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A Rotating Quantum Vacuum

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ABSTRACT

We investigate which mapping we have to use to compare measurements made in a rotating frame to those made in an inertial frame. Using a non-Galilean coordinate transformation we obtain that creation-anihilation operators of a massive scalar field in the rotating frame are not the same as those of an inertial observer. This leads to a new vacuum state (a rotating vacuum) which is a superposition of positive and negative frequency Minkowski particles. After this, introducing an apparatus device coupled linearly with the field we obtain that there is a strong correlation between number of rotating particles (in a given state) obtained via canonical quantization and via response function of the detector. Finally, we analyse polarization effects in circular accelerators in the proper frame of the electron making a connection with the inertial frame point of view.

Key-words: Rotating vacuum; Radiative processes; Polarization effects.

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1 Introduction

1.1 Introductory Remarks

The purpose of this paper is to discuss the puzzle of the rotating detector [1] and to relate this to polarization effects of electrons in storage rings [2]. We try to avoid many technical difficulties to emphasize only fundamental results.

The use of general coordinate transformations in quantum field theory in flat spacetime introduce a plethora of new phenomena. One of these is the Unruh-Davies effect [3, 4]. An universal definition of the vacuum for a system described by a Hamiltonian is that the vacuum is the lowest energy state. If to describe the system we use a finite number of degrees of freedom, all representations of the operator's algebra are unitarily equivalents, i.e., diferent vacua lie in the same Hilbert space. This means that the physical description of the system will not depend on the choice of representation. However, if to describe the system we have to make use of infinite degrees of freedom, there are an infinite number of unitary inequivalent representations of the commutation relations [5]. Different inequivalent representations will in general give rise to different pictures with different physical implications.

A well known example of this situation arises in the study of the quantization of a field by observers with linear proper acceleration using the Rindler's coordinate system [6]. If we quantize a field in the Rindler's manifold one finds quantization structure identical to the quantization obtained using the usual cartesian coordinates adapted to inertial observers: in Rindler's manifold there is a time-like Killing vector and the symmetry generated by this vector field is implemented by a unitary operator group. The generator of this unitary group is positive definite and the construction of eingenstates of this operator allows a particle interpretation [7]. The Minkowski and the Rindler vacua are non-unitarily equivalents. It is possible to show that the Minkowski vacuum can be expressed into a set of EPR type of Rindler's particles [8]. As a natural consequence of this fact is that a particle detector at rest in Rindler's spacetime interacting with a massless scalar field prepared in the Minkowski vacuum responds as though is were at rest in Minkowski spacetime immersed in a bath of thermal radiation. Many authors claim that this case of linear acceleration is physically not very interesting since we need an eternal phase of constant acceleration.

A more treactable case (at least experimentally) is the case of transverse acceleration found in circular movement. This particular situation introduce some interesting questions related with the meaning of particles in non-inertial frames of references. To understand the problem of the rotating detector we have to go back to the problem of the rotating disc, i.e., the problem of rotation in relativity. A question that has interested many authors is whether the intrinsic geometry of a rotating disc is Euclidean or not. Infeld, using Einstein arguments [9] sustained that a rigid disc under uniform angular rotation Ω relative to an inertial frame will exhibit a non-Euclidean geometry (by a rigid body one understood a body in which during the motion no elastic stresses arises). The argument is that the circunference will suffer a Lorentz contraction although the radius r will not. Consequently, the circunference of the rotating disc relative to an inertial frame is less than $2\pi r$. Lorentz had a opposed point of view [10] and claimed that the intrinsic geometry of the rotating disc is Euclidean since the radius and the circunference of the rotating disc contract by the same amount.

A different approach to study this problem based on kinematic arguments has been presented by Hill long time ago [12]. If the speed of any point in a uniform rotating disc is a *linear* function of the radius, distant points have speeds exceeding the velocity of light. Hence this author concluded that the speed-distance law must be non-linear and approach the velocity of light when the radius goes to infinity. Even today these are open questions and no definite answer has been given. Of course Hill's approach is related with the question: which mapping we have to assume to compare measurements made in an inertial and in a rotating frame?

If we assume a "Galilean" transformation relating the inertial and the non-inertial frame it is possible to show that the rotating vacuum is just the Minkowski vacuum. Nevertheless an aparatus device (a detector coupled with a field) which gives information about the particle content of the state of the field can be excited if it is prepared in the ground state with the field in the Minkowski vacuum [13]. This is an odd result. One would expect the rotating detector *not to be excited* by the rotating vacuum. Futhermore, the rotating detector in the (inertial) Minkowski vacuum can be experimentaly implemented, e.g. electrons in storage rings. Therefore a zero rate of excitation in the rotating vacuum and the measurement of transitions of rotating detectors (electrons, atoms, etc.) in the Minkowski vacuum are theoretical and experimental results that test the correctness of the mapping relating the inertial and the rotating frames. In this paper we discuss these problems. We will show how the rate of spontaneous excitation of atoms can give the correct transformation law and the intrinsic geometry of the rotating disc. The same idea has been developed by Svaiter and Svaiter [14] and Iliadakys, Jasper and Audretsch [15], to perceive the existence of cosmic strings. Svaiter and Svaiter assumed the string quantization [16] and obtained the probability of transition per unit proper time for finite time measurements. Iliadakys, Jasper and Audretsch didn't assume the string quantization (thus physically realistic strings are included), and obtained asymptotic results. It is possible to use the same idea, examinating radiating "atoms" to find the intrinsic geometry of the rotating disc.

We would like to stress that we are not interested in discussing the subtle problem of how to decode the information stored in the composite system (detector and scalar field) to convert in a classical sign. The modern treatment of this problem is the following: both the detector and the scalar field are not closed systems but they are open systems interacting with the enviroment. In this way certain phase relations dissapear, i.e., loss of coherence to its enviroment (Decoherence). This idea allows that the composite system (detector and the scalar field) be described by a diagonal matrix density [17]. For an application of such ideas in the Unruh-Davies effect see for example Ref. [18].

1.2 Synopsis

The paper is organized as follows. In section 2 we discuss the possible mappings that we could assume to compare measurements made in the rotating frame and those in the inertial frame. After use a Galilean-like transformationn we present the well known result that a rotating vacuum defined by these transformations is just the Minkowski vacuum.

Some disturbing situations are analysed. In section 3 we discuss radiactive processes in a frame of reference comoving with the monopole detector. Due to interpretational difficulties associated with the Galilean-like transformations we consider a Lorentz-like transformation and second quantize the scalar field in this rotating coordinate system. It is shown that this rotating vacuum is not the Minkowski vacuum. In section 4 we perform the second quantization of the total Hamiltonian of the system to show that the process of an absorption (emission) of a rotating particle and excitation (decay) of the detector in the non inertial frame is interpreted as an excitation (decay) of the detector with emission of a Minkowski particle in the inertial frame. Conclusions are given in section 5. In this paper we use $\hbar = c = 1$.

2 The Rotating Coordinate System

The problem of the rotating disc have been investigated by many authors and can be posed in the following way. Suppose the Minkowski spacetime with line element in the cylindrical coordinate system $x'^{\mu} = (t', r', \theta', z')$ given by

$$ds^2 = dt'^2 - dr'^2 - r'^2 d\theta'^2 - dz'^2. \tag{1}$$

Suppose a disc rotating uniformly about the z axis with angular velocity Ω. Which coordinate transformation we have to use to connect the inertial frame to the rotating frame? In other words, which mapping we have to assume to compare measurements made in those frames? Eddington [19], Rosen [20] and Landau and Lifshitz [21] adopted as the transformation between the inertial $x'^{\mu} = (t', r', \theta', z')$ and rotating frame $x^{\mu} = (t, r, \theta, z)$ the following equations:

$$t = t', \tag{2}$$
$$r = r', \tag{3}$$
$$\theta = \theta' - \Omega t', \tag{4}$$
$$z = z'. \tag{5}$$

In the rotating coordinate system $x^{\mu} = (t, r, \theta, z)$ the line element can be written as

$$ds^2 = (1 - \Omega^2 r^2) dt^2 - dr^2 - r^2 d\theta^2 - dz^2 + 2\Omega r^2 d\theta dt. \tag{6}$$

The line element in the rotating frame is stationary but not static. The world line of a point of the disc is an integral curve of the Killing vector $\xi = (1 - \Omega^2 r^2)^{-1/2} \partial/\partial t$ which is timelike only for $\Omega r < 1$. Rosen claimed that using the transformations given by eqs.(2-5) the speed-distance law is linear and this put a limit on the size of the disc that rotate with a given angular velocity. The same point of view was given by Landau and Lifshitz when they restricted the transformation law for $r < 1/\Omega$.

