

Algebraic Renormalization of Parity-Preserving QED₃ Coupled to Scalar Matter II: Broken Case

by

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ABSTRACT

In this letter the algebraic renormalization method, which is independent of any kind of regularization scheme, is presented for the parity-preserving QED₃ coupled to scalar matter in the broken regime, where the scalar assumes a finite vacuum expectation value, $\langle\varphi\rangle=v$. The model shows to be stable under radiative corrections and anomaly free.

Key-words: QED₃; Algebraic renormalization; Broken symmetry.

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In the present letter, the model proposed in ref. [1] is renormalized, in the broken regime (where the scalar assumes a nonvanishing vacuum expectation value), by using the algebraic renormalization method [2, 3, 4]. This algebraic approach is based on the BRS-formalism [2] together with the Quantum Action Principle [5], which leads to a regularization independent scheme. The stability of the model under radiative corrections is analyzed as well as the possible presence of anomalies. The algebraic renormalization of the model in the symmetric regime was presented in ref. [6].

The gauge invariant action for the parity-preserving QED₃ coupled to scalar matter [1, 6] is given by

$$\begin{aligned} \Sigma_{\text{inv}} = \int d^3x \left\{ -\frac{1}{4} F^{mn} F_{mn} + i\bar{\psi}_+ \not{D}\psi_+ + i\bar{\psi}_- \not{D}\psi_- - m_0(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-) + \right. \\ \left. - y(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-)\varphi^*\varphi + D^m\varphi^*D_m\varphi - \mu^2\varphi^*\varphi - \frac{\zeta}{2}(\varphi^*\varphi)^2 - \frac{\lambda}{3}(\varphi^*\varphi)^3 \right\} , \end{aligned} \quad (1)$$

where the mass dimensions of the parameters m_0 , μ , ζ , λ and y are respectively 1, 1, 1, 0 and 0.

The covariant derivatives are defined as follows :

$$\not{D}\psi_{\pm} \equiv (\not{\partial} + iqq\not{A})\psi_{\pm} \quad \text{and} \quad D_m\varphi \equiv (\partial_m + iQgA_m)\varphi , \quad (2)$$

where g is a coupling constant with dimension of (mass)^½ and q and Q are the $U(1)$ -charges of the fermions and scalar, respectively. In the action (1), F_{mn} is the usual field strength for A_m , ψ_+ and ψ_- are two kinds of fermions (the \pm subscripts refer to their spin sign [7]) and φ is a complex scalar.

The form of the potential is chosen such as to ensure the broken regime, where $\langle\varphi\rangle=v$. Imposing that it must be bounded from below and yields only stable vacua, we get the following conditions on the parameters :

$$\lambda > 0 , \quad \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3}{16} \frac{\zeta^2}{\lambda} . \quad (3)$$

The vacuum expectation value for the $\varphi^*\varphi$ -product, v^2 , is chosen as the solution

$$\langle\varphi^*\varphi\rangle = v^2 = -\frac{\zeta}{2\lambda} + \left[\left(\frac{\zeta}{2\lambda} \right)^2 - \frac{\mu^2}{\lambda} \right]^{\frac{1}{2}} , \quad (4)$$

of the equation

$$\mu^2 + \zeta v^2 + \lambda v^4 = 0 \quad (5)$$

expressing the minimization of the potential. The complex scalar φ is parametrized by

$$\varphi = v + H + i\theta , \quad (6)$$

where θ is the would-be Goldstone boson and H is the Higgs scalar, both with vanishing vacuum expectation values. It should be noticed that the parametrization given by Eq.(6) does not introduce nonrenormalizable interactions, to the contrary of the unitary gauge parametrization [8].

