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Ruderman-Kittel interaction in one dimension for arbitrary coupling constant

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Abstract

The Ruderman-Kittel interaction in one dimension is presented for arbitrary coupling constant as a byproduct of previous considerations for three-dimensions. For coupling constants which are smaller than or even comparable to the Fermi energy, the interaction oscillates as a function of distance like the perturbation result, however, with a reduced amplitude. The interaction between two parallel magnetic lines in two dimensions is also discussed.

Key-words: Ruderman-Kittel interaction; One dimension.

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The Ruderman-Kittel interaction in one dimension has a curious history: Kittel [1] used standard perturbation theory and obtained a result with infinite range. In a addendum (quoted by Yafet [2]) he indicated a different way to treat a singularity which leads to a finite range. Yafet [2] later justified this method with a delicate argument. Actually, the point is that an attractive Dirac δ-function in one dimension has a bound state. Therefore the problem has an essential singularity at vanishing coupling constant, and perturbation theory does not apply. A recent paper [3] discusses the rôle of the bound states in the interaction of two magnetic layers. It contains an alternative derivation of formulas first obtained by Bruno [4] and valid for arbitrary coupling constant. The one dimensional case is obtained by trivial modifications. While this extends the previous result to finite coupling constant, we stress that the weak coupling limit can only be derived consistently by a non perturbative method. The interaction of magnetic layers in three dimensions is different in this respect, since the weak coupling result can be obtained from the Ruderman-Kittel interaction integrating over the positions of the spins, one on each plane [2,5].

Now we translate the formulas of ref. [3] to the one dimensional case using its notation and equation numbers. The Hamiltonian, Eq. (31), remains valid, just as the bound state energies E_1, E_2 and E_3 given by Eqs. (32-36) and plotted in Fig. 1 of [3]. The grand potential por the parallel spin configuration (+ sign in Eq. (31)) corresponding to Eq. (62) becomes

$$\Xi_p^{(1)} = E_1 + E_2 - \frac{E_F}{\pi}\int_0^1 dy\, y\, \Omega_{f,p}(yk_F) \tag{1}$$

with $\Omega_{f,p}$ given by Eq. (55). For the antiparallel spin configuration Eq. (83) translates into

$$\Xi_a^{(1)} = 2E_3 - \frac{E_F}{\pi}\int_0^1 dy\, y\, \Omega_{f,a}(yk_F) \tag{2}$$

with $\Omega_{f,a}$ given by Eq. (78). The energy difference between the two configurations, $\Delta\Xi^{(1)} = \Xi_p^{(1)} - \Xi_a^{(1)}$, determines their relative stability. Fig. 1 shows $\Delta\Xi^{(1)}$ as a function of the distance L between the two spins for the same values of the reciprocal coupling constant l_0 as in fig. 3 of ref. [3]. The shape of the curves looks quite similar. Note, however, that the amplitudes of the oscillations decrease more slowly with distance in the one dimensional case. The dimensions of $\Delta\Xi^{(1)}$ and $\Delta\Xi$, and therefore the units in the two figures are different.

The two dimensional system analogous to that of ref. [3] consists of two parallel magnetic lines in a plane. The formulas of the grand potentials per unit length are:

$$\Xi_p^{(2)} = \tfrac{2E_F k_F}{3\pi}\Big\{1 - \left(1 - \tfrac{E_1}{E_F}\right)^{3/2} + \Theta(L - l_0)\left[1 - \left(1 - \tfrac{E_2}{E_F}\right)^{3/2}\right] - \tfrac{3}{2\pi}\int_0^1 dy\, y\sqrt{1-y^2}\,\Omega_p(yk_f)\Big\} \tag{3}$$

for parallel magnetizations and

$$\Xi_a^{(2)} = \frac{2E_F k_F}{3\pi}\left\{2 - 2\left(1 - \frac{E_3}{E_F}\right)^{3/2} - \frac{3}{2\pi}\int_0^1 dy\, y\sqrt{1-y^2}\,\Omega_a(yk_f)\right\} \tag{4}$$

for antiparallel magnetizations. $\Delta\Xi^{(2)} = \Xi_p^{(2)} - \Xi_a^{(2)}$ is plotted in Fig. (2). The weak coupling limit of $\Delta\Xi^{(2)}$ can be obtained either from Eqs. (3-4), or by integrating the Ruderman-Kittel coupling [Eq. (4) of Ref. [6]] over the positions on the two lines.