A second possibility trying to avoid the disc problem in the core of the discussion is to follow Hill's arguments. This author presented a different answer for the problem. He raised the question if it is possible to find a group of transformation between the inertial and the non-inertial frame in such a way that for small velocities we obtain the linear speed-distance law and for large distance approach the speed of light. Such a transformation frames was presented by Trocheries [22] and also Takeno using a group theoretical approach [23]. The coordinate transformations are given by

$$t = (t' - r'\theta' \tanh \Omega r') \cosh \Omega r', \tag{7}$$

$$r = r', \tag{8}$$

$$\theta = (\theta' - \frac{t'}{r'} \tanh \Omega r') \cosh \Omega r', \tag{9}$$

$$z = z'. \tag{10}$$

Note that if we assume this mapping to connect measurements made in the rotating frame and those made in the inertial frame, in the rotating coordinate system the line element assume a non-stationary form

$$ds^2 = dt^2 - (1+P)dr^2 - r^2 d\theta^2 - dz^2 + 2Q dr d\theta + 2S dr dt, \tag{11}$$

where P, Q and S are given by

$$P = (\frac{Y}{r^2} + 4\Omega\theta t) \sinh^2 \Omega r - \frac{\Omega}{r}(t^2 + r^2\theta) \sinh^2 2\Omega + \Omega^2 Y, \tag{12}$$

$$Q = r\theta \sinh^2 \Omega r - \frac{1}{2} t \sinh 2\Omega r + \Omega r t, \tag{13}$$

$$S = \frac{t}{r} \sinh^2 \Omega r - \frac{1}{2}\theta \sinh 2\Omega r - \Omega r \theta, \tag{14}$$

and

$$Y = (t^2 - r^2\theta^2). \tag{15}$$

Before starting to analise the detector problem we would like to present some experimental and theoretical arguments against an in favour of Trocheries and Takeno's coordinate transformation. The Special Theory of Relativity show us that different inertial frames are connected by Lorentz transformations. Why we use a Lorentz-like transformation to connect measurements in the inertial and the non-inertal frame? We should mention that it is possible to write the transformations defined by eq.(7-10) making a analogy with the Lorentz transformations. Let us define $l = r\theta$ and $\gamma = (1 - v^2)^{-1/2}$. It is straightforward to show that eqs.(7) and (9) becomes

$$t = \gamma(t' - vl')$$

and

$$l = \gamma(l' - vt').$$

In other words the transformations defined by Trocheries and Takeno are "Lorentz-like" transformation. The fundamental difference is that in this case the velocity is $v = \tanh \Omega r'$. It has been sugested by Phipps [24] that the Takeno's velocity distribution does not agree with the experimental data. Strauss [11] also adopted a Lorentz-like transformation, but with a linear $v = \Omega r$ speed-distance law. The important consequence is that the light velocity on the rotating frame is one. Again, some authors claim that this result is in contradiction with the Sagnac's effect [25, 26]. The only way to have results consistents with this effect is to use a "Galilean" transformation given by eq.(2-5). We would like to stress that the above arguments does not establish conclusively that we have to use the Galilean transformations. As we will show, direct supports of Lorentz-like transformation between both frames are suplied by the detector puzzle and the depolarization effect of electrons in a circular accelerator.

Let us now discuss some implications of the Galilean transformation in quantum field theory. To investigate the meaning of particle in an arbitrary cordinate system in a flat spacetime we have two different routs. The first is to canonical quantize the field and obtain the number of particles operator for each mode $N_R(\omega) = b^\dagger(\omega)b(\omega)$ in the arbitrary frame. For static line elements (Rindler, for example) this can be done in a unambiguosly way. For time dependent line elements (Milne, for example) it is possible to define instantaneous positive and negative frequency modes and diagonalize an instantaneous Hamiltonian operator. The second rout is to introduce an measuring device, i.e., a detector (atoms) with a coupling with the field. Experimentalists detect photons in laboratories. They are absorbed at fixed instants and cause the electrons in the atoms to jump from a ground state to an excited state. Glauber an others produced a theory of photodetection using the rotating-wave approximation (RWA). In this approximation the detector (square-law detector) must gives information about the particle content of some state [27][28]. In other words, square-law detectors goes to excited state by absorption of quanta of the field.

Before continue let us discuss some arguments pro and con of the Glauber's detector. Bykov and Takarskii [29] showed that this detector model violates the causality principle for short observations times. If we assume that the observation time is large compared with E^{-1}, everething is in order. Note that it is possible to consider measurements of finite durations only for $\Delta T > 1/E$. Of short time intervals we cannot even define the two-level system. Nevertheless there are some situations were we can not use the RWA, for example in resonant interaction between two atoms [30]. As we will see the RWA can not be used only to find the rate of spontaneous decay. The same situation occur in a semi-classical theory of spontaneous emission where an atom in the excited state is stable since there is no vaccum fluctuations.

Going back to our problem, let us discuss these two routs that are usually used to investigate the meaning of particles in a curvilinear coordinate system. Let first perform the quantization of a massless real scalar field in the rotating frame assuming the first mapping giving by eqs.(2-5). First we have to solve the Klein-Gordon equation in the $x^\mu = (t, r, \theta, z)$ coordinate system given by

$$\left[(\frac{\partial}{\partial t} - \Omega\frac{\partial}{\partial \theta})^2 - \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\partial^2}{\partial z^2}\right]\varphi(t, r, \theta, z) = 0 \tag{16}$$

to find the normal modes that satisfies

$$L_{\bar{K}} u_{\bar{q}mk_z}(t, r, \theta, z) = -i\bar{\omega} u_{\bar{q}mk_z}(t, r, \theta, z), \tag{17}$$

where $\bar{K}$ is a time-like Killing vector. It is not difficult to show that the modes are given by [13, 31]

$$u_{\bar{q}mk_z}(t,r,\theta,z) = \frac{1}{2\pi\left[2(\bar{\omega}+m\Omega)\right]^{\frac{1}{2}}} e^{-i\bar{\omega}t} e^{im\theta} e^{ik_z z} J_m(qr) \tag{18}$$

where

$$(\bar{\omega}+m\Omega)^2 = k_z^2 + q^2, \tag{19}$$

$$(\bar{\omega}+m\Omega) > 0, \tag{20}$$

and the radial part $J_m(qr)$ are the Bessel functions of first kind [32]. To continue the canonical quantization, the field operator $\varphi(t,r,\theta,z)$ have to be expanded using these modes and the complex conjugates $\{u_{\bar{q}mk_z}(t,r,\theta,z), u^*_{\bar{q}mk_z}(t,r,\theta,z)\}$, i.e.,

$$\varphi(t,r,\theta,z) = \sum_m \int \bar{q}d\bar{q}dk_z \left[a_{\bar{q}mk_z} u_{\bar{q}mk_z}(t,r,\theta,z) + a^{\dagger}_{\bar{q}mk_z} u^*_{\bar{q}mk_z}(t,r,\theta,z)\right]. \tag{21}$$

Of course, in stationary coordinate systems the definition of positive and negative frequency modes has no ambiguities. To compare both quantizations i.e., in the inertial and in the rotating frame, we have to find the Bogoliubov coefficients between the inertial modes (cilindrical waves) $\{\psi_k(t',r',\theta',z'), \psi^*_k(t',r',\theta',z')\}$ and the non-inertial ones given by eqs.(18). Since the Bogoliubov coefficients $\beta_{k\nu} = -(u_{qmk_z}, \psi_k)$ are zero, Letaw and Pfautsch concluded that the vacuum defined by

$$a(\bar{q},m,k_z)|0,R>=0 \qquad \forall \qquad \bar{q},m,k_z, \tag{22}$$

i.e., the rotating vacuum is just the Minkowski vacuum. Note that we are not interested to discuss complications introduced by infinite-volume divergences. To circunvented this problem the creation and anihilation operators have to be smeared with square integrable test functions (wave-packet).

The introduction of the detector in this quantization scheme raised a fundamental question. If we prepare a detector in the ground state and the field in the Minkowski vacuum there is a non-null probability to find the detector in the excited state if the detector travel in a rotating world line, parametrized by eqs. (2-5). The orbiting detector will "measure" quanta of the field although there is no rotating quanta in the Minkowski vacuum. How is possible to a detector being excited if is traveling in a rotating disc if we prepare the field in the Minkowski vacuum? After the absorption, the field will be in a lower energy level than the "original vacuum". Therefore this "original vacuum" is not the true vacuum of the field. Another way to formulate the problem is the following one: our physical intuition say that a a rotating particle detector in the ground state interacting with the scalar field prepared in the rotating vacuum must stay in the ground state. Nevertheless, assuming the Galilean transformation, the Minkowski vacuum $|0,M>$ is exactly the rotating vacuum $|0,R>$ and the rate of excitations instead to be zero is different from zero. The detector behaves as if it is not coupled to the vacuum, concluded Davies, Dray and Manogue [1]. This is the so called rotating detector puzzle. Some time ago Grove and Ottewill trying to sheed some light for these problem studied extended detectors [33]. Letaw and Pfautsch, Padmanabhan [34] and also Padmanabhan and Singh [35] concluded that the correlation between vacuum states defined via canonical quantization and via detector is broken in this particular situation. We cannot agree with this conclusion. The preceding considerations suggest that the Galilean transformation is not correct to connect measurements in both frames. In the next section we will remember the formalism and discuss some possibilities to solve the rotating detector puzzle and the interpretational difficulties associated with the Galilean-like transformation.

