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Dimensional Transmutation and Symmetry Breaking in Maxwell-Chern-Simons Scalar QED

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Abstract

The mechanism of dimensional transmutation is discussed in the context of Maxwell-Chern-Simons scalar QED. The method used is non-perturbative. The effective potential describes a broken symmetry state. It is found that the symmetry breaking vacuum is more stable when the Chern-Simons mass is different from zero..

Key-words: Effective potential; Maxwell-Chern-Simons scalar QED.

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1 Introduction

The idea of spontaneous symmetry breaking allows one to describe a wide class of phenomena in both condensed matter and particle physics. In condensed matter physics it furnishes a good phenomenological description of many interesting phenomena. For instance, the magnetic materials and superconduncting compounds are among the physical systems susccessfully described by the broken symmetry picture [1]. The case of superconductivity is of particular interest since it involves a local gauge symmetry. Indeed, the idea of spontaneously broken gauge theories have its origin in superconductivity theory [2]. Phenomenologically, it is well described by a Landau-Ginsburg action with the charged scalar field minimally coupled to Abelian gauge fields. This is just the action of scalar QED. In order to describe a broken symmetry solution, an imaginary mass is attributed to the scalar field. Thus, the gauge field acquire mass through the Higgs mechanism [3]. This is the origin of the well known Meissner effect. The Higgs mechanism was shown to give also a good description of particle physics phenomenology through the so called standard model of elementary particles. In fact, it is on the basis of the experimentally tested electroweak theory. According to the standard model, the Higgs mechanism is responsible for the masses of the vector bosons, the $W^{\pm}$ and Z^0.

Another path to a broken symmetry state is the Coleman-Weinberg mechanism [4]. The Coleman-Weinberg mechanism consists in induce the symmetry breaking via radiative corrections. Thus, it is not attributed an imaginary mass to the scalar particle. Instead, the renormalized mass is zero and the symmetry breaking is not manifest at the tree level as is the case of the Higgs mechanism. The Coleman-Weinberg mechanism is very useful in the construction of grand unified theories and in cosmological models [5]. A very interesting feature of this method is the phenomenon of dimensional transmutation. Dimensional transmutation occurs as a consequence of the breaking of the symmetry. It consists of a reduction in the number of dimensionless couplings which are replaced by corresponding dimensionful parameters. For example, in the case of massless scalar $(QED)_4$ the theory has two dimensionless couplings, namely, e^2 and λ. Here, e^2 correspond to the electromagnetic coupling while λ is the coupling of the scalar particle self-interaction. The symmetry breaking through the Coleman-Weinberg mechanism allows the elimination of the scalar self-coupling in favour of the electromagnetic coupling. This is done at the price of the introduction of a dimensionful parameter in replacement of λ. It turns out that this parameter corresponds to the vacuum expectation value of the scalar field. Thus, we have just two parameters as before but only one is dimensionless.

In order to the phenomenon of dimensional transmutation takes place it is necessary the presence of at least one dimensionless parameter in the theory. This means that we cannot obtain a similar situation in scalar $(QED)_3$ if we restrict ourselves to a $(\phi^\dagger\phi)^2$ interaction in the scalar sector. Therefore, it is necessary the inclusion of a $(\phi^\dagger\phi)^3$ interaction. The resulting coupling will be dimensionless and the theory is renormalizable rather than super-renormalizable. Another important point concerning tridimensional QED is that it admits the inclusion of a Chern-Simons term [6]. With all these terms collected we can build the more general renormalizable scalar QED in $d = 3$.

