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On the Finite Temperature $\lambda\varphi^4$ Model. Is There a First Order Phase Transition in $(\lambda\varphi^4)_3$?

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Abstract

We investigate the behavior at finite temperature of the massive $\lambda\varphi^4$ model in a D-dimensional spacetime, performing a renormalization up to the order of one loop. In this approximation we show that the thermal mass increase with the temperature, while the thermal coupling constant decrese with the temperature. We establish that in the $(\lambda\varphi^4)_3$ model there is a temperature $\beta_\star^{-1}$ above which the coupling constant becomes negative. We argue that the system could develop a first order phase transition, where the origin corresponds to a metastable vacuum.

Key-words: Effective potential; Finite temperature QFT.

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1 Introduction

In the last years, there has been much interest in the nature of the electroweak phase transition. The high temperature effective potential in the standard and in the $(\lambda\varphi^4)_4$ models have been calculated by many authors, where the contribution from multiloops diagrams has been taking into account. Several authors [1] have pointed out the importance to known whether in $(\lambda\varphi^4)_4$ model the phase transition is of first or second order.

The possibility of a first order phase transition in $(\lambda\varphi^4)_4$ has been discussed by Arnold and Spinoza [2]. Using the ring-improved one-loop effective potential these authors showed that even for temperature independent coupling constant, the $(\lambda\varphi^4)_4$ model can develop at the first sight a first order phase transition. Nevertheless, these authors sustain that the phase transition is of the second order and that the one-loop ring improved potential cannot be trusted to distinguish between a first or second order phase transition. They claim that the contribution of higher loop corrections shall dominates over the one-loop ring improved contributions in the phase transition region. Intending to shed some light on this matter Tetradis and Wetterich investigated the order of the phase transition in the N-component $(\lambda\varphi^4)_4$ model using the ideas of the renormalization group (running coupling constant) and obtained a second order phase transition in the model [3]. Different answer has been obtained by other authors. Evaluating the ring diagrams Carrington and Takahashi independently obtained in a pure scalar model at $D = 4$ a first order phase transition [4]. The same answer was obtained by Carmelia and Pi [5] using the technique developed by Cornwall, Jackiw and Tomboulis [6]. Nevertheless the authors of ref. [5] claims that this result (a first order phase transition in the model) must be wrong.

Our interest in these issues was stimulated by some results of Ford and Svaiter [7] concerning the thermal dependence of the mass and coupling constant in $(\lambda\varphi^4)_4$ model defined in a non-simple connected spacetime. In the aforementioned paper these authors studied a neutral scalar field using the one-loop effective potential. The cases of trivial and non-trivial topology of the spacelike sections were discussed. The dependence of the renormalized mass and coupling constant on temperature and topology were derived using the analytic regularization [8] and a modified minimal subtraction renormalization procedure [9]. In addition they have also discussed the possibility of vanishing the renormalized coupling constant in this model, as well as the limits of validity of the one-loop approximation. The $(\lambda\varphi^4)_4$ model in non-trivial topologies of the spacelike sections and finite temperature has recently also been studied by Elizalde and Kirsten [10] and Villareal [11]. These author studied the behavior of the renormalized mass using the one-loop approximation [10] and the two-loops approximation [11], but the behavior of the renormalized coupling constant with the temperature was not analysed.

The two goals of this paper are the following. The first one is to extend the discussion of the massive self-interacting $\lambda\varphi^4$ model to an arbitrary D-dimensional spacetime, assuming trivial topology of the spacelike sections. The second one is to discuss the possibility of a first order phase transition in the massive $(\lambda\varphi^4)_3$ model.

In many papers studying second order phase transition in the $\lambda\varphi^4$ model the temperature dependence of the coupling constant is neglected. This approach is reasonable for the description of a second order phase transition because in this case the variation of the mass with the temperature is the most important fact. It is enough to consider the renormalized coupling constant as constant and the thermal mass drives the second order phase transition [12]. As we will see, in the one-loop effective potential of the $\lambda\varphi^4$ model all even powers of the vacuum expectation value of the field ϕ_0 does appears. Therefore this model may display a first order phase transition if the ϕ_0^4 coefficient changes its sign while the ϕ_0^2 and ϕ_0^6 coefficients are positive.