3 Radiative Processes of the Monopole Detector and a New Rotating Vacuum

Let us consider a system (a detector) endowed with internal degrees of freedom defining two energy levels with energy ω_g and ω_e, $(\omega_g < \omega_e)$ and respective eigenstates $|g\rangle$ and $|e\rangle$ [4, 36, 37]. This system is weakly coupled with a hermitian massless scalar field $\varphi(x)$ with interaction Lagrangian

$$L_{int} = \lambda m(\tau)\varphi(x(\tau)), \tag{23}$$

where $x^\mu(\tau)$ is the world line of the detector parametrized using the proper time τ, $m(\tau)$ is the monopole operator of the detector and λ is a small coupling constant between the detector and the scalar field. For different couplings between the detector and the scalar field see for example Ref.[38] and also Ford and Roman [39].

In order to discuss radiative processes of the whole system (detector plus the scalar field), let us define the Hilbert space of the system as the direct product of the Hilbert space of the field $\mathbf{H_F}$ and the Hilbert space of the detector $\mathbf{H_D}$

$$\mathbf{H} = \mathbf{H_D} \otimes \mathbf{H_F}. \tag{24}$$

The Hamiltonian of the system can be written as:

$$H = H_D + H_F + H_{int}, \tag{25}$$

where the unperturbed Hamiltonian of the system is composed of the noninteracting detector Hamiltonian H_D and the free massless scalar field Hamiltonian H_F. We shall define the initial state of the system as:

$$|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle \,, \tag{26}$$

where $|j\rangle$, $(j = 1, 2)$ are the two possible states of the detector ($|1\rangle = |g\rangle$ and $|2\rangle = |e\rangle$) and $|\Phi_i\rangle$ is the initial state of the field. In the interaction picture, the evolution of the combined system is governed by the Schrodinger equation

$$i\frac{\partial}{\partial\tau}\,|\mathcal{T}\rangle = H_{int}\,|\mathcal{T}\rangle \,, \tag{27}$$

where

$$|\mathcal{T}\rangle = U(\tau, \tau_i)\,|\mathcal{T}_i\rangle \,, \tag{28}$$

and the evolution operator $U(\tau, \tau_i)$ obeys

$$U(\tau_f, \tau_i) = 1 - i\int_{\tau_i}^{\tau_f} H_{int}(\tau')U(\tau', \tau_i)d\tau'. \tag{29}$$

In the weak coupling regime, the evolution operator can be expanded in power series of the interaction Hamiltonian. To first order, it is given by

$$U(\tau_f,\tau_i) = 1 - i\int_{\tau_i}^{\tau_f} d\tau' H_{int}(\tau'). \tag{30}$$

The probability amplitude of the transition from the initial state $|\mathcal{T}_i\rangle = |j\rangle \otimes |\Phi_i\rangle$ at the hypersurface $\tau = 0$ to $\left|j'\right\rangle \otimes |\Phi_i\rangle$ at τ is given by

$$\left\langle j'\Phi_f\right| U(\tau,0)\,|j\Phi_i\rangle = -i\lambda\int_0^{\tau} d\tau' \left\langle j'\Phi_f\right| m(\tau')\varphi(x(\tau'))\,|j\Phi_i\rangle\,, \tag{31}$$

where $|\Phi_f\rangle$ is an arbitrary state of the field and $\left|j'\right\rangle$ is the final state of the detector. The probability of the detector being excited at the hypersurface τ, assuming that the detector was prepared in the ground state is:

$$P_{eg}(\tau) = \lambda^2 |\left\langle e\right| m(0)\left|g\right\rangle|^2 \int_0^{\tau} d\tau' \int_0^{\tau} d\tau'' e^{iE(\tau''-\tau')} \left\langle \Phi_i\right| \varphi(x(\tau'))\varphi(x(\tau''))\left|\Phi_i\right\rangle, \tag{32}$$

where $E = \omega_e - \omega_g$ is the energy gap between the eigenstates of the detector. Note that we are interested in the final state of the detector and not that of the field, so we sum over all the possible final states of the field $|\Phi_f\rangle$. Since the states are complete, we have

$$\sum_f |\Phi_f\rangle\langle\Phi_f| = 1. \tag{33}$$

Eq.(32) shows us that the probability of excitation is determined by an integral transform of the positive Wightman function.

Before starting to analyze radiative processes, we would like to point out that a more realistic model of particle detector must also have a continuum of states. This asumption allows us to use a first order perturbation theory without taking into account higher order corrections. Although we will use in this paper the two-state model, the case of a mixing between a discrete and a continuum eigenstates deserves further investigations. For a complete discussion of the detector problem see for example Refs. [40, 41, 42]. In this section we will use a different notation. Two distincts spacetime points in the rotating coordinate system will be given by $x^\mu = (\eta,\xi)$ and $x'^\mu = (\eta',\xi')$. Note that we are not interested in the z and r dependence of the response function and we will use only ξ as angular coordinate (this is exactly the situation in storage rings). Since we are interested in finite time measurements let us follow Svaiter and Svaiter [43], and also Ford, Svaiter and Lyra [44] defining

$$\eta - \eta' = \zeta \tag{34}$$

and

$$\eta_f - \eta_i = \Delta T. \tag{35}$$

We would like to stress that Levin, Peleg and Peres [45] also used the same technique to study radiative processes in finite observation times. Substituting eqs.(34) and (35) in eq.(32) and defining $F(E,\Delta T)$ by

$$P_{12}(E,\Delta T,\xi,\xi') = \lambda^2 |\left\langle 2\right| m \left|1\right\rangle|^2 F(E,\Delta T,\xi,\xi')$$

we have

$$F(E,\Delta T,\xi,\xi') = \int_{-\Delta T}^{\Delta T} d\zeta(\Delta T - |\tau|)e^{iE\zeta}\,\langle 0,M|\,\varphi(\eta',\xi')\varphi(\eta,\xi)\,|0,M\rangle\,. \tag{36}$$

It is clear that $F(E,\Delta T)$ is the probability of excitation normalized by the selectivity of the detector. The same can be done for decay processes and the probability of decay $P_{21}(E,\Delta T)$ is given by

$$P_{21}(E,\Delta T) = \lambda^2|\,\langle 1|\,m\,|2\rangle\,|^2 F(-E,\Delta T).$$

Let us define the rate $R(E,\Delta T,\xi,\xi')$, i.e., this probability transition per unit proper time as:

$$R(E,\Delta T,\xi,\xi') = \frac{d}{d(\Delta T)}F(E,\Delta T,\xi,\xi'). \tag{37}$$

Writting in a concise form we have:

$$R(E,\Delta T,\xi,\xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta}\,\langle 0,M|\,\varphi(\eta',\xi')\varphi(\eta,\xi)\,|0,M\rangle\,. \tag{38}$$

This important result shows that asymptotically the rate of excitation (decay) of the detector is given by the Fourier transform of the positive frequency Wightman function. This is exactly the quantum version of the Wiener-Khintchine theorem which asserts that the spectral density of a stationary random variable is the Fourier transform of the two point-correlation function. Spliting the field operator in positive and negative frequency parts, the rate becomes:

$$\begin{aligned} R(E,\Delta T,\xi,\xi') \;=\; & \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta}\Big[\langle 0,M|\,\varphi^{(+)}(\eta',\xi')\varphi^{(+)}(\eta,\xi)\,|0,M\rangle \\ & + \langle 0,M|\,\varphi^{(-)}(\eta',\xi')\varphi^{(-)}(\eta,\xi)\,|0,M\rangle \\ & + \langle 0,M|\,\varphi^{(-)}(\eta',\xi')\varphi^{(+)}(\eta,\xi)\,|0,M\rangle \\ & + \langle 0,M|\,\varphi^{(+)}(\eta',\xi')\varphi^{(-)}(\eta,\xi)\,|0,M\rangle\Big]\,. \end{aligned} \tag{39}$$

The last matrix element can be writen as

$$\begin{aligned} \langle 0,M|\,\varphi^{(+)}(\eta',\xi')\varphi^{(-)}(\eta,\xi)\,|0,M\rangle \;=\; & \langle 0,M|\,\varphi^{(-)}(\eta,\xi)\varphi^{(+)}(\eta',\xi')\,|0,M\rangle \\ + \; & [\varphi^{(+)}(\eta',\xi'),\varphi^{(-)}(\eta,\xi)]. \end{aligned} \tag{40}$$

The commutator is a c-number independent of the initial state of the field. Many authors in quantum optics claim that this contribution has no great physical interest. So the matrix elements determining the detection of quanta of the field are of the form

$$\langle 0,M|\,\varphi^{(-)}(\eta',\xi')\varphi^{(+)}(\eta,\xi)\,|0,M\rangle + \langle 0,M|\,\varphi^{(-)}(\eta,\xi)\varphi^{(+)}(\eta',\xi')\,|0,M\rangle\,. \tag{41}$$

Substituting the modes given by eq.(18) in eq.(39) it is possible to show that the rotating detector has non-zero probability of excitation. Since the contribution given by eq.(41) is zero (there are no rotating particles in Minkowski vacuum), the non-zero rate is cause by the last term in eq.(40). A disagreable situation emerges. Our apparatus device is not measuring the particle content of some state.