With the parametrization (6), the action (1) reads:

$$\begin{aligned} \Sigma_{\text{inv}} = \int d^3x \left\{ -\frac{1}{4} F^{mn} F_{mn} + i\bar{\psi}_+ \not{D}\psi_+ + i\bar{\psi}_- \not{D}\psi_- - m_0(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-) + \right. \\ \left. - y(\bar{\psi}_+\psi_+ - \bar{\psi}_-\psi_-)((v + H)^2 + \theta^2) + \partial^m H \partial_m H + \partial^m \theta \partial_m \theta + \right. \\ \left. + 2vQgA^m \partial_m \theta + 2QgA^m (H \partial_m \theta - \theta \partial_m H) + Q^2 g^2 A^m A_m ((v + H)^2 + \theta^2) + \right. \\ \left. - \mu^2((v + H)^2 + \theta^2) - \frac{\zeta}{2}((v + H)^2 + \theta^2)^2 - \frac{\lambda}{3}((v + H)^2 + \theta^2)^3 \right\} . \end{aligned} \quad (7)$$

The masses arising from the action (7) for ψ_{\pm} , A_m and H , are respectively given by $m=m_0 + yv^2$, $M_A^2=2v^2Q^2g^2$ and $M_H^2=2v^2(\zeta + 2\lambda v^2)$.

In order to quantize the system (7) one has to add a gauge-fixing action Σ_{gf} – we choose the ξ -gauge – and an action term Σ_{ext} for the coupling of the BRS transformations to external sources :

$$\Sigma_{\text{gf}} = \int d^3x \left\{ B \partial^m A_m + \frac{\xi}{2} B^2 + \bar{c} \square c \right\} , \quad (8)$$

$$\Sigma_{\text{ext}} = \int d^3x \{ \bar{\Omega}_+ s\psi_+ - \bar{\Omega}_- s\psi_- - s\bar{\psi}_+ \Omega_+ + s\bar{\psi}_- \Omega_- + s\theta \Theta + sH \Xi \} \quad . \quad (9)$$

The BRS transformations are given by :

$$\begin{aligned} sH &= -Qc\theta \quad , \quad s\theta = Qc(v + H) \quad , \\ s\psi_{\pm} &= iq_c\psi_{\pm} \quad , \quad s\bar{\psi}_{\pm} = -iq_c\bar{\psi}_{\pm} \quad , \\ sA_m &= -\frac{1}{g}\partial_m c \quad , \quad sc = 0 \quad , \\ s\bar{c} &= \frac{1}{g}B \quad , \quad sB = 0 \quad , \end{aligned} \quad (10)$$

where c is the ghost, \bar{c} is the anti-ghost and B is the Lagrange multiplier field.

The complete action, Σ , we are considering here is

$$\Sigma = \Sigma_{\text{inv}} + \Sigma_{\text{gf}} + \Sigma_{\text{ext}} \quad . \quad (11)$$

The QED₃-action (11) is invariant under the reflexion symmetry, P , whose action on the fields and external sources is fixed as below :

$$\begin{aligned} x_m &\xrightarrow{P} x_m^P = (x_0, -x_1, x_2) \quad , \\ \psi_{\pm} &\xrightarrow{P} \psi_{\pm}^P = -i\gamma^1\psi_{\mp} \quad , \quad \bar{\psi}_{\pm} \xrightarrow{P} \bar{\psi}_{\pm}^P = i\bar{\psi}_{\mp}\gamma^1 \quad , \\ A_m &\xrightarrow{P} A_m^P = (A_0, -A_1, A_2) \quad , \\ \phi &\xrightarrow{P} \phi^P = \phi \quad , \quad \phi = H, \theta, c, \bar{c}, B \quad , \\ \Omega_{\pm} &\xrightarrow{P} \Omega_{\pm}^P = -i\gamma^1\Omega_{\mp} \quad , \quad \bar{\Omega}_{\pm} \xrightarrow{P} \bar{\Omega}_{\pm}^P = i\bar{\Omega}_{\mp}\gamma^1 \quad , \\ \Theta &\xrightarrow{P} \Theta^P = \Theta \quad , \quad \Xi \xrightarrow{P} \Xi^P = \Xi \quad . \end{aligned} \quad (12)$$

The ultraviolet and infrared dimensions, d and r respectively, as well as the ghost numbers, $\Phi\Pi$, and the Grassmann parity, GP , of all fields and sources are collected in Table 1.