It is noteworthy that for the Ruderman-Kittel interaction in three dimensions only the weak coupling limit exists. The spectrum of an electron in an attractive $\delta^{(3)}$ potential has no lower bound. This becomes evident when a Gaussian function is used as variational function: the energy goes to minus infinity as the range of the function vanishes. In general, when a δ^3-function is used in solid state physics to represent a highly localized potential, it is understood that only the first Born approximation applies. In contrast, the coupling of a magnetic plane to conduction electron spins is represented by a δ-function which depends on one coordinate only, which allows an exact treatment and actually requires it so that the bound states are obtained.

FIGURES

FIG. 1. Energy difference $\Delta\Xi^{(1)}$ between the parallel and antiparallel configurations of two spins on a line as a function of their separation L for various values of the coupling constant. $\Delta\Xi^{(1)}$ in units of $2\epsilon_0/\pi$, and L in units of k_F^{-1}. For vanishing coupling constant (full line) $\Delta\Xi^{(1)} = -\pi$ at $L = 0$ in these units. This curve differs little from a case of weak coupling, $k_F l_0 = 8$, corresponding to $E_F/\epsilon_0 = 32$ (long dashes). Intermediate dashes belong to intermediate coupling: $k_F l_0 = \sqrt{2}$ i.e. $E_F/\epsilon_0 = 1$. Strong coupling (short dashes) with $k_F l_0 = 0.5$, $E_F/\epsilon_0 = 0.125$ leads to a rapidly damped interaction.

FIG. 2. Energy difference per unit length $\Delta\Xi^{(2)}$ for two parallel lines in a plane as a function of their separation L for various values of the coupling constant. $\Delta\Xi^{(2)}$ in units of $2\epsilon_0 k_F/\pi^2$, and L in units of k_F^{-1}. In these units for vanishing coupling constant (full line) $\Delta\Xi^{(2)} = -\pi$ at $L = 0$. The parameter values are the same as in Fig. 1.