The first solution of the the puzzle of the rotating detector was given a few months ago by Davies, Dray and Manogue [1]. These authors assumed that the field is defined only in the interior of a cilinder of radius a in such a way that the rotating Killing vector $\partial_t - \Omega\partial_\theta$ is always timelike. Consequently the response function of the rotating detector is zero. Of course if the angular velocity of the detector is above some threshold, excitation occurs. Clearly the excitation of the rotating detector is related with the "Galilean" transformation to rotating coordinates which is not valid above certain radius.

Since the vacuum state of quantized field is a *global* object we will present a different solution for the problem. Let us assume that the coordinate transformation between the inertial and the rotating frame is that defined by Trocheries and Takeno [22, 23]. The advantage of this coordinate transformation is that the velocity of a rotating point is $v = \tanh\Omega r$ (for small radius or angular velocities we recovered the situation $v = \Omega r$). This coordinate transformation cover all the Minkowski manifold for all angular velocities. Although we will be not able to calculate explicity the Bogoliubov coefficients between the inertial and the rotating modes we will prove that these coefficients are non-zero and in this case the answer obtained calculating the Bogoliubov coefficients between cartesian and rotating modes and the response function of the detector will agree.

To prove the above assumption, first we have to canonical quantize a massless scalar field assuming the second mapping given by eqs.(7-10). It is an human impossible task to solve exactly the Klein-Gordon equation in Takeno's coordinate system. Making a Taylor expansion for $\cosh\Omega r$ and $\tanh\Omega r$ and retaining terms up the first order in Ωr the line element becomes

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - 4r\Omega\theta dr dt - dz^2.$$

We point out that although we will consider only the case $\Omega r < 1$, the low-velocity limit of Takeno's transformation does not give the "Galilean" transformation since we have $t = t' - \Omega r'^2\theta'$. In this approximation the metric is stationary by not static. This means that although there is a timelike Killing vector field K, the spatial sections putting $t =$ constant are not orthogonal to the time lines putting $r, theta$ and z constants, i.e., the Killing vector K is not orthogonal to the spatial section. This line element describe a physical situation in which world lines infinitesimally close to a given world line are spatially rotating with respect this world line [46]. In this simplified case, the Kein-Gordon equation reads

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - 4\Omega\theta\frac{\partial}{\partial t} - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} - 4\Omega\theta r\frac{\partial^2}{\partial r\partial t} - \frac{\partial^2}{\partial z^2}\right)\varphi(t,r,\theta,z) = 0. \quad (42)$$

The solution can be found using partial separation of variables

$$\varphi(t,r,\theta,z) = T(t)Z(z)f(r,\theta). \quad (43)$$

We obtain:

$$Z(z) = e^{ik_z z} \quad (44)$$

and

$$T(t) = e^{-i\omega t}. \quad (45)$$

Finally, defining $\omega^2 = k_z^2 + q^2$ we obtain the equation for $f(r, \theta)$

$$\left[\frac{\partial^2}{\partial r^2} + (\frac{1}{r} - 4i\omega\Omega\theta r)\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + (q^2 - 4i\omega\Omega\theta)\right] f(r, \theta) = 0. \qquad (46)$$

The general solution of this equation was derived in the appendix A and is given by

$$\begin{aligned} f(y, \theta) &= Ce^{i\mu\theta}\left[J_\mu(y) + le^{i\lambda\theta}J_{\mu+\lambda}(y)\right] \\ &+ \frac{l}{2}\int d\theta' \int dy' G(y, \theta; y', \theta')\theta' \left[y'^3 J_{\mu-1}(y') + 2y'^2 J_\mu(y') - y'^3 J_{\mu+1}(y')\right]. \end{aligned}$$

where l, y and $G(y, \theta; y', \theta')$ are also defined in the appendix A, and C is a normalization factor. Although the spatial part of the solution of eq.(42) is extremely complicated, there is not ambiguity in the definition of positive and negative rotating modes since the temporal part is given by eq.(45) and the world line of the detector is an integral curve of the Killing vector $K = \partial/\partial t$ that generates a one-parameter group of isometries.

Note that we have problems to define the Hamiltonian in the rotating frame if we work with the Takeno coordinate transformation without assume $\Omega r < 1$. The metric given by eqs.(11-15) is not invariant under time translations. Usualy the Hamiltonian is defined as

$$H = \int T^{\mu\nu}\xi_\mu d\sigma_\nu \sqrt{-g}$$

where ξ_μ is a timelike Killing vector field. Since in the rotating frame the line element is not stationary it is a complicated question how to define H_R. A possible solution of this problem is to use the same idea that we use in expanding universes where there is no timelike Killing vector field. It is possible to introduce the definition of particles at each time. This procedure introduce the difficult of particle creation [47, 48]. An alternative idea is to define the energy as the integral over all modes

$$H = \int_0^\infty d\omega \frac{1}{2}\omega N(\omega)$$

where $N(\omega)d\omega$ is the number of modes with frequency between ω and $\omega + d\omega$ [49]. To circumvent the ultraviolet divergences of the equation above it is convenient to quantize the field in the interior of a box and define the renormalized mode sum energy

$$< H >_{ren} = \int_0^\infty d\omega \frac{1}{2}\omega \left[N(\omega) - N_0(\omega)\right]$$

where now $N(\omega)d\omega$ is the number of modes with frequency between ω and $\omega + d\omega$ in the presence of the boundaries and $N_0(\omega)d\omega$ is the number of modes with frequency between ω and $\omega + d\omega$ in the empty space-time. $< H >_{ren}$ represent the physically observable change in the vaccum energy of the field (Casimir energy). The motivation of the $< H >_{ren}$ definition is that there is some frequency above which the boundary is transparent. This definition eliminate the ultraviolet divergences.

Going back to the low-limit velocity case, we have that the line element is stationary and there is no ambiguity to define the rotating vacuum $|0, R>$ is such a way that:

$$b_{q\mu k_z}|0, R> = 0. \qquad (47)$$

where

$$\varphi(t,r,\theta,z)=\sum_{\mu}\int qdqdk_z\left[b_{q\mu k_z}v_{q\mu k_z}(t,r,\theta,z)+b^{\dagger}_{q\mu k_z}v^{*}_{q\mu k_z}(t,r,\theta,z)\right]. \tag{48}$$

By sake of simplicity let use the following notation:

$$\varphi(t,r,\theta,z)=\sum_{\nu}b_{\nu}v_{\nu}(t,r,\theta,z)+b^{\dagger}_{\nu}v^{*}_{\nu}(t,r,\theta,z), \tag{49}$$

where $\nu\equiv\{q,\mu,k_z\}$ is a collective index.