	A_m	H	θ	ψ_{\pm}	c	\bar{c}	B	Θ	Ξ	Ω_{\pm}
d	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	2
r	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	0	3	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	2
$\Phi\Pi$	0	0	0	0	1	-1	0	-1	-1	-1
GP	0	0	0	1	1	1	0	1	1	0

Table 1: UV and IR dimensions, d and r , ghost numbers, $\Phi\Pi$, and Grassmann parity, GP .

The BRS invariance of the action is expressed in a functional way by the Slavnov-Taylor identity

$$\begin{aligned} \mathcal{S}(\Sigma) = \int d^3x \left\{ -\frac{1}{g}\partial^m c \frac{\delta\Sigma}{\delta A^m} + \frac{1}{g}B \frac{\delta\Sigma}{\delta \bar{c}} + \frac{\delta\Sigma}{\delta \bar{\Omega}_+} \frac{\delta\Sigma}{\delta \psi_+} - \frac{\delta\Sigma}{\delta \bar{\Omega}_-} \frac{\delta\Sigma}{\delta \psi_-} - \frac{\delta\Sigma}{\delta \Omega_+} \frac{\delta\Sigma}{\delta \bar{\psi}_+} + \frac{\delta\Sigma}{\delta \Omega_-} \frac{\delta\Sigma}{\delta \bar{\psi}_-} + \right. \\ \left. - \frac{\delta\Sigma}{\delta \Theta} \frac{\delta\Sigma}{\delta \theta} - \frac{\delta\Sigma}{\delta \Xi} \frac{\delta\Sigma}{\delta H} \right\} = 0 \quad . \quad (13) \end{aligned}$$

In addition to the Slavnov-Taylor identity (13) the gauge condition, the ghost equation and the antighost equation read

$$\frac{\delta\Sigma}{\delta B} = \partial^m A_m + \xi B \quad ; \quad (14.a)$$

$$\frac{\delta\Sigma}{\delta \bar{c}} = \square c \quad ; \quad (14.b)$$

$$-i\frac{\delta\Sigma}{\delta c} = i\square \bar{c} - i\frac{\delta\Sigma_{\text{ext}}}{\delta c} \quad . \quad (14.c)$$

We notice that, the right-hand sides of Eqs.(14.a - 14.c) are linear in the quantum fields, then are not subjected to renormalizations.

The solution for the Eqs.(14.a - 14.b) is simply

$$\Sigma = \bar{\Sigma}(\psi_{\pm}, H, \theta, A_m, c, \Omega_{\pm}, \Theta, \Xi) + \int d^3x \left\{ B \partial^m A_m + \frac{\xi}{2} B^2 + \bar{c} \square c \right\} . \quad (15)$$

Putting these informations into (13) we find that the constraint on $\bar{\Sigma}$ is given by

$$\begin{aligned} \bar{\mathcal{S}}(\bar{\Sigma}) = \int d^3x \left\{ -\frac{1}{g} \partial^m c \frac{\delta \bar{\Sigma}}{\delta A^m} + \frac{\delta \bar{\Sigma}}{\delta \bar{\Omega}_+} \frac{\delta \bar{\Sigma}}{\delta \psi_+} - \frac{\delta \bar{\Sigma}}{\delta \bar{\Omega}_-} \frac{\delta \bar{\Sigma}}{\delta \psi_-} - \frac{\delta \bar{\Sigma}}{\delta \Omega_+} \frac{\delta \bar{\Sigma}}{\delta \bar{\psi}_+} + \frac{\delta \bar{\Sigma}}{\delta \Omega_-} \frac{\delta \bar{\Sigma}}{\delta \bar{\psi}_-} + \right. \\ \left. - \frac{\delta \bar{\Sigma}}{\delta \Theta} \frac{\delta \bar{\Sigma}}{\delta \theta} - \frac{\delta \bar{\Sigma}}{\delta \Xi} \frac{\delta \bar{\Sigma}}{\delta H} \right\} = 0 . \end{aligned} \quad (16)$$