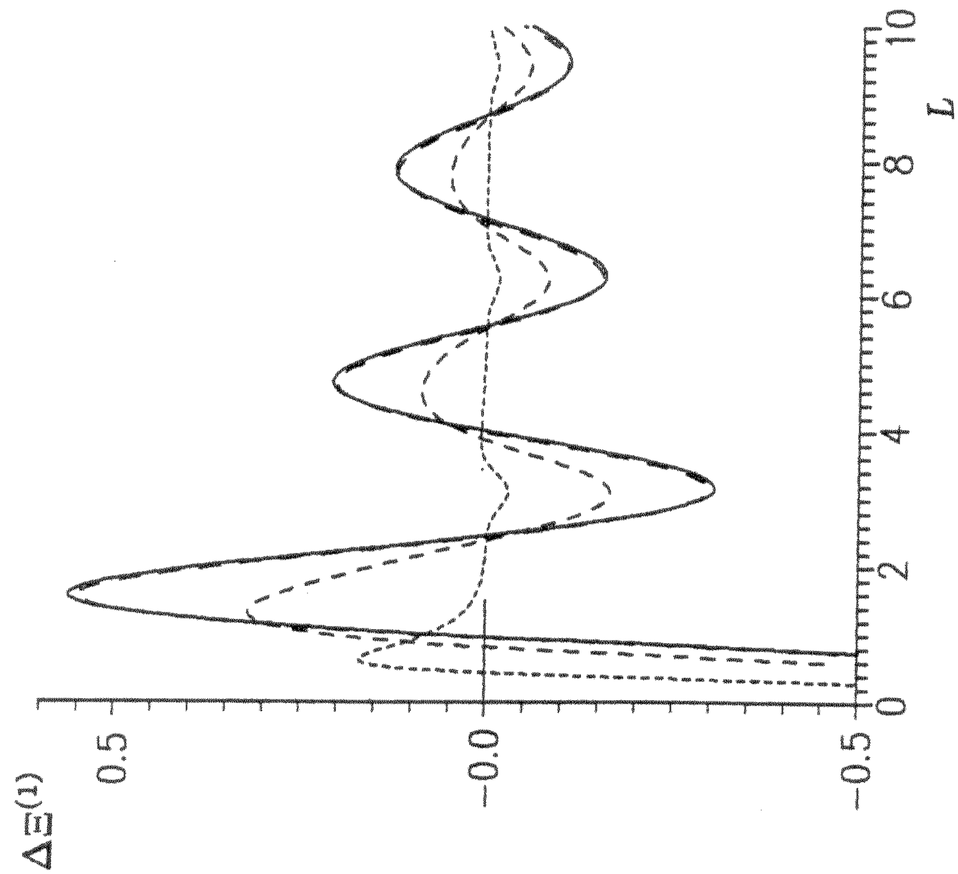

FIG. 1

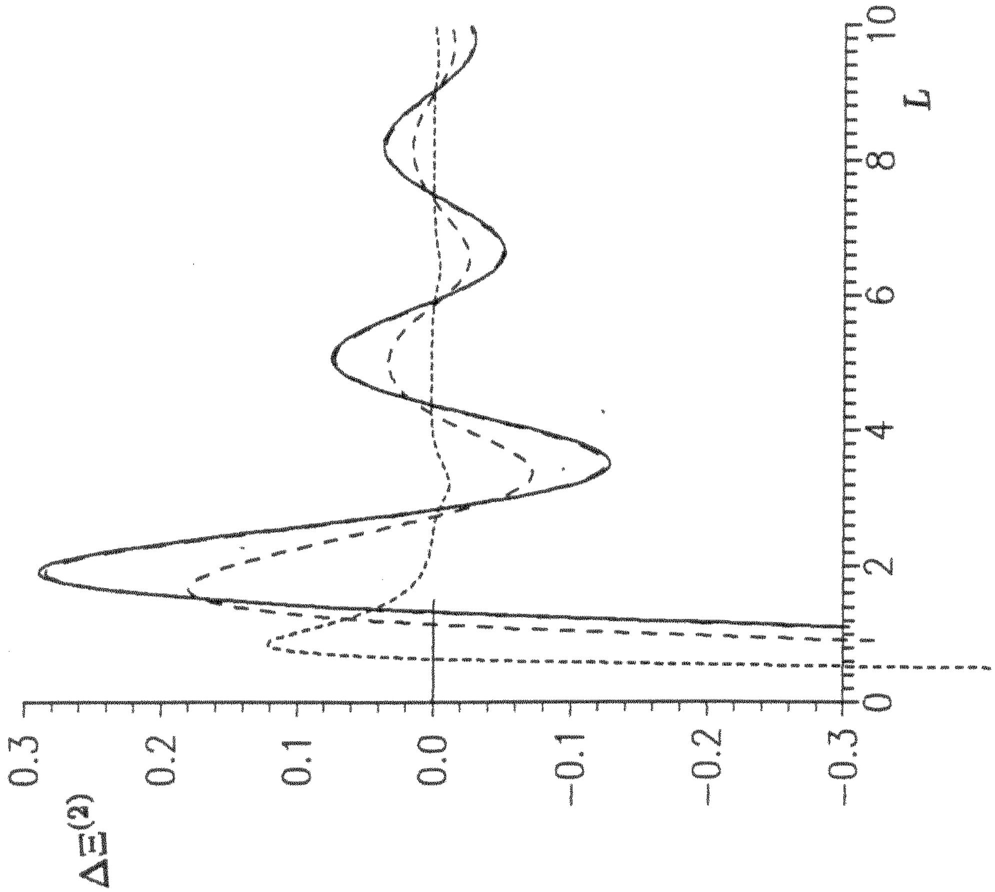

FIG. 2

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