It is straighforward to show that the Minkowski vacuum can be expressed as a many rotating-particles state. By comparing the expansion of the field operator using the inertial modes and the rotating modes it is possible to obtain the expression comparing both vacua, i.e $|0,M>$ and $|0,R>$:

$$|0,M>=e^{\frac{i}{2}\sum_{\mu,\nu}b^{\dagger}(\mu)V(\mu,\nu)b^{\dagger}(\nu)}|0,R> \tag{50}$$

where

$$V(\mu,\nu)=i\sum_{k}\beta^{*}_{\mu k}\alpha^{-1}_{k\nu}, \tag{51}$$

and the Bogoliubov coefficients are given by $\beta_{\nu k}=-(v_{\nu},\psi^{*}_{k})$ and $\alpha_{\nu k}=(v_{\nu},\psi_{k})$. It is clear that the number of rotating particles in a specific mode in the Minkowski vacuum is given by

$$<0,M|N_R(\nu)|0,M>=\sum_{k}|\beta_{k\nu}|^2. \tag{52}$$

Let us choose the hypersurface $t'=$ constant to find the Bogoliubov coeficients, i.e.,

$$\beta_{\nu k}=i\int_0^{2\pi}d\theta'\int_{-\infty}^{\infty}dz\int_0^{\infty}rdr\left\{v_{\nu}(x')\left[\partial_{t'}\psi_k(x')\right]-\left[\partial_{t'}v_{\nu}(x')\right]\psi_k(x')\right\}.$$

The Bogoliubov coefficients $B_{\nu k}$ must be non-zero since the positive and negative frequency rotating modes are mixture between positive and negative inertial modes. The important conclusion from the above arguments is that the Minkovski and this rotating vacuum are not the same. Now we will show that the expectation value of the number operator of rotating particles is proportional to the response function, recovering the old idea that rate of spontaneous excitation is proportional to the number of particles (in the mode of interest) in the state of the field. Note that the rate given by eq. (39) can be written as:

$$R(E,\Delta T,\xi,\xi')=R_I(E,\Delta T,\xi,\xi')+R_{II}(E,\Delta T,\xi,\xi')$$

where

$$R_I(E,\Delta T,\xi,\xi')=\int_{-\Delta T}^{\Delta T}d\zeta e^{iE\zeta}\Big(\langle 0,M|\,\varphi^{(-)}(\eta',\xi')\varphi^{(+)}(\eta,\xi)\,|0,M\rangle + \langle 0,M|\,\varphi^{(-)}(\eta,\xi)\varphi^{(+)}(\eta',\xi')\,|0,M\rangle\Big). \tag{53}$$

and

$$R_{II}(E,\Delta T,\xi,\xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \langle 0,M| \left[\varphi^{+}(\eta^{'},\xi^{'}),\varphi^{-}(\eta,\xi)\right] |0,M\rangle \,. \tag{54}$$

The former equation is independent of the state. It is a vacuum fluctuations contribution and in the case of spontaneous excitation ($E > 0$) in the asymptotic limit gives zero (stability of the detector's ground state). In the case if spontaneous decay ($E < 0$) we can substitute the Minkowski vacuum by the rotating vacuum i.e.,

$$R_{II}(E,\Delta T,\xi,\xi') = \int_{-\Delta T}^{\Delta T} d\zeta e^{iE\zeta} \langle 0,R| \left[\varphi^{+}(\eta^{'},\xi^{'}),\varphi^{-}(\eta,\xi)\right] |0,R\rangle \,. \tag{55}$$

A straighforward calculation gives us a general expression in both cases for the detector at rest in the rotating frame i.e.

$$R_{II}(E,\Delta T) = \frac{1}{2\pi}\left[-E\Theta(-E) + \frac{\cos(E\Delta T)}{\pi\Delta T} + \frac{|E|}{\pi}\left(Si(|E|\Delta T) - \frac{\pi}{2}\right)\right]$$

and, in the asymptotic limit for spontaneous decay we have:

$$\lim_{\Delta T\to\infty} R_{II}(E,\Delta T) = \frac{|E|}{2\pi}$$

It is not dificult to show that the term between the parentesis in eq.(53) gives

$$\langle 0,M|\,\varphi^{(-)}(\eta^{'},\xi^{'})\varphi^{(+)}(\eta,\xi)\,|0,M\rangle + \langle 0,M|\,\varphi^{(-)}(\eta,\xi)\varphi^{(+)}(\eta',\xi')\,|0,M\rangle =$$
$$\sum_{\mu\nu}\sum_{k}\beta^{*}_{\nu k}\beta_{k\mu}\left(v_{\nu}(\eta',\xi')v^{*}_{\mu}(\eta,\xi) + v_{\nu}(\eta,\xi)v^{*}_{\mu}(\eta',\xi')\right).$$

Consequently the rate of excitation for the detector at rest in the rotating frame is:

$$R_{I}(E,\Delta T,\xi) = \Delta T|Z|^2|f(r,\xi)|^2\,\langle 0,M|N_R(\mu)|0,M\rangle\left[\frac{\sin(E-\mu)\Delta T}{(E-\mu)\Delta T} + \frac{\sin(E+\mu)\Delta T}{(E+\mu)\Delta T}\right].$$

In the asymptotic limit the rate of excitation becomes

$$\lim_{\Delta T\to\infty} R_{I}(E,\Delta T,\xi) = \Delta T|Z|^2|f(r,\xi)|^2\,\langle 0,M|N_R(\mu)|0,M\rangle\,\delta(E-\mu). \tag{56}$$

Thus the rate of excitation will be proportional to the number of rotating particles with energy E in the Minkowski vacuum multiplied by the square of the "wavefunction" ($|Z||f(r,\xi)|$) in the world line of the detector.

Bell and Leinaas studied the depolarization problem in accelerators trying to use the idea of a Unruh-Davies effect. The electron in a accelerated ring is a magnetic version of the monopole detector, since there is a linear coupling between the magnetic field B and the magnetic moment of the electron. To see this result let us define the invariant operator

$$H = \frac{e}{2m^2}F^{*}_{\mu\nu}p^{\mu}s^{\nu}$$

where m is the electron mass, $F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ and s^ν is the four vector spin operator. In the frame in which the electron is at rest the operator H describes the interaction between the spin magnetic moment of the electron with the magnetic field,

$$H = -\vec{\mu}.\vec{B}.$$

To understand the depolarization problem let us suppose an ensemble of electrons (detectors) in equilibrium with a thermal bath. The probability to find the detector in the state $|i>$ is:

$$P_i = \frac{e^{-\beta\omega_i}}{Z} \tag{57}$$

or

$$\frac{P_e}{P_g} = e^{-\beta E} \tag{58}$$

defining the occupation number $N(e)$ and $N(g)$ we have

$$N(e) = N(g)e^{-\beta E}. \tag{59}$$

Since the electron in a accelerator is a magnetic version of the monopole detector in the equilibrium, the rate between spin up and spin down will be given by the above equation. Thus if we introduce a complete unpolarized electron beam, it will suffer a polarization until the equilibrium is reached. The asymptotic rate of spin flip will be proportional to the asymptotic limit of the rate $R_\beta(E,\Delta T)$ i.e.,

$$\lim_{\Delta T\to\infty} R_\beta(E,\Delta T) = \frac{|E|}{2\pi}\left[\Theta(-E)\left(1+\frac{1}{e^{\beta|E|}-1}\right)+\Theta(E)\frac{1}{e^{\beta E}-1}\right].$$

Note that although the situation is similar to the Rindler's case where the detector goes to excited state by absorption of Rindler's particles (the Minkowski vacuum is a thermal state of Rindler's particles), there is a fundamental difference. In the Rindler's case there is an past and future horizont. Part of information which would have an inertial observer is inaccessible for accelerated observers. Although the Minkowski vacuum $|0,M>$ is a pure state, for accelerated observers it must be described by a statistical operator. This is the origin of the thermal distribution of particles. As was noted by Bell and Leinaas in the case of circular motion the measurements of the polarization *does not agree* with the calculations if we interpret the polarization by thermal effects. In our approach, depolarization is related with the fact that the Minkowski vacuum is a many particle state of rotating particles. Let us try to improve this ideas using Einstein's arguments [50]. All calculations will be held in the rotating frame. Suppose that the probability to find the detector in the state $|i>$ is given by

$$P(\omega_i) = \frac{f(\omega_i)}{Z} \tag{60}$$

where the partition function Z is given by

$$Z = \sum_{i=1}^{2} f(\omega_i). \tag{61}$$

Still following Einstein's arguments we have three different processes: absortion of rotating particles, induced emission and spontaneous emission (stimulated emission by the $|0, R>$ vacuum fluctuations) of rotating particles. Defining the rate of spontaneous decay by $A_{2\to1}(E, \Delta T)$ we have

$$dW_{2\to1}(E, \Delta T) = A_{2\to1}(E, \Delta T)dt \tag{62}$$

For the induced emission $R_{2\to1}(E, \Delta T)$ we have

$$dW_{2\to1}(E, |\Delta T) = R_{2\to1}(E, \Delta T)dt, \tag{63}$$

and finally for the rate of absorption $R_{1\to2}(E, \Delta T)$ we have

$$dW_{1\to2}(E, \Delta T) = R_{1\to2}(E, \Delta T)dt. \tag{64}$$

In the *equilibrium situation* between an ensemble of rotating detectors and the scalar field in the Minkowski vacuum (asymptotic limit) we have

$$f(\omega_1)\rho(E)R_{1\to2}(E) = f(\omega_2)\left(\rho(E)R_{2\to1}(E) + A_{2\to1}(E)\right) \tag{65}$$

where ρ is the number of rotating particle in the mode E in the Minkowski vacuum i.e.

$$\rho(E) =< 0, M|N_R(E)|0, M> . \tag{66}$$

Although the spectrum of the rotating particles in the Minkowski vacuum is not known, at the equilibrium we have $R_{1\to2}(E) = R_{2\to1}(E)$. In the equilibrium situation the this hipotesis must hold. Note that this is not in principle fundamental for our conclusions. A straightforward calculations gives

$$\rho(E) = \frac{A_{2\to1}(E)}{R_{1\to2}(E)} \frac{1}{\frac{f(\omega_1)}{f(\omega_2)} - 1} \tag{67}$$

The knowledge of the Bogoliubov coefficients $\beta_{k\mu}(\nu)$ give us both $\rho(E)$ and $R_{2\to1}(E)$. A second step in our analysis is to use the result that $A_{2\to1}(E)$ is exactly the rate of spontaneous decay of a inertial detector interacting with the field in the Minkowski vacuum. Thus we have

$$\frac{f(\omega_1)}{f(\omega_2)} = \left(\frac{ER^{-1}_{1\to2}(E)}{< 0, M|N(E)|0, M >} - 1\right). \tag{68}$$

This result show us the conection between the the rate between up and down spins as a function of the mean life of the excited state and $\rho(E)$ after the equilibrium situation is reached.