We would like to stress that the study of the dependence of the coupling constant with the temperature it is not new in the literature. Many authors have studied such dependence in the $\lambda\varphi^4$ model and also in a abelian model like QED [13]. Instead of using perturbative arguments, the use of the renormalization group equations allowed the investigation on the mass and coupling constant thermal dependence. Such program was implemented by Fujimoto, Ideura, Nakano and Yoneyama [14]. These authors obtained that the behavior of the mass and coupling constant with the temperature are opposite, i.e. when the temperature increases, the renormalized mass increases while the renormalized coupling constant decreases.

The result of our analysis is that for $D < 4$, there is a temperature $\beta_\star^{-1}$ where the effective coupling constant vanishes. For temperatures $\beta^{-1} > \beta_\star^{-1}$, the renormalized coupling constant becomes negative and the system may suffer a first order phase transition. We should note that at $\beta^{-1} = \beta_\star^{-1}$ the system is still in an interacting phase. At this temperature only the effective coupling constant ($\lambda(\beta) = \lambda - \lambda^2 f(\beta)$) vanishes. All the higher 2n-points correlation functions do not vanish, therefore the model is not gaussian at the temperature $\beta_\star^{-1}$. This is an important point that was stressed by Weldon [15]. The non-triviality of the model can be verified analysing the beta coefficient of the Callan-Zymanzik equation. Since for $D = 3$ there is a ultraviolet atractive fixed point, the model is not Gaussian. For $D > 4$ Aizeman and Frohlich proved the triviality of the model [16].

One may argue that $\lambda\varphi^4$ cannot exhibit a first order phase transition because the Ising model displays only a second order phase transition and $\lambda\varphi^4$ belongs to the same universality class of the Ising model. This argument breaks down if we assume a positive tree level mass square because in the one-loop approximation the thermal correction to the mass is positive and the Ising model at the phase transition belongs to the same universality class of the *massless* $\lambda\varphi^4$ model [17]. Therefore starting with a positive tree level mass squared the renormalized mass will always remain positive and the model will not be massless at any temperature (second order phase transition is easily obtained working with a negative tree level squared mass).

One important point is whether one-loop approximation results can be trusted. In fact, the behavior of the thermal correction to the coupling constant changes in the two-loops approximation. It was been shown by Funakubo and Sakamoto [13] that only for *low* temperatures the behavior of the thermal coupling constant remains the same as the obtained in the one-loop approximation. For high temperatures ($\beta^{-1} >> m$) the behavior is opposite i.e., the thermal correction to the coupling constant is positive. Nevertheless this fact does not exclude the possibility of a first order phase transition at intermediate temperatures in $(\lambda\varphi^4)_3$ also for the two-loops approximation. One interesting result in favour of the one-loop approximation was given by Stevenson [18] for the massless model. The extension of the Stevenson result for the massive model deserves investigations.

The paper is organized as follows. In section II, the massive self-interacting $\lambda\varphi^4$ model is analised. Conclusions are given in section III . In this paper we use $\frac{h}{2\pi} = c = 1$.

2 The one-loop effective potential in the $\lambda\varphi^4$ model at zero and finite temperature.

Let us assume the following Lagrange density associated with a massive neutral scalar field:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 + \frac{1}{2}\delta Z(\partial_\mu\varphi)^2 - \frac{1}{2}\delta m^2\varphi^2 - \frac{1}{4!}\delta\lambda\varphi^4, \tag{1}$$

where δZ, δm^2, and $\delta\lambda$ are the wave function, mass and coupling constant counterterms of the model. Defining the vacuum expectation value of the field $\varphi_0 = \frac{<0|\varphi|0>}{<0|0>}$, after the Wick rotation, in the one-loop aproximation, the effective potential is given by [19]:

$$V(\varphi_0) = V_I(\varphi_0) + V_{II}(\varphi_0) \tag{2}$$

where,

$$V_I(\varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \frac{\lambda}{4!}\varphi_0^4 - \frac{1}{2}\delta m^2\varphi_0^2 - \frac{1}{4!}\delta\lambda\varphi_0^4, \tag{3}$$

and

$$V_{II}(\varphi_0) = \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s}\left(\frac{1}{2}\lambda\varphi_0^2\right)^s \int \frac{d^Dq}{(2\pi)^D}\frac{1}{(\omega^2 + \vec{q}^{\;2} + m^2)^s}. \tag{4}$$

Before continuing, we would like to discuss one important point. Performing analytic or dimensional regularization, we must introduce a mass parameter μ, in terms of which dimensional analysis gives to the field a dimension $[\varphi] = \mu^{1/2(D-2)}$ and to the coupling constant a dimension $[\lambda] = \mu^{4-D}$. Mass has dimension of inverse length, i.e. $[\mu] = [m] = L^{-1}$, and the effective potential (the energy density per unit volume) has dimension of L^{-D}.

