would of course also have to change the title of the old paper from "Einstein causality and Artin braids" to "Huygens causality and Artin braids" referring to the timelike ordering for which the conformal observables fulfill the Huygens principle of vanishing commutators. The "on-vacuum" structure of commutation relations follows from the structure of the conformal 3-point functions (here the F, G, H fields are not observable fields but are as the F,G of the previous section)

$$\langle H^*(x_3)G(x_2)F(x_1)\rangle = c_{FGH}\frac{1}{[-(x_{12})_\varepsilon^2]^{\delta_3}}\frac{1}{[-(x_{13})_\varepsilon^2]^{\delta_2}}\frac{1}{[-(x_{23})_\varepsilon^2]^{\delta_1}} \tag{3.1}$$
$$\delta_1 = \frac{1}{2}(\delta_F+\delta_H-\delta_G),\ \delta_2 = \frac{1}{2}(\delta_G+\delta_H-\delta_F),\ \delta_3 = \frac{1}{2}(\delta_F+\delta_G-\delta_H)$$

where the ε-prescription was explained in the introduction. For spacelike and timelike distances one concludes

$$G(x_2)F(x_1)\Omega = \begin{cases} F(x_1)G(x_2)\Omega,\ (x_2-x_1)^2 < 0 \\ e^{\pi i(\delta_G+\delta_F)}Z^*F(x_1)G(x_2)\Omega,\ (x_2-x_1)^2 > 0,\ (x_2-x_1)_0 > 0 \end{cases} \tag{3.2}$$

since this relation is valid on all quasiprimary composites H. They consist of the equal point limit of the associated primary H_{min} (lowest scale dimension operator in the same charge class) multiplied with a polynomial in the observable field. These composites applied to the vacuum form a dense set in the respective charge sector[6] and hence the on-vacuum formula is a consequence of the structure of 3-point functions. The spacelike local commutativity off-vacuum is consistent with that on-vacuum since for y timelike with respect to the spacelike pair x_1, x_2 we have (c_F=superselected charge of F)

$$\begin{aligned} P_\alpha F(x_1)G(x_2)H(y)\Omega &= \sum_\beta P_\alpha F(x_1)P_\beta G(x_2)H(y)\Omega \\ &= \sum_\beta P_\alpha F(x_1)P_\beta e^{i\pi(\delta_G+\delta_H-\delta_\beta)}H(y)G(x_2)\Omega \\ &= \sum_{\beta\beta'} R^{(\alpha\gamma)}_{\beta\beta'}(c_F,c_G)e^{i\pi(\delta_G+\delta_H-\delta_\beta)}P_\alpha H(y)P_{\beta'}F(x_1)P_\gamma G(x_2)\Omega \end{aligned} \tag{3.3}$$

and therefore the off-vacuum vanishing of the F-G commutator is consistent with the on-vacuum vanishing of this commutator if there holds a certain relation between $R(c_F,c_G)$ and $R(c_G;c_F)$ which is identically fulfilled for $c_F = c_G$. Similarly one does not run into inconsistencies if one tries to obtain a timelike off-vacuum F-G situation from the on-vacuum placement by commuting through a H which is spacelike to the timelike F-G pair

[6] With a bit more work and lengthier formulas one can avoid the colliding point limit and use correlation functions containing 3 charged fields and an arbitrary number of neutral observable fields. The dependence on the observable coordinates is described by a rational function on $\bar{M}$.

$$P_\alpha F(x_1)G(x_2)H(y)\Omega = P_\alpha H(y)F(x_1)G(x_2)\Omega = P_\alpha H(y)e^{i\pi(\delta_G+\delta_H-\delta_\beta)}G(x_2)F(x_1)\Omega$$
$$= \sum_\beta R^{(\alpha\gamma)}_{\beta\beta'}(c_F,c_G)P_\alpha G(x_2)P_{\beta'}F(x_1)P_\gamma H(y)\Omega = \sum_{\beta\beta'} R^{(\alpha\gamma)}_{\beta\beta'}(c_F,c_G)P_\alpha G(x_2)P_{\beta'}H(y)F(x_1)\Omega \qquad (3.4)$$

where in the second line we commuted F through G before trying to bring both to the vacuum. Since their is no rule to commute the $P_\alpha G(x_2)P_{\beta'}$ with $P_{\beta'}H$ for $(x_2-y)^2<0$, there is no way to get to the same HGF order as in the first line and hence no consistency relation to be checked. The absence of rules for spacelike commutations for projected fields protects the formalism to run into inconsistencies.