The corresponding linearized Slavnov-Taylor operator for any functional $\bar{\mathcal{F}}$ reads

$$\begin{aligned} \bar{\mathcal{S}}_{\bar{\mathcal{F}}} = \int d^3x \left\{ -\frac{1}{g} \partial^m c \frac{\delta}{\delta A^m} + \frac{\delta \bar{\mathcal{F}}}{\delta \bar{\Omega}_+} \frac{\delta}{\delta \psi_+} - \frac{\delta \bar{\mathcal{F}}}{\delta \bar{\Omega}_-} \frac{\delta}{\delta \psi_-} + \frac{\delta \bar{\mathcal{F}}}{\delta \psi_+} \frac{\delta}{\delta \bar{\Omega}_+} - \frac{\delta \bar{\mathcal{F}}}{\delta \psi_-} \frac{\delta}{\delta \bar{\Omega}_-} + \right. \\ \left. - \frac{\delta \bar{\mathcal{F}}}{\delta \Omega_+} \frac{\delta}{\delta \bar{\psi}_+} + \frac{\delta \bar{\mathcal{F}}}{\delta \Omega_-} \frac{\delta}{\delta \bar{\psi}_-} - \frac{\delta \bar{\mathcal{F}}}{\delta \bar{\psi}_+} \frac{\delta}{\delta \Omega_+} + \frac{\delta \bar{\mathcal{F}}}{\delta \bar{\psi}_-} \frac{\delta}{\delta \Omega_-} + \frac{\delta \bar{\mathcal{F}}}{\delta \Theta} \frac{\delta}{\delta \theta} + \frac{\delta \bar{\mathcal{F}}}{\delta \theta} \frac{\delta}{\delta \Theta} + \right. \\ \left. - \frac{\delta \bar{\mathcal{F}}}{\delta \Xi} \frac{\delta}{\delta H} - \frac{\delta \bar{\mathcal{F}}}{\delta H} \frac{\delta}{\delta \Xi} \right\} . \end{aligned} \quad (17)$$

The following nilpotency identities holds :

$$\bar{\mathcal{S}}_{\bar{\mathcal{F}}} \bar{\mathcal{S}}(\bar{\mathcal{F}}) = 0 , \quad \forall \bar{\mathcal{F}} ; \quad (18.a)$$

$$\bar{\mathcal{S}}_{\bar{\mathcal{F}}} \bar{\mathcal{S}}_{\bar{\mathcal{F}}} = 0 \quad \text{if} \quad \bar{\mathcal{S}}(\bar{\mathcal{F}}) = 0 . \quad (18.b)$$

The operation of $\bar{\mathcal{S}}_{\bar{\Sigma}}$ over the fields and the external sources is given by

$$\begin{aligned} \bar{\mathcal{S}}_{\bar{\Sigma}} \phi &= s \phi , \quad \phi = \psi_{\pm}, \bar{\psi}_{\pm}, H, \theta, A_m, c, \bar{c} \text{ and } B , \\ \bar{\mathcal{S}}_{\bar{\Sigma}} \bar{\Omega}_+ &= \frac{\delta \Sigma}{\delta \psi_+} , \quad \bar{\mathcal{S}}_{\bar{\Sigma}} \bar{\Omega}_- = -\frac{\delta \Sigma}{\delta \psi_-} , \\ \bar{\mathcal{S}}_{\bar{\Sigma}} \Omega_+ &= -\frac{\delta \Sigma}{\delta \bar{\psi}_+} , \quad \bar{\mathcal{S}}_{\bar{\Sigma}} \Omega_- = \frac{\delta \Sigma}{\delta \bar{\psi}_-} , \\ \bar{\mathcal{S}}_{\bar{\Sigma}} \Theta &= -\frac{\delta \Sigma}{\delta \theta} , \quad \bar{\mathcal{S}}_{\bar{\Sigma}} \Xi = -\frac{\delta \Sigma}{\delta H} . \end{aligned} \quad (19)$$