On the basis of the above discussion we can legitimately ask the following question. Is it possible to implement the Coleman-Weinberg mechanim in a renormalizable Maxwell-

Chern-Simons QED? It is the main aim of this paper to show that the answer to this question is affirmative. In order to achieve this goal we will use a simple non-perturbative approach. Recently the Coleman-Weinberg mechanism in massless scalar $(QED)_4$ was studied non-perturbatively [7]. It is shown that the one-loop result of Coleman and Weinberg can be established beyond the range of validity of perturbation theory. For definiteness we will rederive briefly the Coleman-Weinberg effective potential in the approach of ref.[7]. This will help to fix the ideas and will be done in section 2. In section 3 we use the method of section 2 to obtain the effective potential for the Maxwell-Chern-Simons scalar QED. We use the more general renormalizable action which means that a $(\phi^\dagger\phi)^3$ interaction term is included. We establish then the symmetry breaking and dimensional transmutation in the massless case. Finally, we discuss the results in section 4. In this paper we use $\hbar = c = 1$.

2 Warm up: scalar $(QED)_4$

The aim of this section is mainly to introduce the method that will be used in the next section. The essence of the method is that it is almost a tree level manipulation. It is worth to point out that the Higgs mechanism works at the tree level if one chooses the unitary gauge. In this case symmetry breaking is manifest at tree level. We will make almost the same thing with respect to the Coleman-Weinberg mechanism. This is achieved by noting that in an Abelian theory the action is quadratic in the gauge fields allowing a straightforward Gaussian integration. This nice feature is also a weakness of our method since it is not possible to generalize the procedure to non-Abelian gauge fields. Just like in the case of the Higgs mechanism we will find convenient to work in the unitary gauge. The unitary gauge parametrization is obtained by integrating out exactly the gauge freedom. The scalar sector is then rewritten in terms of the real scalar field $\rho(x)$ where $\rho^2 = \phi_1^2 + \phi_2^2$ where ϕ_1 and ϕ_2 are respectively the real and imaginary parts of the field ϕ. Thus, after straightforward integration of the gauge fields one obtains the following Euclidean effective action [7]:

$$
\begin{aligned}
S_{eff}[\rho] &= \frac{1}{2}\ln\det[\delta_{\mu\nu}(-\Box + e^2\rho^2) + \partial_\mu\partial_\nu] - \delta^4(0)\int d^4x \ln(e\rho) \\
&\quad + \int d^4x \left[\frac{1}{2}\rho(-\Box + m^2)\rho + \frac{\lambda}{4!}\rho^4\right]. \qquad (1)
\end{aligned}
$$

This is an exact expression. The factor $-\delta^4(0)\int d^4x \ln(e\rho)$ arises from the exponentiation of the Jacobian $\det(e\rho)$ which occurs in the functional measure as a result of the unitary gauge parametrization. Looking for a constant saddle point $<\rho>$ we find

$$
\int d^4x \left(m^2 <\rho> + \frac{\lambda}{3!} <\rho>^3 - \frac{\delta^4(0)}{<\rho>}\right) + e^2 <\rho> TrD_{\mu\nu}(x - x') = 0, \qquad (2)
$$

where $D_{\mu\nu}(x - x')$ is the propagator of the massive vector field with mass $e^2 < \rho >^2$. By evaluating explicitly the trace of the propagator we obtain an exact cancellament of the divergent factor proportional to $\delta^4(0)$. The solution to Eq.(2) that will concern us consists of the gap equation:

$$< \rho >^2 = -\frac{6m^2}{\lambda} - \frac{18e^2}{\lambda}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+e^2<\rho>^2}. \tag{3}$$

After evaluation of the integral above using a cutoff Λ the gap equation becomes

$$\lambda_R < \rho >^2 = -6m_R^2 - \frac{9e^4}{8\pi^2} < \rho >^2 \ln\frac{e^2<\rho>^2}{\mu^2}, \tag{4}$$

where we have defined the renormalized parameters:

$$m_R^2 = m^2 + \frac{3e^2}{16\pi^2}\Lambda^2, \tag{5}$$

$$\lambda_R = \lambda + \frac{9e^4}{8\pi^2}\ln\frac{\mu^2}{\Lambda^2}. \tag{6}$$

In above μ is an arbitrary renormalzation scale. Now we demand that all stationary points are solutions to Eq.(4). A broken symmetry solution corresponds to a local maximum at the origin and two degenerate absolute minima of the effective potential. The solution corresponding to the local maximum $<\rho>_{max}=0$ is a solution to Eq.(3) only if $m_R^2=0$. Now, let the solution corresponding to the minimum be given by $<\rho>_{min}=\sigma$. We obtain that

$$\lambda_R = -\frac{9e^4}{8\pi^2}\ln\frac{e^2\sigma^2}{\mu^2}. \tag{7}$$

By considering a x-independent background field $\overline{\rho}$ in the expression for the effective action, Eq.(1), and evaluating explicitly the logarithm of the determinant we obtain the following expression for the effective potential:

$$V(\overline{\rho}) = \frac{\lambda_R}{24}\overline{\rho}^4 + \frac{3e^4}{64\pi^2}\overline{\rho}^4\ln\frac{e^2\overline{\rho}^2}{\mu^2} - \frac{3e^4}{128\pi^2}\overline{\rho}^4, \tag{8}$$

where we have assumed that $m_R^2=0$. Substituting Eq.(7) in Eq.(8) one obtains

$$V(\overline{\rho}) = \frac{3e^4}{64\pi^2}\overline{\rho}^4\left(\ln\frac{\overline{\rho}^2}{\sigma^2} - \frac{1}{2}\right). \tag{9}$$

Eq.(9) is just the Coleman-Weinberg potential [4]. It is important to stress that we established non-perturbatively the one-loop result of ref.[4]. With the usual perturbative scheme it is assumed that $\lambda_R \sim e^4$ and corrections $\sim \lambda_R^2$ are neglected. This assumption is not necessary here. Note that we rederive the Coleman-Weinberg result in a mean field like approximation with the help of a gap equation. The result is obtained as the 'tree level' of the effective action, Eq.(1). Note also that the derivation given here is not restricted to a small value of e^2.

3 Maxwell-Chern-Simons scalar QED

In this section we use the method of the previous section applied to the case of the Maxwell-Chern-Simons scalar QED. We work in the unitary gauge as in the last section. The Euclidean effective action resulting from the exact integration of the vector fields is given by

$$S^{CS}_{eff}[\rho] = \frac{1}{2}\ln\det[\delta_{\mu\nu}(-\Box + e^2\rho^2) + \partial_\mu\partial_\nu + i\theta\epsilon_{\mu\lambda\nu}\partial_\lambda] - \delta^3(0)\int d^3x \ln(e\rho) + \int d^3x\left[\frac{1}{2}\rho(-\Box + m^2)\rho + \frac{\lambda}{4!}\rho^4 + \frac{\eta}{6!}\rho^6\right]. \quad (1)$$

In above, θ is the Chern-Simons mass and η is a dimensionless coupling. Note that in $d = 3$ the parameters e^2 and λ have dimension of mass. Eq.(1) corresponds to the more general renormalizable action. Without the term ρ^6 the action would be super-renormalizable and we would have only dimensionful couplings. The presence of at least one dimensionless coupling is crucial for the phenomenon of dimensional transmutation. Otherwise, there is nothing to transmute.

As before, we look for a constant saddle point solution to the effective action. Then, stationarity of the action with respect to this saddle point implies the following gap equation:

$$e^2\left(1 + \frac{|\theta|}{\sqrt{\theta^2 + 4e^2 < \rho >^2}}\right)\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2 + M^2_+(< \rho >^2)} + e^2\left(1 - \frac{|\theta|}{\sqrt{\theta^2 + 4e^2 < \rho >^2}}\right)\int\frac{d^3p}{(2\pi)^3}\frac{1}{p^2 + M^2_-(< \rho >^2)} + m^2 + \frac{\lambda}{6} < \rho >^2 + \frac{\eta}{120} < \rho >^4 = 0, \quad (2)$$

where the $M^2_\pm$ are defined by

$$M^2_\pm(< \rho >^2) = e^2 < \rho >^2 + \frac{\theta^2}{2} \pm \frac{|\theta|}{2}\sqrt{\theta^2 + 4e^2 < \rho >^2}. \quad (3)$$