We still have to answer some questions. Where does the energy of excitation come from if we analyse the process from the point of view of the inertial observer? The non-inertial observer does not meet any dificulty. At some initial time we prepare the detector in the ground state and the field in the Minkowski vacuum. Since the Minkowski vacuum is a many rotating-particles state the detector goes to excited state absorbing a positive energy particle. For large time intervals energy conservation holds. For the point of view of the inertial observer the field is in the empty state. How is possible the excitation? A natural answer is to say that it is necessary an external accelerating agency to suplly energy to keep the detector in the rotating world-line. It is possible to show that the detector goes to excited state with the emission of a Minkowski particle. In the next section we will perform the second quantization of the detector Hamiltonian to analyse the absorption and emission processes from the inertial point of view.

4 Second Quantization of the Total Hamiltonian and Polarization Effects on Electrons and Positrons in Storage Rings

In this section we will prove that the process: absorption (emission) of positive energy rotating particle with excitation (decay) of the detector (from the non-inertial point of view) is interpreted as a emission of a Minkowski particle with excitation (decay) of the detector from the inertial point of view. This simple result express the fact that electrons (positrons) experience a gradual polarization orbiting in a storage ring. This mechanism lead to the emission of spin-flip synchronon radiation [51]. It is important to stress that the amount of spin-flip radiation is extremely small compared with the non-flip radiation. An open question is why the polarization is not complete after the system reach the equilibrium? We will try to answer this question applying the ideas developed by us in the preceding sections. Of course again we have a oversimplified description of the phenomenon. Before start the second quantization of the detector and interaction Hamiltonian let us remember the fundamental results of the preceeding section (we will use a different notation in this section).

In Minkowski space time it is possible to quantize a massless scalar field using the cartesian coordinate adapted to inertial observers. Thus the scalar field can be expanded using an orthonormal set of modes

$$\varphi(x) = \sum_i a_i u_i(x) + a_i^\dagger u_i^*(x) \tag{69}$$

where

$$a_i|0, M >= 0 \qquad \forall\, i. \tag{70}$$

There is an inequivalent quantization using coordinates adapted to a rotating observer. The scalar field can be expanded using a second set of orthonormal modes

$$\varphi(x) = \sum_j b_j v_j(x) + b_j^\dagger v_j^*(x) \tag{71}$$

where

$$b_j|0, R >= 0 \qquad \forall\, j. \tag{72}$$

As both sets are complete, the non-inertial modes can be expanded in terms of the inertial ones, i.e.

$$v_j(x) = \sum_i \alpha_{ji} u_i(x) + \beta_{ji} u_i^*(x) \tag{73}$$

or

$$u_i(x) = \sum_j \alpha_{ji}^* v_j(x) - \beta_{ji} v_j^*(x). \tag{74}$$

Using these equations and the orthonormality of the modes it is possible to write the annihilation and creation operators of inertial particles in the mode i as a linear combination of non-inertial creation and annihilation operators [52], i.e.

$$a_i = \sum_j \alpha_{ji} b_j + \beta^*_{ji} b^\dagger_j \tag{75}$$

or

$$b_j = \sum_i \alpha^*_{ji} a_i - \beta^*_{ji} a^\dagger_i . \tag{76}$$

Let us use the notation introduced in section 3, i.e. $|g>=|1>$ and $|e>=|2>$. Thus we have

$$H_D|i>=\omega_i|i> \qquad\qquad i=1,2. \tag{77}$$

Using the above equation and the orthonormality of the energy eigenstates of the detector Hamiltonian, we can write

$$H_D = \sum_{i=1}^{2} \omega_i |i>< i|. \tag{78}$$

To second quantize the detector Hamiltonian we have to introduce the Dicke operators [53]

$$S^+ = |2>< 1|, \tag{79}$$

$$S^- = |1>< 2|, \tag{80}$$

and finally

$$S_z = \frac{1}{2}(|2>< 2| - |1>< 1|). \tag{81}$$

In the case of n eigenstates of the (atom) detector Hamiltonian we have to work with the atomic operators, i.e. the multilevel generalization of the Dicke spin operators for the two level system. The detector Hamiltonian in the two level case can be written as

$$H_D = ES_z + \frac{1}{2}(\omega_1+\omega_2). \tag{82}$$

The operators S^+, S^- and S_z satisfy the angular momentum commutation relations corresponding to spin 1/2 value, i.e.

$$\left[S^+, S^-\right] = 2S_z, \tag{83}$$

$$\left[S_z, S^+\right] = S^+, \tag{84}$$

$$\left[S_z, S^-\right] = -S^-. \tag{85}$$

It is clear that S^+ and S^- are respectivelly raising and lowering operators of the detector states ($S^+|1>=|2>, S^+|2>=0, S^-|2>=|1>, S^-|1>=0$). The interaction Hamiltonian given by eq.(23) can be written as

$$H_{int} = \lambda[m_{21}S^+ + m_{12}S^- + S_z(m_{22}-m_{11})]\varphi(x), \tag{86}$$

where

$$< i|m(0)|j >= m_{ij}. \tag{87}$$

We should simplify the discussion choosing $m_{11} = m_{22}$. As we will see the part if the interaction hamiltonian with the S_z operator is responsible for the non-flip synchroton radiation. Substituting eq.(71) in eq.(86) we see that there are different processes with absorption or emission of rotating particles with excitation or decay of the detector. It is possible to show that some of these processes are virtual, and only processes with energy conservation survive in the asymptotic limit, i.e., excitation of the detector with absorption of a rotating particle (process involving $b_j S^+$) and decay of the detector with emission of a rotating particle (process involving $b_j^\dagger S^-$).

The first process is generated by the following operators:

$$m_{12} \sum_j v_j(x) b_j S^+. \tag{88}$$

Substituting eq.(73) and eq.(76) in eq.(88) it is clear that the above process of absorption of a rotating particle in the mode j is the following:

$$\sum_{ijk} \left[\beta_{ji}^* \alpha_{jk} u_k(x) + \beta_{ji}^* \beta_{jk} u_k^*(x)\right] a_i^\dagger S^+. \tag{89}$$

Therefore this process for the inertial observer is an excitation of the detector with creation of Minkowski particles.

The second process is generated by the following operators:

$$m_{21} \sum_j v_j^*(x) b_j^\dagger S^-. \tag{90}$$

Substituting eq.(73) and eq.(76) in eq.(90) we see that the above process of emission of a rotating particle in the mode j is the following:

$$\sum_{ijk} \left[\alpha_{ij} \alpha_{jk}^* u_k^*(x) + \alpha_{ij} \beta_{jk}^* u_k(x)\right] a_i^\dagger S^-. \tag{91}$$

Therefore this process for the inertial observer is a decay of the detector with creation of Minkowski particles.

Now we are able to understand the problem of the synchroton radiation. In the emision of synchroton radiation by electrons moving along a circular orbit, there are two kinds of processes: the first is the emission of photons without spin flip of the electron and the second is emission with spin flip. We will restrict our discussion to the second case. To make a parallel with the detector's problem we have to assume that the electron trajectory is "classical" (there is no fluctuation of the electron path) or even after the photon emission there is no recoil (as was stressed by Bell and Leinaas, the results does not depend on position fluctuations of the electron trajectory). There are two differents results in the literature depending on the value of the Landé-g factor of the electron. Jackson showed that the rate of transition from an initial state with the spin of the electron directed along the magnetic field (high energy state) to a state with the electron spin in opposite to the magnetic field (lower energy state) is lower than the opposite situation if the Landé-g factor of the electron obeys $0 < g < 1.2$. It is important to stress that the situation is opposite of the naive description where polarization arises from spontaneous emission as the spin move from its "upper" (high energy state) to its "lower" (low energy state) in the magnetic field. For the case where $1.2 < g < 2$ Jackson and also Sokolov et al [54] obtained that after the photon emission the electron spin will tends to orient themselves in opposite to the magnetic field (going to the lower energy state). Of course, positrons spins will have an oposite behavior. These both results are consistent with our interpretation that absorption (emission) of a rotating particle with excitation (decay) of the detector in the non-inertial frame is interpreted as emission of a Minkowski particle with excitation (decay) of the detector in the inertial frame.