In order to study the stability [2] of the action (11) under the radiative corrections, we perturb the classical action by local functional, Σ^c , having the same quantum numbers as $\bar{\Sigma}$:

$$\bar{\Sigma} \longrightarrow \bar{\Sigma}' = \bar{\Sigma} + \epsilon \Sigma^c \quad (20)$$

where ϵ is an infinitesimal parameter. Then requiring that the perturbed action Σ' satisfies the same conditions as $\bar{\Sigma}$ we obtain :

$$\bar{\mathcal{S}}_{\bar{\Sigma}} \Sigma^c = 0 , \quad (21)$$

$$\frac{\delta \Sigma^c}{\delta B} = 0 , \quad \frac{\delta \Sigma^c}{\delta \bar{c}} = 0 , \quad \frac{\delta \Sigma^c}{\delta c} = 0 , \quad (22)$$

which follow from the Slavnov-Taylor identity and from the conditions (14.a - 14.c), and, moreover:

$$W_{\text{rigid}} \Sigma^c = 0 , \quad (23)$$

where W_{rigid} is the Ward operator of rigid $U(1)$ symmetry defined by

$$\begin{aligned} W_{\text{rigid}} = \int d^3x \left\{ q \psi_+ \frac{\delta}{\delta \psi_+} + q \psi_- \frac{\delta}{\delta \psi_-} - q \bar{\psi}_+ \frac{\delta}{\delta \bar{\psi}_+} - q \bar{\psi}_- \frac{\delta}{\delta \bar{\psi}_-} - iQ(v+H) \frac{\delta}{\delta \theta} + iQ\theta \frac{\delta}{\delta H} + \right. \\ \left. + q \Omega_+ \frac{\delta}{\delta \Omega_+} + q \Omega_- \frac{\delta}{\delta \Omega_-} - q \bar{\Omega}_+ \frac{\delta}{\delta \bar{\Omega}_+} - q \bar{\Omega}_- \frac{\delta}{\delta \bar{\Omega}_-} - iQ\Xi \frac{\delta}{\delta \Theta} + iQ\Theta \frac{\delta}{\delta \Xi} \right\} . \end{aligned} \quad (24)$$

Eq.(23) follows from the rigid $U(1)$ invariance of the action¹ in the Landau gauge:

$$W_{\text{rigid}}\Sigma = 0 \quad . \quad (25)$$

The BRS consistency condition in the ghost number sector zero, given by Eq.(21), constitutes a cohomology problem due to the nilpotency (18.b) of the linearized Slavnov-Taylor operator (17). Its solution can always be written as a sum of a trivial cocycle $\bar{\mathcal{S}}_{\Sigma}\hat{\Sigma}$, where $\hat{\Sigma}$ has ghost number -1 , and nontrivial elements Σ_{phys} belonging to the cohomology of $\bar{\mathcal{S}}_{\Sigma}$ (17) in the sector of ghost number zero:

$$\Sigma^c = \Sigma_{\text{phys}} + \bar{\mathcal{S}}_{\Sigma}\hat{\Sigma} \quad , \quad (26)$$

where the trivial cocycle $\bar{\mathcal{S}}_{\Sigma}\hat{\Sigma}$ corresponds to field renormalizations, which are unphysical. On the other hand, the nontrivial perturbation Σ_{phys} corresponds to a redefinition of the physical parameters – coupling constants and masses. An explicit computation, yields the following solution for Eq.(26):

$$\begin{aligned} \Sigma_{\text{phys}} = & z_g \left(g \frac{\partial}{\partial g} - N_A + N_B - 2\xi \frac{\partial}{\partial \xi} \right) \Sigma + z_m m \frac{\partial \Sigma}{\partial m} + \\ & + z_y y \frac{\partial \Sigma}{\partial y} + z_{M_H^2} M_H^2 \frac{\partial \Sigma}{\partial M_H^2} + z_{\zeta} \zeta \frac{\partial \Sigma}{\partial \zeta} + z_{\lambda} \lambda \frac{\partial \Sigma}{\partial \lambda} \quad , \end{aligned} \quad (27.a)$$