By using an ultraviolet cutoff Λ it is straightforward to compute the integrals in Eq.(2). The gap equation becomes

$$-\frac{e^2}{4\pi}|M_+(< \rho >^2)|\left(1 + \frac{|\theta|}{\sqrt{\theta^2 + 4e^2 < \rho >^2}}\right) - \frac{e^2}{4\pi}|M_-(< \rho >^2)|\left(1 - \frac{|\theta|}{\sqrt{\theta^2 + 4e^2 < \rho >^2}}\right) + \frac{\lambda}{6} < \rho >^2 + \frac{\eta}{120} < \rho >^4 = 0. \quad (4)$$

We have assumed, just as in the previous section, that the renormalized mass is zero.

The effective potential is obtained from Eq.(1) and is given by

$$V(\overline{\rho}) = -\frac{1}{12\pi}[|M_+(\overline{\rho}^2)|^3 + |M_-(\overline{\rho}^2)|^3] + \frac{\lambda}{4!}\overline{\rho}^4 + \frac{\eta}{6!}\overline{\rho}^6. \tag{5}$$

Note that, in contrast to the calculations performed in the previous section, no renormalization scale arises here. This is a special feature of the $d = 3$ case.

Let us consider the solution to the gap equation associated to the symmetry breaking minimum of the potential, $<\rho>_{min}=\sigma$. Solving Eq.(4) for η and substituting the result in Eq.(5) we get

$$\begin{aligned} V(\overline{\rho}) \;=\; & -\frac{1}{12\pi}[|M_+(\overline{\rho}^2)|^3 + |M_-(\overline{\rho}^2)|^3] + \frac{\lambda}{24}\overline{\rho}^4 \\ & +\frac{1}{12\sigma^2}\left\{\frac{e^2}{2\pi\sigma^2}\left[|M_+(\sigma^2)|\left(1+\frac{|\theta|}{\sqrt{\theta^2+4e^2\sigma^2}}\right)\right.\right. \\ & \left.\left. +|M_-(\sigma^2)|\left(1-\frac{|\theta|}{\sqrt{\theta^2+4e^2\sigma^2}}\right)\right] - \frac{\lambda}{3}\right\}\overline{\rho}^6. \end{aligned} \tag{6}$$

Note that dimensional transmutation has occurred. We had before four parameters, θ, e^2, λ and η, the parameter η being dimensionless. Now, we remain with the same number of parameters but the dimensionless parameter η has disappeared and has been replaced by the parameter σ^2, which has the dimension of mass.

The above effective potential corresponds to a broken symmetry phase with one local maximum at $<\rho>_{max}=0$ and with two degenerate absolute minima, at $\pm\sigma$. If $\lambda > 0$ we have that the stability condition $\lambda \leq \lambda_c$ must holds, where the critical parameter λ_c is given by

$$\begin{aligned} \lambda_c \;=\; & \frac{3e^2}{2\pi\sigma^2}\left[|M_+(\sigma^2)|\left(1+\frac{|\theta|}{\sqrt{\theta^2+4e^2\sigma^2}}\right)\right. \\ & \left. +|M_-(\sigma^2)|\left(1-\frac{|\theta|}{\sqrt{\theta^2+4e^2\sigma^2}}\right)\right]. \end{aligned} \tag{7}$$