It is not difficult to extend the results given by eqs.(3) and (4) to finite temperature states. After a Wick rotation, the functional integral runs over the fields that satisfy periodic boundary conditions in Euclidean time. Than, we perform as usual the following replacement in the Euclidean region:

$$\int \frac{d\omega}{2\pi} \to \frac{1}{\beta}\sum_n \tag{5}$$

and

$$\omega \to \frac{2\pi n}{\beta} \tag{6}$$

where $\omega_n = \frac{2\pi n}{\beta}$ are the Matsubara frequencies. Defining the dimensionless quantities:

$$c^2 = \frac{m^2}{4\pi^2\mu^2}, \tag{7}$$

$$(\beta\mu)^2 = a^{-1}, \tag{8}$$

and

$$g = \frac{\lambda}{8\pi^2} \tag{9}$$

$$\frac{\varphi_0}{\mu} = \phi \tag{10}$$

$$k^i = \frac{q^i}{2\pi\mu} \tag{11}$$

the Born terms plus one-loop terms contributing to the effective potential gives,

$$V(\beta, \varphi_0) = V_I(\varphi_0) + V_{II}(\beta, \varphi_0)$$

where

$$V_{II}(\beta, \phi) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{2s} g^s \phi^{2s} \sum_{n=-\infty}^{\infty} \int d^d k \frac{1}{(an^2 + c^2 + \vec{k}^{\,2})^s}. \tag{12}$$

To evaluate the Matsubara sum in eq.(12) different methods are used in the literature. A very elegant method was emphasized by Kapusta [20], where eq.(12) is expressed as a contour integral. In the method the Matsubara frequency sum separates in a piece which is temperature independent and a temperature dependent part. Since we are interested to make a paralel with the tricritical phenomena in the paper we will apply an alternative method using a mix between dimensional and zeta function analytic regularizations. Let us first use dimensional regularization[21]. Using the well known result,

$$\int \frac{d^d k}{(k^2 + a^2)^s} = \frac{\pi^{\frac{d}{2}}}{\Gamma(s)} \Gamma(s - \frac{d}{2}) \frac{1}{a^{2s-d}}, \tag{13}$$

and defining,

$$f(D, s) = f(d+1, s) = \frac{(-1)^{s+1}}{2s} \pi^{\frac{d}{2}} \Gamma(s - \frac{d}{2}) \frac{1}{\Gamma(s)} \tag{14}$$

we obtain,

$$V_{II}(\beta, \phi) = \mu^D \sqrt{a} \sum_{s=1}^{\infty} f(D, s) g^s \phi^{2s} A_1^{c^2}(s - \frac{d}{2}, a), \tag{15}$$

where the function $A_N^{c^2}(s, a_1, a_2..a_N)$ is the inhomogeneous Epstein zeta function defined by

$$A_N^{c^2}(s, a_1, a_2, .., a_N) = \sum_{n_1, n_2..n_N=-\infty}^{\infty} (a_1 n_1^2 + a_2 n_2^2 + ... + a_N n_N^2 + c^2)^{-s}. \tag{16}$$

The terms $s \le \frac{D}{2}$ are divergent and we will regularize the one-loop effective potential using the principle of the analytic extension. Let us assume that each term in the series of the one-loop effective potential $V(\beta, \phi)$ is the analytic extension of these terms, defining in the beginning in an open connected set.

After some calculations using a Melin transform [22] it is possible to find the analytic continuation of eq.(15) to the whole complex s plane. The result is a meromorphic function with simple poles at the points $s = 1, 2.., \frac{D}{2}$. Substituting the analytic extension in eq.(15) yields

$$V_{II}(\beta,\phi) = \mu^D \sum_{s=1}^{\infty} g^s \phi^{2s} h(D,s)\Big(\frac{1}{2^{\frac{D}{2}-s+2}}\Gamma(s-\frac{D}{2})(\frac{m}{\mu})^{D-2s} + \sum_{n=1}^{\infty}\Big(\frac{m}{\mu^2\beta n}\Big)^{\frac{D}{2}-s} K_{\frac{D}{2}-s}(mn\beta)\Big) \tag{17}$$

where:

$$h(D,s) = \frac{1}{2^{\frac{D}{2}-s-1}}\frac{1}{\pi^{\frac{D}{2}-2s}}\frac{(-1)^{s+1}}{s}\frac{1}{\Gamma(s)}, \tag{18}$$

and $K_\mu(z)$ is the Kelvin function [23].