$$\begin{aligned} \bar{\mathcal{S}}_{\Sigma}\hat{\Sigma} = & \bar{\mathcal{S}}_{\Sigma} \int d^3x \{ z_{\psi} (\bar{\psi}_+ \Omega_+ - \bar{\Omega}_+ \psi_+ - \bar{\psi}_- \Omega_- + \bar{\Omega}_- \psi_-) + z_H [\theta \Theta + (v + H)\Xi] \} \\ = & z_{\psi} \left(N_{\psi_+} + N_{\bar{\psi}_+} + N_{\psi_-} + N_{\bar{\psi}_-} - N_{\Omega_+} - N_{\bar{\Omega}_+} - N_{\Omega_-} - N_{\bar{\Omega}_-} \right) \Sigma \\ & + z_H \left(N_{\theta} + \hat{N}_H - N_{\Theta} - N_{\Xi} \right) \Sigma \quad , \end{aligned} \quad (27.b)$$

where the counting operators are defined by

$$\begin{aligned} N_{\phi} = & \int d^3x \phi \frac{\delta}{\delta \phi} \quad , \quad \phi = \psi_{\pm}, \bar{\psi}_{\pm}, \theta, \Omega_{\pm}, \bar{\Omega}_{\pm}, \Theta, \Xi, A_m \text{ and } B \quad , \\ \hat{N}_H = & \int d^3x (v + H) \frac{\delta}{\delta H} \quad . \end{aligned} \quad (28)$$

The stability proof we have given corresponds, at the quantum level, to the multiplicative renormalizability of the model: all the possible counterterms induced by the radiative corrections correspond to a redefinition of the parameters of the starting classical theory. The parameters $z_g, z_m, z_y, z_{M_H^2}, z_{\zeta}, z_{\lambda}, z_H$ and z_{ψ} are then renormalization constants, which are fixed by the following normalization conditions – expressed on the vertex functional Γ , which coincides with the classical action Σ in the classical limit:

$$\begin{aligned} \Gamma_{HH}(p^2) \Big|_{p^2=M_H^2} &= 0 \quad , \quad \frac{\partial}{\partial p^2} \Gamma_{HH}(p^2) \Big|_{p^2=\kappa^2} = 1 \quad , \\ \Gamma_{HHHH}(p) \Big|_{p=\bar{p}(\kappa)} &= -\zeta \quad , \quad \Gamma_{HHHHHH}(p) \Big|_{p=\bar{p}(\kappa)} = -\lambda \quad , \\ \Gamma_{\bar{\psi}_{\pm}\psi_{\pm}}(\not{p}) \Big|_{\not{p}=\pm m} &= 0 \quad , \quad \frac{\partial}{\partial \not{p}} \Gamma_{\bar{\psi}_{\pm}\psi_{\pm}}(\not{p}) \Big|_{\not{p}=\kappa} = 1 \quad , \\ \frac{\partial}{\partial p^2} \Gamma_{A^T A^T}(p^2) \Big|_{p^2=\kappa^2} &= 1 \quad , \quad \Gamma_{\bar{\psi}_{\pm}\psi_{\pm}HH}(p) \Big|_{p=\bar{p}(\kappa)} = \mp y \quad , \end{aligned} \quad (29)$$

where κ is an energy scale and $\bar{p}(\kappa)$ some reference set of 4-momenta at this scale.

To complete the proof of the renormalizability of the model, we show that all the symmetries defining the model can be extended to the quantum level, for the vertex functional Γ

$$\Gamma = \Sigma + \mathcal{O}(\hbar) \quad . \quad (30)$$

¹Rigid invariance itself follows from the antighost equation (14.c) and from the validity of the Slavnov-Taylor identity (13).