Let us also briefly look at the compatibility of the timelike plektonic structure with the conformal structure of the 4-point function of 4 identical hermitean fields

$$W(x_4,x_3,x_2,x_1) := \sum_\gamma \langle F(x_4)F(x_3)P_\gamma F(x_2)F(x_1)\rangle \qquad (3.5)$$
$$= \left[\frac{x^2_{42}x^2_{31}}{(x_{43})^2_\varepsilon (x_{32})^2_\varepsilon (x_{21})^2_\varepsilon (x_{14})^2_\varepsilon}\right]^{\delta_A}\sum_\gamma w_\gamma(u,v),$$
$$u=\frac{x^2_{43}x^2_{21}}{(x_{42})^2_\varepsilon (x_{31})^2_\varepsilon},\ v=\frac{x^2_{32}x^2_{41}}{(x_{42})^2_\varepsilon (x_{31})^2_\varepsilon}$$

Whereas the spacelike commutations leads to functional relations for $w=\sum_\gamma w_\gamma(u,v)$ with the exchange of two fields causing a rational transformation of the u,v (apart from multiplying the w by rational u,v factors), the timelike commutation of the off-vacuum fields produces rational transformation together with monodromy R-matrix mixing of the γ-components [9] (in addition to multiplying the w with noninteger powers of u and v which depend on the scale dimension δ_F). Despite some similarities with the chiral case, the dependence of w_γ on two cross ratios probably requires the use of more elaborate techniques than the hypergeometric formalism which is sufficient for the chiral one variable cross ratio dependence Here we will not pursue this matter.

In order to incorporate these observations on correlation functions into the algebaic approach one should start from a theorem [18] which shows that a locally conformal field net on M allows a natural extension $\widetilde{\mathcal{F}}$ to a Haag dual net on $\bar{M}$. The difficult step is to prove that there exists a nontrivial (observable) subalgebra $\mathcal{A}$ on $\bar{M}$. The geometric complement of a double cone $\mathcal{O}$ which is relevant for Haag duality of $\mathcal{A}$ consists of all points on $\bar{M}$ which are not lightlike to $\mathcal{O}$ [3].

An attempt to show the existence of $\mathcal{A}$ by modular method shows the difficulty. Consider the inclusion

$$(\mathcal{F}(\mathcal{V}^t_+)\subseteq\mathcal{F}(\mathcal{V}_+),\Omega) \qquad (3.6)$$

where $\mathcal{F}(\mathcal{V}^t_+)$ is the forward lightcone algebra shifted upward in time by t. One easily checks that this inclusion is modular, i.e. that the modular group of $\mathcal{F}(\mathcal{V}_+)$ (the dilation group) in one direction compresses $\mathcal{F}(\mathcal{V}^t_+)$ further into $\mathcal{F}(\mathcal{V}_+)$. As a result the relative commutant

$$\mathcal{F}(\mathcal{V}^t_+)'\cap\mathcal{F}(\mathcal{V}_+) \qquad (3.7)$$

together with the time translation and dilation turns out to define a bosonic net on the timelike line. The application to the vacuum generates the vacuum sector H_0 (by definition) and the covariantized net (using Poincaré transformations) of relative commutants if nontrivial, could serve as definition of the conformal observable net $\mathcal{A}$ on $\bar{M}$. There is also the net $P_0\tilde{\mathcal{F}}P_0$ which contains $\mathcal{A}$ and has the same modular group. The consistency of the above timelike braid group structure would suggest that these two nets are equal. A triviality of $\mathcal{A}$ actually appears quite pathological, but ultimately this problem of existence of nontrivial anomalous dimension has to be solved by constructive examples.

By cutting $\bar{M}$ open at $\bar{M}\backslash M$ one looses Haag duality, but one regains it together with a new net after redefining double cone algebras as intersections of the forward with the (time-shifted) backward light cone, which amounts to a timelike dualization. The mechanism, which involves diluting the net at infinity and offsetting this by making finitely localized double cones bigger, has been nicely explained for free fields in [21].