If we suppose that $D = 4$, the model is perturbatively renormalizable and an appropriate choice of δm^2 and $\delta\lambda$ will render the analytic extension of the terms of the series in s in the effective potential analytic functions in the neighbourhood of the poles $s = 1$ and $s = 2$ respectively.

To find the exact form of the counterterms let us use the renormalization conditions

$$\frac{\partial^2}{\partial\phi^2}V(\beta,\phi)|_{\phi=0} = m^2\mu^2 \tag{19}$$

and

$$\frac{\partial^4}{\partial\phi^4}V(\beta,\phi)|_{\phi=0} = \lambda\mu^4. \tag{20}$$

Since the vacuum expectation value of the field has been chosen to be constant, there is no need for wave function renormalization. Substituting eqs.(3),(17) and (18) in eqs.(19) and (20) it is possible to find the exact form of the countertems in such a way that they cancel the polar parts of the analytic extension of the terms $s = 1$ and $s = 2$. Note that we are using a "modified" minimal subtraction renormalization scheme where the mass and coupling constant counterterms are poles at the physical values of s. The temperature dependent mass is proportional to the regular part of the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 1$. The same argument can be applied to the renormalized coupling constant. The thermal contribution to the renormalized coupling constant is proportional to the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of the pole $s = 2$. The choice of the renormalization point $\phi = 0$ implies that only the regular part in the neighborhood of the pole $s = 1$ will appear in the renormalized mass. From the above discussion we can write

$$m^2(\beta) = m^2 + \Delta m^2(\beta) \tag{21}$$

and

$$\lambda(\beta) = \lambda + \Delta\lambda(\beta), \tag{22}$$

where $m^2(\beta)$ and $\lambda(\beta)$ are respectively the temperature dependent renormalized mass squared and coupling constant. A straightforward calculation of the thermal contribution to the renormalized mass squared using eq.(17), (18) and (21) gives

$$\Delta m^2(\beta) - \Delta m^2(\infty) = \frac{1}{8\pi^2}\lambda\sum_{n=1}^{\infty}\frac{m}{\beta n}K_1(mn\beta). \tag{23}$$

Using the asymptotic representation of the Bessel function $K_n(z)$ for small arguments we obtain that at high temperatures the temperature dependent mass squared is proportional to $\lambda\beta^{-2}$ [24]. The result given by eq.(23) was also obtained by Braden [25] using Schwinger's proper time method. This author also discussed the two-loop effective potential and the problem of overlapping divergences where the possibility of temperature dependent counterterms appears. Nevertheless these divergences must cancel as it was stressed by Kislinger and Morley [26].

Based uppon the same arguments previously used, the thermal contribution to the renormalized coupling constant is given by:

$$\Delta\lambda(\beta) - \Delta\lambda(\infty) = -\frac{3}{8\pi^2}\lambda^2 \sum_{n=1}^{\infty} K_0(mn\beta). \tag{24}$$

The Bessel function $K_0(z)$ is positive and decreases for $z > 0$. Therefore let us present an interesting result: the renormalized coupling constant attains its maximum at zero temperature ($\beta^{-1} = \infty$) and decreases monotonically as the temperature increases. In other words, the thermal contribution to the renormalized coupling constant $\Delta\lambda(\beta) - \Delta\lambda(\infty)$ is negative, and increases in modulus with the temperature. The same result was obtained by Fujimoto, Ideura, Nakano and Yoneyama using the renormalization group equations at finite temperature [14]. Once we are discussing thermal effects, in the limit of zero temperature the thermal contribution to the mass and coupling constant must vanish.This can be easily seen from eqs.(23) and (24). Since the thermal contribution to the renormalized coupling constant is negative one could enquiry: is it possible for the renormalized coupling constant to vanish? Once $\Delta\lambda(\beta)$ is $O(\lambda^2)$ and we assume $D = 4$, it is not possible to implement such a mechanism for finite temperatures. For $D < 4$ the renormalized coupling constant is not necessarily a small quantity. In the strong coupling constant regime ($D = 3$) we expect to find a finite temperature $\beta_\star^{-1}$ such that the renormalized coupling constant vanishes.