Now, it is trivial to verify that the solution of Eqs.(14.a – 14.b), that are linear in the quantized fields, is given by

$$\Gamma = \bar{\Gamma}(\psi_{\pm}, H, \theta, A_m, c, \Omega_{\pm}, \Theta, \Xi) + \int d^3x \left\{ B \partial^m A_m + \frac{\xi}{2} B^2 + \bar{c} \square c \right\} \quad (31)$$

As a consequence we have the following conditions on $\bar{\Gamma}$ – defined from Γ similarly to (15):

$$\frac{\delta \bar{\Gamma}}{\delta B} = 0 \quad ; \quad (32.a)$$

$$\frac{\delta \bar{\Gamma}}{\delta \bar{c}} = 0 \quad ; \quad (32.b)$$

$$-i \frac{\delta \bar{\Gamma}}{\delta c} = -i \frac{\delta \Sigma_{\text{ext}}}{\delta c} \quad ; \quad (32.c)$$

$$W_{\text{rigid}} \bar{\Gamma} = 0 \quad , \quad (32.d)$$

where W_{rigid} has already been defined by (24), and where (32.c) is the quantum extension of Eq.(14.c).

According to the Quantum Action Principle [4, 5] the Slavnov-Taylor identity (13) gets a quantum breaking

$$\mathcal{S}(\Gamma) = \bar{\mathcal{S}}(\bar{\Gamma}) = \Delta \cdot \Gamma = \Delta + \mathcal{O}(\hbar \Delta) \quad , \quad (33)$$

where Δ is a local integrated functional with ghost number one.

The nilpotency identity (18.a) together with

$$\bar{\mathcal{S}}_{\Gamma} = \bar{\mathcal{S}}_{\Sigma} + \mathcal{O}(\hbar) \quad (34)$$

implies the following consistency condition for the breaking Δ :

$$\bar{\mathcal{S}}_{\Sigma} \Delta = 0 \quad . \quad (35)$$

In order to identify other constraints for Δ , we use the following algebraic relations, valid for any functional $\bar{\mathcal{F}}$ with zero GP :

$$\frac{\delta \bar{\mathcal{S}}(\bar{\mathcal{F}})}{\delta B} - \bar{\mathcal{S}}_{\bar{\mathcal{F}}} \frac{\delta \bar{\mathcal{F}}}{\delta B} = 0 \quad ; \quad (36.a)$$

$$\frac{\delta \bar{\mathcal{S}}(\bar{\mathcal{F}})}{\delta \bar{c}} + \bar{\mathcal{S}}_{\bar{\mathcal{F}}} \frac{\delta \bar{\mathcal{F}}}{\delta \bar{c}} = 0 \quad ; \quad (36.b)$$

$$-i \int d^3x \frac{\delta}{\delta c} \bar{\mathcal{S}}(\bar{\mathcal{F}}) + \bar{\mathcal{S}}_{\bar{\mathcal{F}}} \int d^3x \left(-i \frac{\delta}{\delta c} \bar{\mathcal{F}} + i \frac{\delta \Sigma_{\text{ext}}}{\delta c} \right) = W_{\text{rigid}} \bar{\mathcal{F}} \quad ; \quad (36.c)$$

$$W_{\text{rigid}} \bar{\mathcal{S}}(\bar{\mathcal{F}}) - \bar{\mathcal{S}}_{\bar{\mathcal{F}}} W_{\text{rigid}} \bar{\mathcal{F}} = 0 \quad . \quad (36.d)$$