If $\lambda > \lambda_c$ the potential is unbounded below and the vacuum is unstable. However, if $\lambda < 0$ no stability condition is necessary because the potential will be always be bounded from below. instability with respect to the value of η, rather than λ, was considered non-perturbatively in ref.[8] in the context of a $O(N)$ symmetric ϕ_3^6 theory. It was found that the vacuum is unstable for $\eta > \eta_c$, η_c being the critical value of η. Here we have a similar situation. The vacuum is unstable for $\lambda > \lambda_c$. The stability condition (7) implies that in the pure scalar limit $e^2 = \theta = 0$ we must have necessarily $\lambda \leq 0$. The limit $\lambda = \lambda_c$ corresponds to a Maxwell-Chern-Simons theory without the term $(\phi^\dagger\phi)^3$. This situation with $\theta = 0$ was already studied from a perturbative point of view [9]. The authors of ref.[9] argued that the symmetry breaking obtained by the one-loop result is spurious. Their calculations were also performed at the critical point $m_R^2 = 0$. In this case the radiative corrections failed in induce the symmetry breaking because of the absence of the $(\phi^\dagger\phi)^3$ term and, therefore, of a dimensionless coupling. Here we have made a pseudo-tree-level analysis and, therefore, our result correspond to the leading contribution to the effective potential. For this reason, we argue that the fluctuations cannot turn the asymmetric phase we found into a symmetric one. Our picture of the Coleman-Weinberg mechanism is similar to the case of the Higgs mechanism where formal manipulations performed at the tree level in the unitary gauge produce a broken symmetry state. Since the Coleman-Weinberg mechanism is based on the idea that quantum fluctuations may induce symmetry breaking, we must perform the tree level analysis in an effective field theory, described in the case $d = 4$ by the effective action, Eq.(1), and in the case $d = 3$ by the effective action, Eq.(1).

A further feature of the effective potential given by Eq.(6) is that $V(0) = 0$ only if $\theta = 0$. If $\theta \neq 0$ we have that $V(0) < 0$. The scalar field ρ have the same vacuum expectation value for all values of θ. However, the vacuum energy is lower in the case $\theta \neq 0$ than in the case $\theta = 0$. This means that the renormalizable (that is, with the term $(\phi^\dagger\phi)^3$ included) Maxwell-Chern-Simons scalar QED is more stable than the non-topological scalar QED in $d = 3$.

Let us compare our results with the one-loop calculation. The one-loop effective potential is given by

$$\begin{aligned} V(\overline{\rho}) &= -\frac{1}{12\pi}[M_+^3(\overline{\rho}^2) + M_-^3(\overline{\rho}^2)] - \frac{1}{12\pi}\left(\frac{\lambda}{2}\overline{\rho}^2 + \frac{\eta}{24}\overline{\rho}^4\right)^{3/2} \\ &\quad + \frac{\lambda}{4!}\overline{\rho}^4 + \frac{\eta}{6!}\overline{\rho}^6. \end{aligned} \tag{8}$$

Here, η is an independent parameter, that is, it is not written in terms of the other parameters. When $\eta = \theta = 0$, Eq.(8) agrees with ref.[9]. The situation which $\eta = 0$ but $\theta \neq 0$ corresponds in our case to $\lambda = \lambda_c$. In this case the one-loop result have a term which is absent in our result, a term proportional to $\lambda^{3/2}|\overline{\rho}|^3$. However, in the loop expansion given here λ is not written as a function of other parameters of the theory, as in Eq.(7). Moreover, if $\theta = 0$, λ_c is given by

$$\lambda_c = \frac{3e^3}{\pi|\sigma|}. \tag{9}$$

Thus, it seems that in contrast to the case $d = 4$ treated in the previous section, here the one-loop result is not reproduced. However, it is important to remember that the one-loop computation of Coleman and Weinberg neglects terms proportional to λ_R^2. It was argued that under the plausible hypothesis that $\lambda_R \sim e^4$, terms proportional to e^4 and λ_R correspond to the leading contributions. The non-perturbative approach used in the previous section confirm this hypothesis. Therefore, the $d = 4$ result does not agree exactly with the complete one-loop result. In fact, it agrees with the Coleman-Weinberg expression of the one-loop result, which neglects some few terms. The same thing happens in $d = 3$. In order to have some insight with respect to the order of magnitudes neglected, let us consider for simplicity the case $\eta = \theta = 0$. In our approach we have that the quartic self-coupling is given by Eq.(9). According to the one-loop result, a term $\sim e^{9/2}$ is being neglected.