We note that there is no discontinuity in the behavior between the cases $D = 4$ and $D < 4$ as we will see later. For $D < 4$ the model becomes superrenormalizable and only a finite number set of graphs need overall counterterms. In the one-loop aproximation for $D = 4$ there are only two divergent graphs and for $D < 4$ there is only one. This result can be easily obtained by investigating eq.(17). In this equation the divergent part of the effective potential is given by $\Gamma(s - \frac{D}{2})$ and for $D < 4$ only the $s = 1$ pole will appear. In other words, for $D < 4$ there is only finite coupling constant renormalization at the one-loop aproximation. The graph $s = 2$ gives a finite and negative contribution to the coupling constant. For $D \geq 4$ the renormalization of the coupling constant is obligatory (note the presence of the pole in $s = 2$).

Going back to the generic D-dimensional case, the renormalization conditions also are given by eqs.(19) and (20). Using the renormalization conditions we can find the regular part of the analytic extension which gives a finite contribution to the renormalized mass squared $\Delta m^2(D, m, \lambda, \beta)$ and coupling constant $\Delta\lambda(D, m, \lambda, \beta)$ in a D-dimensional flat spacetime. We will simplify the notation writing $\Delta m^2(\beta)$ and $\Delta\lambda(\beta)$. The thermal contribution to the mass and coupling constant are respectively:

$$\Delta m^2(\beta) - \Delta m^2(\infty) = \frac{\mu^{D-2}\lambda}{2(2\pi)^{D/2}} \sum_{n=1}^{\infty} \left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta) \tag{25}$$

and

$$\Delta\lambda(\beta) - \Delta\lambda(\infty) = -\frac{3}{2}\frac{\mu^{D-4}\lambda^2}{(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \tag{26}$$

These are the main results of the paper. Since $\Delta\lambda(\beta) - \Delta\lambda(\infty) < 0$ we may have a temperature $\beta_\star^{-1}$ where $\lambda(\beta)$ vanish for $D < 4$.

Before discussing a existence of a first order phase transition in the case $D = 3$, we would like to point out that the investigation of the $(\lambda\varphi^4)_4$ model with a negative bare coupling constant has recently been done by Langfeld et al, where an analytic continuation of the model with positive λ to negative values was presented [27]. Although several authors claim that the renormalized coupling constant of the $(\lambda\varphi^4)_4$ model must be positive, a definitive supporting argument is still lacking [28].

Going back to the discussion of a first order phase transition, let us define a dimensionless effective potential $v = \frac{V}{\mu^D}$, as:

$$\begin{aligned} v(\beta,\phi) &= \frac{1}{2}m^2\mu^{2-D}\phi^2 + \frac{\lambda}{4(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta)\phi^2 \\ &+ \frac{\lambda}{4!}\mu^{4-D}\phi^4 - \frac{1}{16}\frac{\lambda^2}{(2\pi)^{D/2}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta)\phi^4 \\ &+ \ high\ order\ terms\ in\ \phi. \end{aligned} \tag{27}$$

The previous results can be used to demonstrate a first order phase transition in the massive $(\lambda\varphi^4)_3$ model. To simplify our discussion let us assume that is possible to truncate the series of the effective potential in $s = 3$. These does not imply the assumption that high order powers of the field gives vanishing contributions. They are simply neglected as compared to the leading terms, since we are interested in the profile of the effective potential near the origin. The coefficient of φ^6 is positive (one requires this to ensure that the truncated effective potential is bounded from below). For the sake of simplicity, let us also assume that the coefficient of the φ^6 is constant and given by σ for the case $D = 3$. In this case the leading contributions to the effective potential is

$$\begin{aligned} v(\beta,\phi) &= \left(\frac{1}{2}m^2 + \frac{\lambda}{4(2\pi)^{\frac{3}{2}}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{\frac{1}{2}} K_{\frac{1}{2}}(mn\beta)\right)\phi^2 \\ &+ \left(\frac{\lambda}{4!} - \frac{\lambda^2}{16(2\pi)^{\frac{3}{2}}}\sum_{n=1}^{\infty}\left(\frac{m}{\mu^2\beta n}\right)^{-\frac{1}{2}} K_{\frac{1}{2}}(mn\beta)\right)\phi^4 + \sigma\phi^6. \end{aligned} \tag{28}$$