To find the degree of polarization before the equilibrium situation is achieved let us define the occupation number of electrons with spins directed in oposition to the magnetic field (lower energy state) by N_1, and N_2 is the number of electrons with spins directed to the magnetic field. Of course we have $N_1(t)+N_2(t) = N$, where N = constant is the total numbers of electrons in the ring. We will do all the calculations in the rotating frame. The degree of polarization of an ensemble of electrons in the beam is defined as

$$P(t) = \frac{N_1(t) - N_2(t)}{N_1(t) + N_2(t)}. \tag{92}$$

The equation of the evolution of N_1 and N_2 are given by

$$\frac{dN_1}{dt} = N_2\left[\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T)\right] - N_1\left[\rho(E)R_{1\to2}(E,\Delta T)\right] \tag{93}$$

and

$$\frac{dN_2}{dt} = N_1\left[\rho(E)R_{1\to2}(E,\Delta T)\right] - N_2\left[\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T)\right] \tag{94}$$

Let us avoid the dificult to find $R_{1\to2}(E,\Delta T))$ and $R_{2\to1}(E,\Delta T)$ and using the following approximation i.e,

$$\rho(E)R_{2\to1}(E,\Delta T) + A_{2\to1}(E,\Delta T) = \sigma_{21} = \text{constant} \tag{95}$$

and

$$\rho(E)R_{1\to2}(E,\Delta T) = \sigma_{12} = \text{constant}. \tag{96}$$

Then, starting from a situation where there is no polarization, i.e., $P(t=0) = 0$ it is possible to find the polarization until the equilibrium situation is achieved. It is necessary only to integrate the above equations. A straightforward calculation gives

$$N_1(t) = \frac{N}{2}\left(\frac{\sigma_{12}-\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right)e^{-(\sigma_{12}+\sigma_{21})t} + N\left(\frac{\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right) \tag{97}$$

and

$$N_2(t) = -\frac{N}{2}\left(\frac{\sigma_{12}-\sigma_{21}}{\sigma_{12}+\sigma_{21}}\right)e^{-(\sigma_{12}+\sigma_{21})t} + N\left(\frac{\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right). \tag{98}$$

The degree of polarization of the beam is

$$P(t) = \left(\frac{\sigma_{21}-\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right)\left(1 - e^{-(\sigma_{12}+\sigma_{21})t}\right). \tag{99}$$

We obtained that if $R_{1\to 2}(E,\Delta T)$, $R_{2\to 1}(E,\Delta T)$ and $A_{2\to 1}(E,\Delta T)$ are independent of time the asymptotic degree of polarization is constant i.e.,

$$\lim_{t\to\infty} P(t) = \left(\frac{\sigma_{21}-\sigma_{12}}{\sigma_{12}+\sigma_{21}}\right).$$

Experimental results show us a not complete polarization. Why there is residual depolarization? This is the puzzle stressed by Jackson [51] and also Bell and Leinas [2]. From the former equation it is easy to see that the polarization can not be complete. The process absorption of a rotating particle with excitation of the detector has always non null probability. In the asymptotic limit we have that if

$$R_{21}+A_{21} > 3R_{12},$$

the lower energy state is prefered ($1.2 < g < 2$, for the Landé-g factor), and if

$$R_{21}+A_{21} < 3R_{12},$$

the higher energy state is prefered ($0 < g < 1,2$ for the Landé-g factor).

We remark that the results that the polarization can not be complete was obtained in a very crude approximation where the rates $R_{1\to 2}(E,\Delta T)$, $R_{2\to 1}(E,\Delta T)$ and $A_{2\to 1}(E,\Delta T)$ does not depend on time (see eq.(53) and eq.(54)). A more realistic result can be obtained assuming that this rates does depend on time. Defining $n_1 = N_1/N$ and $n_2 = N_2/N$ and also

$$\rho(E)R_{2\to 1}(E,\Delta T) + A_{2\to 1}(E,\Delta T) = \sigma_{21}(t)$$

and

$$\rho(E)R_{1\to 2}(E,\Delta T) = \sigma_{12}(t)$$

we obtain the following equations:

$$n_1(t) + n_2(t) = 1 \tag{100}$$

and

$$\frac{dn_1(t)}{dt} + n_1(t)\left[\sigma_{12}(t)+\sigma_{21}(t)\right] = \sigma_{21}(t) \tag{101}$$

Let us consider the homogeneous linear equation:

$$\frac{dn_1^{(0)}(t)}{dt} = n_1^{(0)}(t)\left[\sigma_{12}(t)+\sigma_{21}(t)\right] = 0 \tag{102}$$

A general solution is

$$n_1^{(0)}(t) = C_1 e^{-\int^t [\sigma_{12}(t')+\sigma_{21}(t')]dt'}. \tag{103}$$

Now let us substitute in the non-homogeneous equation the expression

$$n_1(t) = v(t) e^{-\int^t [\sigma_{12}(t')+\sigma_{21}(t')]dt'}. \tag{104}$$

The equation for $v(t)$ becomes

$$\frac{dv(t)}{dt} e^{-\int_0^t [\sigma_{12}(t')+\sigma_{21}(t')]dt'} = \sigma_{21}(t) \tag{105}$$

consequently we have

$$v(t) = C_2 + \int^t dt'\sigma_{21}(t')e^{\int^{t'}[\sigma_{12}(t'')+\sigma_{21}(t'')]dt''}. \tag{106}$$

The general solution that we are looking for involves two quadratures and it is given by

$$n_1(t) = C_2 e^{-\int^t[\sigma_{12}(t')+\sigma_{21}(t')]dt'} + e^{-\int^t[\sigma_{12}(t')+\sigma_{21}(t')]dt'} \int^t dt'\sigma_{21}(t')e^{\int^{t'}[\sigma_{12}(t'')+\sigma_{21}(t'')]dt''}. \tag{107}$$

With the values of $R_{2\to1}(E,\Delta T)$, $R_{1\to2}(E,\Delta T)$ and $A_{2\to1}(E,\Delta T)$, it is possible to find the degree of polarization.

We would like to point out that there is a diferent approach to study these problems. As it has been pointed out by Milonni and Smith [55] and Ackerhalt, Knight and Eberly [56], it is possible to study radiative processes without using perturbation theory, but using the Heisenberg equations of motion. In this approach it is possible to obtain non-perturbative expressions for radiative processes where the radiation reaction appears in a very simple way: the part of the field due to the atom (detector) that drives the Dicke operators [53]. In this approach it is possible to identify the role of radiation reaction and vacuum fluctuations in spontaneous emission. We would like to stress the fact that the contribution of vacuum fluctuations and radiation reaction can be chosen arbitrarily, depending on the order of the Dicke and field operators. As it was discussed by Dalibard, Dupont-Roc and Cohen-Tannoudji [57], there is a preferred ordering in such a way that the vacuum fluctuations and radiation reaction contribute equally to the spontaneous emission process. More recently this approach was developed by Audretsch and Muller, Audretsch, Mensky and Muller and also Audretsch, Muller and Holzmann [58] to study the Unruh-Davies effect. These authors constructed the following picture of the Unruh-Davies effect. The effect of vacuum fluctuations is changed by the acceleration, although the contribution of radiation reaction is unaltered. Due to the modified vacuum fluctuation contribution, transition to an excited state becomes possible even in the vacuum. It will be interesting to use this formalism to study the rotating detector.

5 Conclusions

In this paper we discuss the relativistic problem of uniform rotation and how this question is related with the puzzle of the rotating detector. After this we discuss the response function of a particle detector traveling in different world lines interacting with a scalar field prepared in two different vacua: the Minkowski and the rotating vacuum. For electrons in storage rings, a residual depolarization has been found experimentally. Bell and Leinaas investigate this effect using the idea of circular Unruh-Davies effect. We propose a alternative solution to the rotating detector puzzle and how this will be related with depolarization effects in circular accelerators.