4 Discussion

The results of the previous sections show that the phenomenon of dimensional transmutation can be established outside of the perturbative framework. The Coleman-Weinberg one-loop result for massless scalar $(QED)_4$ can be obtained non-perturbatively. It has been shown recently that this approach allows a description of the Coleman-Weinberg mechanism free from Landau ghost singularities [7]. The Landau ghost singularity is frequently associated to a trivial behavior of the theory. The one-loop renormalization group analysis shows that the running coupling constant have this problem [4]. In fact, it diverges for finite momenta. The result of section 2 is obviously non-perturbative and this trouble does not occur. The Landau ghost seems to be an artifact of perturbation theory.

The case of the Maxwell-Chern-Simons scalar QED was treated using the prescription of section 2. We found the phenomenon of dimensional transmutation and thence symmetry breaking. Of course, this result was obtained at the expense of the solution to a gap equation. It is well known that mean field theories are characterized by gap equations. Also, mean field theories have the tendency to produce phase transitions. Therefore, it is not surprising that a broken symmetry solution result from our computations. We use the gap equations to eliminate the dimensionless couplings in the previous sections. What has been done is just like mean field theories. Note that mean field theories are necessarily non-perturbative. However, it is not rare mean field theories produce spurious results. A classical example is the Landau-Ginsburg theory for the Ising ferromagnet [10]. It turns out that the mean field result reproduce the correct critical indices only for $d > 4$. For $d < 4$ its predictions are completely wrong. In fact, it predicts that symmetry breaking does occur in $d = 1$ while it is known from the exact $d = 1$ solution that spontaneous magnetization is absent. The case $d = 2$ have exact solution [11] and the Landau-Ginsburg theory disagrees with this exact result although we have spontaneous magnetization in this case. However, the Landau-Ginsburg theory of superconductivity gives very good phenomenological results. This is because the microscopic theory of superconductivity is itself a mean field theory, the BCS theory [12]. Indeed, the Landau-Ginsburg theory of superconductivity can be derived from the BCS theory [13]. The Landau-Ginsburg theory of superconductivity is just the Higgs mechanism applied to scalar QED. We have discussed another path to this theory via the Coleman-Weinberg mechanism. We give to the Coleman-Weinberg mechanism the status of mean field theory. The Maxwell-Chern-Simons scalar QED may be viewed as a phenomenological theory to high temperature superconductors [14]. It can be shown that this system exibits also vortex solutions [15].

Another path to the study of phase transitions is approach the theory by finite temperature field theoretical methods [16]. Recently the finite temperature technique has been applied to the study of the Maxwell-Chern-Simons scalar QED [17]. It was found that a phase transition does in fact occur for an infinitesimally small positive mass. Therefore, it seems that the transition is of the Coleman-Weinberg type. It has been concluded in ref.[17] that it is incorrect to assume that the scalar field have an imaginary mass in order to describe symmetry breaking in this system. This is a characteristic of the $d = 3$ case. It is more consistent to break the symmetry in $d = 3$ via the Coleman-Weinberg mechanism than that with the Higgs machanism which needs an imaginary mass introduced by hand.

Summarizing, we have obtained the phenomenon of dimensional transmutation in Maxwell-Chern-Simons scalar QED. In addition, we obtained that the vacuum is more stable for $\theta \neq 0$ than that for $\theta = 0$. It is important to stress that the approach used in this paper is non-perturbative. Indeed, it is a tree level analysis of an effective theory obtained through an exact integration of the gauge fields. The finite temperature case is under investigation. An important question concerns the order of the phase transition.

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