¿From the above discussion, for $D = 3$ we obtain the following profile for the effective potential in the neighborhood of the origin. Bellow the temperature $\beta_\star^{-1}$, the dimensionless effective potential has only one global minimum. Heating the system above the temperature $\beta_\star^{-1}$, the renormalized coupling constant would become negative and the system can develop a first order phase transition since the vacuum expectation value of the field changes discontinuously by temperature effects. Note the similarity with the tricritical phenomena where in the tree level ($V(\varphi) = m^2\varphi^2 + \lambda\varphi^4 + \sigma\varphi^6$) the model develop a first order phase transition if we allow the coefficient of the quartic term to be negative [29].

3 Conclusions

In this paper we studied the renormalization program assuming that a scalar field is in equilibrium with a thermal reservoir at temperature β^{-1}. We have attempted to analize the consequences of the fact that not only the renormalized mass, but also the renormalized coupling constant acquire thermal corrections.

It was proved that in the $\lambda\varphi^4$ model, in the one-loop aproximation, the thermal correction to the renormalized mass is positive and the thermal correction to the renormalized coupling constant is negative. In this case the renormalized coupling constant attains its maximum at zero temperature and decreases monotonically as the temperature increases. In $D = 4$, $\Delta\lambda(\beta)$ is $O(\lambda^2)$ therefore it is not possible to vanish the renormalized coupling constant at a finite temperature of the thermal bath. In $D = 3$ (in the regime of strong coupling) there is a finite temperature where this can be achieved. For temperatures $\beta^{-1} > \beta_\star^{-1}$ (negative coupling constant) the system can develop a first order phase transition, where the origin is a false vacuum.

As we discussed, the thermal corrections to the coupling constant at high temperatures including higher order loop contributions is positive. It is clear that our conjecture about a first order transition for the $(\lambda\varphi^4)_3$ model concerns the low temperature regime, where the renormalized coupling constant may becomes negative. In other words, if the temperature $\beta_\star^{-1}$ is in a region such that the one-loop corrections to the effective potential are the leading ones, then we could have a first order phase transition in $D = 3$. Of course we are not sure whether the existence of the phase transition as conjectured by us is or not an artifact of one-loop approximation. It would be interesting to have a fully non-perturbative argument to demonstrate or disprove this conjecture in a general way, using resummation methods or for example Constructive Field Theory arguments.

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References

[1] S.J.Chang, Phys.Rev.D **12**, 1671 (1975), ibid **13**, 2278 (1976), S.F.Magruder, Phys.Rev.D **14**,1602 (1976), P.Ginsparg, Nucl.Phys. **B170**, 388 (1980), M.Dine, R.G.Leigh, P.Huet, A.Linde and D.Linde, Phys.Rev.D **46**, 550 (1992).

[2] P.Arnold and O.Spinosa, Phys.Rev.D **47**, 3546 (1993).

[3] N.Tetradis and C.Wetterich, Nucl.Phys. **B398** , 659 (1993), N.Tetradis and C.Wetterich, Int.Jour.of Mod.Phys.A **22**, 4029 (1994).

[4] M.E.Carrington, Phys.Rev.D, **45**, 2933 (1992), K.Takahashi, Z.Phys.C **26**, 601 (1985).

[5] G.A.Carmelina and S.Y.Pi, Phys.Rev.D **47**, 2356 (1993).

[6] Cornwall, Jackiw and Tomboulis, Phys.Rev.D **10**, 2428 (1974).

[7] L.H.Ford and N.F.Svaiter, Phys.Rev.D **51**, 6981 (1995).

[8] E.Speer, J.Math.Phys. **9**, 1404 (1968), C.G.Bollini, J.J.Giambiagi and A.G.Domingues, Il Nuovo Cim. **31**, 550 (1964).

[9] G.'t.Hooft, Nucl.Phys. **B61**, 455 (1973).

[10] E.Elizalde and K.Kirsten, J.Math. Phys. **35**, 1260 (1994).

[11] C.Villarreal, Phys.Rev.D, **51**, 2959 (1995).

[12] G.Parisi, Jour. Stat. Phys. **23**, 49 (1980).