Taking into account Eqs.(32.a – 32.d), Eq.(33) and assuming $\bar{\mathcal{F}} = \bar{\Gamma}$ in Eqs.(36.a – 36.d), the following consistency conditions on the breaking Δ are found :

$$\frac{\delta \Delta}{\delta B} = 0 \quad ; \quad (37.a)$$

$$\frac{\delta \Delta}{\delta \bar{c}} = 0 \quad ; \quad (37.b)$$

$$\int d^3x \frac{\delta}{\delta c} \Delta = 0 \quad ; \quad (37.c)$$

$$W_{\text{rigid}} \Delta = 0 \quad . \quad (37.d)$$

The Wess-Zumino consistency condition (35) constitutes a cohomology problem like in the zero ghost number case (21). Its solution can always be written as a sum of a trivial cocycle $\mathcal{S}_{\Sigma} \hat{\Delta}^{(0)}$, where $\hat{\Delta}^{(0)}$ has ghost number 0, and of nontrivial elements belonging to the cohomology of \mathcal{S}_{Σ} (17) in the sector of ghost number one:

$$\Delta^{(1)} = \hat{\Delta}^{(1)} + \mathcal{S}_{\Sigma} \hat{\Delta}^{(0)} \quad , \quad (38)$$

where $\Delta^{(1)}$ must be even under P -symmetry and obey the conditions imposed by Eqs.(37.a – 37.d). The trivial cocycle $\mathcal{S}_{\Sigma} \hat{\Delta}^{(0)}$ can be absorbed into the vertex functional Γ as a local integrated noninvariant counterterm $-\hat{\Delta}^{(0)}$.

Now, from the condition (37.c), we conclude that

$$\Delta^{(1)} = \int d^3x K_m^{(0)} \partial^m c \quad . \quad (39)$$

By analyzing the Slavnov-Taylor operator $\tilde{\mathcal{S}}_{\Sigma}$ (17) and the Wess-Zumino consistency condition (35), we see that the UV and IR dimensions of the breaking $\Delta^{(1)}$ are bounded by $d \leq \frac{7}{2}$ and $r \geq 2$. Therefore, $K_m^{(0)}$, of ghost number 0, has dimensions bounded by $d \leq \frac{5}{2}$, $r \geq 1$.

Now, rewriting $K_m^{(0)}$ as a linear combination

$$K_m^{(0)} = \sum_{i=1}^8 a_i K_m^{(0)i} \quad , \quad (40)$$

where

$$\begin{aligned} K_m^{(0)1} &= A_m \quad , \quad K_m^{(0)2} = A_m A^n A_n \quad , \\ K_m^{(0)3} &= A_m (A^n A_n)^2 \quad , \quad K_m^{(0)4} = A_m (\bar{\psi}_+ \psi_+ - \bar{\psi}_- \psi_-) \quad , \\ K_m^{(0)5} &= A_m A^n A_n ((v+H)^2 + \theta^2) \quad , \quad K_m^{(0)6} = A_m ((v+H)^2 + \theta^2) \quad , \\ K_m^{(0)7} &= A_m ((v+H)^2 + \theta^2)^2 \quad \text{and} \quad K_m^{(0)8} = \bar{\psi}_+ \gamma_m \psi_+ + \bar{\psi}_- \gamma_m \psi_- \quad , \end{aligned} \quad (41)$$

and solving all the conditions it has to fulfil, we can easily show, with the help of Eqs.(19), that there exist local functionals $\hat{\Delta}^{(0)i}$ such that

$$\int d^3x K_m^{(0)i} \partial^m c = \mathcal{S}_{\Sigma} \hat{\Delta}^{(0)i} \quad , \quad i = 1, \dots, 8 \quad . \quad (42)$$

This means that $\hat{\Delta}^{(1)} = 0$ in (38), which implies the implementability of the Slavnov-Taylor identity to every order through the absorption of the noninvariant counterterm $-\sum_i a_i \hat{\Delta}^{(0)i}$.

In conclusion, the algebraic method of renormalization allowed us to show that the model is perturbatively renormalizable to all orders. The study of the possible counterterms has led to the conclusion that the model is multiplicatively renormalizable, namely that the counterterms can be reabsorbed by a redefinition of the initial parameters. Finally, we have proven that anomalies are absent. We stress that our algebraic analysis does not involve any regularization scheme, nor any particular diagrammatic calculation.

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