Let use the result that the probability of transition per unit proper time depends not only of the world line of the "atom" but also the particular vacuum in which we prepare the field to study four different situations:

i) The response function of an inertial detector interacting with the field in the Minkowski vacuum;

ii) The response function of the rotating detector interacting with the field in the Minkowski vacuum;

iii) The response function of an inertial detector interacting with the field in the rotating vacuum;

iv) The response function of the rotating detector interacting with the field in the rotating vacuum.

The same kind of analysis in a different situation was given by Pinto Neto and Svaiter [59]. The case (i) gives the usual result that an inertial detector in its ground state interacting with the field in the Minkowski vacuum does not excite. It is clear that the situation (iv) will give the same result. The case (ii) can be produced in a laboratory. The case (iii) is more involved. How to produce the rotating vacuum? A possible solution is to use the ideas developed by Denardo and Percacci [31] and also Manogue [60]. This second author consider the case of rotating boundaries to push the vacuum around. Note that we are dealing with a Casimir rotating vacuum. Is it possible to create some kind of rotating vacuum? If the answer is positive we conjecture that the situation (iii) will give the same response function as situation (ii).

It will be of interest to explore the consequences of this paper, in particular to examinate some interesting astrophysical situations. For example, the origin of non-thermal radio-frequency in the Universe can be explained by the mechanism of synchroton radiation? [61]. Some authors discussed the metric of a spinning cosmic string [62]. We conjecture that electrons and positrons in the neighbourhood of such objects must emit synchroton radiation. On the same grounds we conjecture that any rotating astrophysical object (sppining pulsars for example [63]) with a cloud of electrons and positrons is a source of synchroton radiation. We can attempt to justify our conjecture using the well known result that the radiation emited by a pulsar has a high degree of polarization. This fact suggest that the mechanism is similar to the one that generates synchroton radiation.

Before finish we would like to made some coments concerning the Sagnac's effect. This is the optical analogue of the Foulcault pendulum. In the Sagnac's experiment the apparatus device rotates, and the optical experiment can determine the rotation of the frame relative to an inertial frame. This shows the diference between inertial and the rotating (non-inertial) frame. For inertial frames it is impossible to determine the absolute velocity of the apparatus. In the case of the rotating frame the angular velocity can be obtatined. Our criticism of this scheme is the following: to measure the proper spatial line element (in the rotating frame) we have to measure the time taken by the light signal between an emision and also absorption from atoms. The connection with the detector puzzle shows how is intricate the analysis. We conclude that the Sagnac's effect can detect the angular velocity of the apparatus but not conclude that the mapping that connect inertial and the non-inertial frame is the galilean one.

In conclusion, in this paper we have attemp to show that the Minkowski and a rotating vacuum are not the same. Although the Bogoliubov coefficients $\beta_{k\nu}$ between the inertial and the non-inertial modes are non-zero it is very difficult to calculate them. We are forced to admit that we fail to finish our interprize since we meet a basic difficulty to calculate the number of rotating particles in the Minkowski vacuum. Is it possible to go further?

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Appendix A

In this appendix we will present the solution of Eq. (46):

$$\left[\frac{\partial^2}{\partial r^2} + (\frac{1}{r} - 4i\omega\Omega\theta r)\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + (q^2 - 4i\omega\Omega\theta)\right] f_\mu(r,\theta) = 0.$$

Let us define $g(r,\theta)$ by the following equation:

$$f_\mu(r,\theta) = e^{i\mu\theta} g_\mu(r,\theta).$$

A direct substitution gives the equation for $g(r,\theta)$:

$$\left[\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{\mu^2}{r^2} + q^2\right) + \frac{1}{r^2}\left(\frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right) - \sigma\theta\left(r\frac{\partial}{\partial r} + 1\right)\right] g_\mu(r,\theta) = 0,$$

where $\sigma = 4i\omega\Omega$. Define the new quantity $y = qr$ and $l = \sigma/q^2$ the equation becomes

$$\left[\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y} - \frac{\mu^2}{y^2} + 1\right) + \frac{1}{y^2}\left(\frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right) - l\theta\left(y\frac{\partial}{\partial y} + 1\right)\right] g_\mu(y,\theta) = 0.$$

There appear to be no way of solve the above equation exactly. Consequently let us try a perturbative solution given by

$$g_\mu(y,\theta) = J_\mu(y) + \sum_{k=1}^{\infty} l^k P_\mu^{(k)}(y,\theta).$$

By considering only the first order term in the above expansion and for simplicity using the notation $P_\mu^{(1)}(y,\theta) \equiv P_\mu(y,\theta)$ we obtain:

$$\left(\frac{\partial^2}{\partial y^2} + \frac{1}{y}\frac{\partial}{\partial y} - \frac{\mu^2}{y^2} + 1\right) P_\mu(y,\theta) + \frac{1}{y^2}\left(\frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right) P_\mu(y,\theta) - \theta\left(y\frac{\partial}{\partial y} + 1\right) J_\mu(y) = 0.$$

Defining

$$\frac{1}{2}y^3 J_{\mu-1}(y) + y^2 J_\mu(y) - \frac{1}{2}y^3 J_{\mu+1}(y) = h(y),$$

we get:

$$\left[\left(y^2\frac{\partial^2}{\partial y^2} + y\frac{\partial}{\partial y} - \mu^2 + y^2\right) + \frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right] P_\mu(y,\theta) = \theta h(y).$$

It is possible to use the Green's functions method to find the general solution for $P_\mu(y,\theta)$. Thus,

$$P_\mu(y,\theta) = P_\mu^{(0)}(y,\theta) + \int d\theta' \int dy' G(y,\theta;y',\theta')\theta' h(y'),$$

where $P_\mu^{(0)}(y,\theta)$ is the solution of the homogeneous equation, and $G(y,\theta;y',\theta')$ satisfy

$$\left[\left(y^2\frac{\partial^2}{\partial y^2} + y\frac{\partial}{\partial y} - \mu^2 + y^2\right) + \frac{\partial^2}{\partial\theta^2} + 2i\mu\frac{\partial}{\partial\theta}\right] G(y,\theta;y',\theta') = \delta(y-y')\delta(\theta-\theta').$$

It is straightforward to find the solution of the homogeneous equation using separation of variables method defining:

$$P_\mu^{(0)}(y,\theta) = e^{i\lambda\theta} Q_\mu^{(0)}(y).$$

Then,

$$Q_\mu^{(0)}(y) = J_{\mu+\lambda}(y).$$

Finally the general solution is given by:

$$\begin{aligned} f(y,\theta) &= e^{i\mu\theta}\left[J_\mu(y) + l e^{i\lambda\theta} J_{\mu+\lambda}(y)\right] \\ &+ \frac{l}{2}\int d\theta' \int dy' G(y,\theta;y',\theta')\theta' \left[y'^3 J_{\mu-1}(y') + 2y'^2 J_\mu(y') - y'^3 J_{\mu+1}(y')\right] \end{aligned}$$

Appendix B

An orthonormal set is defined through a scalar product in the vector space of the solutions of some equation of motion. In the case of Klein-Gordon field this scalar product is Hermitian but not positive definite. Let be $f(x)$ and $g(x)$ two elements of F, where F is the vector space of the solutions of the Klein-Gordon equation with the scalar product defined by

$$(f,g) = -i\int_\Sigma \sqrt{-g}\, d\Sigma^\mu \left[f(x)(\partial_\mu g^*(x)) - (\partial_\mu f(x))g^*(x)\right]$$

where $d\Sigma^\mu = \eta^\mu d\Sigma$ with η^μ a future directed unit vector orthogonal to the space-like hypersurface Σ and $d\Sigma$ is the volume element in Σ. An orthonormal set (u_k, u_k^*) is said to be complete if every solution $f(x)$ of F can be written as

$$f(x) = \sum_k a_k u_k(x) + b_k u_k^*(x)$$

where the coeficients a_k and b_k are given by

$$a_k = (u_k, f)$$

and

$$b_k = -(u_k^*, f).$$

Let G be a subset of F. If(v_j, v_j^*) and (u_i, u_i^*) are two orthonormal sets such that the expand every element of G, then they are called equivalents. In this case

$$v_j(x) = \sum_i \alpha_{ji} u_i(x) + \beta_{ji} u_i^*(x)$$

and

$$u_i(x) = \sum_j \alpha_{ji}^* v_j(x) - \beta_{ji} v_j^*(x).$$

They are said to be complete only if $F = G$. The quantum field $\varphi(x)$ can be expanded using either of the two complete sets (u_i, u_i^*) or (v_j, v_j^*) that would lead to two diferent vacua $|0>$ and $|0'>$ respectively. When $\sum_{ij} |\beta_{ij}|^2$ converges, the representations are said to be unitarily equivalent. If it diverges they are non-unitarily equivalents and they are not related to any unitary operator in the Fock space. For a very interesting introduction to this subject see for example the Miransky [64] and also the Umezawa book [65].

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