This sort of situation where points or subsets are cut out from a localization region of a net is much better understood in chiral theories. There the above situation corresponds to punching a hole into the circle (say at ∞) in which case the new net lost the conformal rotation and only retains translation and dilation. One then recovers Haag duality as well as full conformal invariance (with an L_0 with a different low-lying spectrum) by suitably extending those algebras in the net which do not contain ∞. The first observation was made in [19] and extended by a more algebraic analysis, including a partial classification of all such extensions in [18]. One expects that a similar construction in the higher dimensional case will confirm the compatibility between the spacelike DHR structure and the present ideas on the level of AQFT.

4 Outlook

If one wants to use constructions of chiral models as a guide for higher dimensional conformal models, one must avoid ideas which are obviously limited to low dimensions, as representation theory of

the diffeomorphism- (Virasoro algebra) or loop-groups (current algebras). Rather one should use the spacelike (permutation group statistics) and timelike (braid group) structure in the process of classifying and constructing models.

For observable fields the correlation functions are meromorphic on M and rational on $\bar{M}$ as functions of the Poincaré invariants. But past experience shows that to base a construction on the (linear) properties of Wightman functions does not really work because the nonlinear positivity requirements from quantum theory are not controllable in such an approach.

All successful low dimensional model constructions start with concrete operators in Hilbert space and keep the positivity under control throughout the whole construction procedure. But even having opted for operator methods, one still faces the question of whether one should first aim for the observable algebras and follow the dichotomy of observables-charged fields or pass directly at the latter. The division into observables/field algebras is useful for structural investigations and for situations where mathematicians already have studied algebras (loop groups, Virasoro-diffeomorpisms,..) which are candidates for observable algebras.

Which objects are more fundamental, observable- or (superselected) field-algebras? This kind of question is a bit reminiscent of what was first the hen or the egg. From a historical point the fields were there first[7] before Haag realized already at the end of the 50^{ies} almost single-handed that it would be a good idea to view fields in their role of charge-carrying operators as representation theoretical objects carrying generalized superselected charge. This thought was extremely fruitful and led 10 years later to the DHR approach and another 20 years later to the DR-theorem. It provided structural insight into the inner workings of local quantum physics which Lagrangian QFT was unable to unravel and although it was not designed to lead to instant predictions, it became a valuable long term investment into QFT. Logically the central position in the structural analysis belongs to the observable algebras.

I would like to advocate the thesis that for higher dimensional conformal theories the best constructive strategy is to take the most advanced mathematical and conceptual tools and return to the old program of constructing charged fields directly. It appears to me more natural to explain the rather complicated quantization phenomena of observables (e.g. the Friedan-Qiu-Shenker c-quantization) in terms of the conceptually simpler quantization which is inherent in the Makov traces on the braid group. I am convinced that such an approach exists and that the Tomita-Takesaki modular theory will play an important role. Relations to the isomorphism between anti de Sitter and conformal spacetime as well as to perturbative attempts (conformal supersymmetric Yang-Mills models) can be found in [9].

The power of the modular localization method is evidenced in recent approaches involving "polarization-free-generators" to low dimensional particle physics problems [22][23].

Another curious aspect of the present ideas is the very radical way theories with braid group structure violate the Coleman-Mandula (C-M) theorem [14]. Braid group structures cannot be encoded into a multiplicities with a group like action which then factorizes with the spacetime actions of the conformal symmetry (as in chiral current algebra representations). The violation in low-dimensional models (chiral models, massive factorizing D=1+1 models) which do not fit the prerequisites of the C-M theorem was of course well known to those authors, but it seems that everybody expected that this could at best occur in D=1+2 massive plektonic models but is excluded in $D > 1+2$ theories. The present work suggests that higher dimensional conformal theories with anomalous dimensions not only do *not* satisfy the particle prerequisites [8], of the C-M theorem but also violate the spacetime/internal factorization of symmetries predicted by the C-M theorem (even after it was adapted to supersymmetries). The C-M prohibition of nontrivial amalgamations of internal and spacetime symmetries applies very much to the DR internal symmetries, but the charge fusion symmetries behind the central projectors in conformal theories in any spacetime dimension is definitely outside the C-M realm. It is not completely excluded that this has consequences for non group like regularities even in massive theories, since there are no exact nonabelian flavor symmetries in nature. But presently one has no idea of how and by what means conformal theories could be related to theories describing scattering of massive particles.

All this shows that the DHR superselection sector structure has come a long way, and the statement that braid group structures are excluded in higher dimensions without any further qualification seem to be on its way out. It appeares that there is a new dynamical role for an extension of the idea of superselected charges of which conformal theories are a foreboding.

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