[13] K.Babu Joseph, V.C.Kuriakose and M.Sabir, Phys Lett **115B**, 120 (1982), O.J.P.Eboli and G.C.Marques, Phys. Lett. **162B**, 189 (1985), K.Funakubo and M.Sakamoto, Phys.Lett. **186B**, 205 (1987), P.Fedley, Phys. Lett. **196B**, 175 (1987), G.Barton, Ann of Phys. **200**, 271 (1990), F.T.Brandt, J.Frenkel and J.C.Taylor, Phys.Rev.D **44**, 1801 (1991), J.Frenkel, A.V.Saa and J.C.Taylor, Phys.Rev.D **46**, 3670 (1992), M.Loewe and J.C.Rojas, Phys.Rev.D **46** 2689 (1992). R.R.Parwani, Phys.Rev.D, **45**, 4695 (1992), R.R.Parwani and H.Singh, Phys.Rev.D **51**, 4518 (1995).

[14] Y.Fujimoto, K.Ideura, Y.Nakano and H.Yoneyama, Phys.Lett. **167B**,406 (1986). H.Matsumoto, Y.Nakano and H.Umezawa, Phys.Rev.D **29**, 1116 (1984).

[15] H.A.Weldon, Phys.Lett. **174B**, 427 (1986).

[16] M.Aizeman, Phys.Rev.Lett **47**, 1 (1981) and Comm.Math.Phys. **86**, 1 (1982), J.Frohlich, Nucl.Phys. **B200**, 281 (1982).

[17] A.M.Polyakov, "Gauge Fields and Strings" (Harwood Academic Publishers, United Kingdom, 1987).

[18] P.M.Stevenson, " The $\lambda\varphi^4$ Miracle: Laticce data and zero-point potential" e-print: hep-ph/9507408

[19] G.Jona Lasinio, Nuov. Cim. **34**, 1790 (1964), S.Coleman and E.Weinberg, Phys. Rev.D **7**, 1888 (1973).

[20] J.I.Kapusta, "Finite temperature field theory, (Cambridge University Press, Cambridge, UK, 1989).

[21] J.F.Ashmore Nuovo Cim.Lett. **9**,289 (1972), G.G.Bolini and J.J.Giambiagi, Nuovo Cim.B **12**, 20 (1972), G.T'Hooft and M.Veltman Nucl.Phys.B **44**, 189 (1972).

[22] K.Kirsten, J.Math.Phys.**32**, 3008 (1991).

[23] Handbook of Mathematical Functions, edited by M.Abramowitz and I.Stegun, Dover New York, 1965.

[24] D.A.Kirzhnitz and A.D.Linde, Phys.Lett.**42B**, 471 (1972), S.Weinberg, Phys.Rev.D **9**, 2257 (1974), C.Dolan and R.Jackiw, Phys. Rev.D **9**, 3320 (1974), D.A.Kirzhnitz and A.D.Linde, Ann.of Phys.**101**,195 (1976), P.Arnold, Phys.Rev.D **46**, 2628 (1992).

[25] H.W.Braden, Phys.Rev.D **25**, 1028 (1982).

[26] M.B.Kislinger and P.D.Morley, Phys.Rev.D **13**, 2771 (1976).

[27] K.Langfeld, F.Schmuser and H.Reinhardt, Phys.Rev.D **51**, 765 (1995).

[28] K.Symanzik, Lett.Nuovo Cim. **6**, 77 (1977), R.A.Brandt, Phys.Rev.D **14**, 3381 (1976), R.A.Brandt, Ng W. and Y.W.Bong, Phys Rev.D **19**, 503 (1979), J.Iliopoulus, C.Itzykson and A.Martin, Rev.Mod.Phys. **47**, 165 (1975).

[29] M.Blume, V.J.Emery and R.G.Griffiths, Phys.Rev.A **4**, 1071 (1971), E.K.Riedel and F.J.Wegner, Phys.Rev.Lett.**29**, 349 (1972), S.Sarbach and T.Schneider, Phys.Rev.B **16**, 347 (1977), S.Sarbach and M.E.Fisher, Phys.Rev.B **18**, 2350 (1978), S.Sarbach and M.E.Fisher, Phys.Rev.B **20**, 2797 (1979). H.Hamer, Phys.Rev.B**21**, 3999 (1980), D.Boyanovsky and L.Masperini, Phys.Rev.D **21**, 1550 (1980).