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Algebraic Renormalization of Parity-Preserving QED_3 Coupled to Scalar Matter I: Unbroken Case

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Abstract

In this letter the algebraic renormalization method, which is independent of any kind of regularization scheme, is presented for the parity-preserving QED_3 coupled to scalar matter in the symmetric regime, where the scalar assumes vanishing vacuum expectation value, $\langle\varphi\rangle=0$. The model shows to be stable under radiative corrections and anomaly free.

Key-words: QED_3; Algebraic renormalization; Unbroken symmetry.

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The study of gauge field theories in 3 space-time dimensions [1] has been well-supported by a possible field-theoretical approach to describe some Condensed Matter phenomena, such as High-T_c Superconductivity and Quantum Hall Effect [2, 3]. Some Abelian models have been proposed in this direction, namely, the QED_3 and $\tau_3\mathrm{QED}_3$ [4, 5].

One of the interesting properties of 3-dimensional gauge field theories is the Landau gauge finiteness of non-Abelian Chern-Simons theories [6].

The confinement of massive electrons in 3 space-time dimensions is a remarkable characteristic of this lower dimensional space [7]. Recently, it was shown by using the Bethe-Salpeter equations that in a parity-preserving QED_3 there are bound states in electron-positron systems, positronium states [8].

In a recent work [9], a parity-preserving QED_3 with spontaneous breaking of a local $U(1)$-symmetry was proposed. The breakingdown is accomplished by a sixth-power potential. It was shown that electrons scattered in D=1+2 can experience a mutual attractive interaction, depending on their spin states, where the intermediate bosons involved in such processes are a massive vector meson and a Higgs scalar. This attractive scattering potential comes from processes in which the electrons are correlated in momentum space with opposite spin polarizations (s-wave state).

One has still to study the renormalizability of this model, with the $U(1)$ gauge invariance spontaneously broken as explained above. However, the present letter is dedicated to the preliminary task of doing that for the simpler case of unbroken gauge invariance. In this symmetric phase, the gauge boson remains massless. The same should occur for the fermion, since, in the broken phase, its mass is completely generated by the Higgs mechanism.

But since a massless spinor might cause infrared singularities due to the presence of super-renormalizable vertices involving the massless fields, we will add a fermion mass term in order to avoid this problem – which anyhow will not appear in the physically interesting broken phase, where the fermion is anyhow massive.

After a very brief summary of the model, we will show that its parametrization is stable under small perturbations. This, together with the proof of the absence of anomalies given in the final part of the paper, will mean the multipicative renormalizability of the theory.

The study of the renormalizability of the broken phase will be presented in a forthcoming paper [10].

The gauge invariant action for the parity-preserving QED_3[1] coupled to scalar matter [9] in the $U(1)$-symmetric regime, $\langle\varphi\rangle$=0, is given by:

$$\Sigma_{inv} = \int d^3x \left\{ -\frac{1}{4}F^{mn}F_{mn} + i\overline{\psi}_+ D\!\!\!\!/\,\psi_+ + i\overline{\psi}_- D\!\!\!\!/\,\psi_- - m_0(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-) + \right.$$
$$\left. - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + D^m\varphi^* D_m\varphi - \mu^2\varphi^*\varphi - \frac{\zeta}{2}(\varphi^*\varphi)^2 - \frac{\lambda}{3}(\varphi^*\varphi)^3 \right\} \quad , \tag{1}$$

where the mass dimensions of the parameters m_0, μ, ζ, λ and y are respectively 1 ,1, 1, 0 and 0. The form of the potential is chosen such as to ensure the symmetric regime, where $\langle\varphi\rangle$=0. Imposing that it must be bounded from below and yield only sable vacua, we get the following conditions on the parameters:

$$\lambda > 0 \quad , \quad \zeta < 0 \quad \text{and} \quad \mu^2 > \frac{3}{16}\frac{\zeta^2}{\lambda} \quad . \tag{2}$$

The covariant derivatives are defined as follows:

$$D\!\!\!\!/\,\psi_\pm \equiv (\partial\!\!\!/ + iqgA\!\!\!/)\psi_\pm \quad \text{and} \quad D_m\varphi \equiv (\partial_m + iQgA_m)\varphi \quad , \tag{3}$$

where g is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$, and q and Q are the $U(1)$-charges of the fermions and scalar, respectively. In the action (1), F_{mn} is the usual field strength for A_m, ψ_+ and ψ_- are two kinds of fermions (the $\pm$ subscripts refer to their spin sign [11]) and φ is a complex scalar. It should be noticed that in the action (1) a parity-preserving mass term for ψ_+ and ψ_- has been added to the original action of ref. [9] in order to avoid potential IR divergences which may be caused by the super-renormalizable interactions.

[1]The metric adopted throughout this work is $\eta_{mn} = (+,-,-)$; m, n=(0,1,2). Note that slashed objects mean contraction with γ-matrices. The latter are taken as γ^m=$(\sigma_x, i\sigma_y, -i\sigma_z)$.

The complete action, Σ, we are considering here, is given by:

$$\Sigma = \Sigma_{inv} + \Sigma_{gf} + \Sigma_{ext} \quad , \tag{4}$$

where Σ_{gf} is the gauge-fixing action and Σ_{ext} is the action for the external sources:

$$\Sigma_{gf} = \int d^3x \left\{ B\partial^m A_m + \frac{\xi}{2}B^2 + \overline{c}\Box c \right\} \quad , \tag{5}$$

$$\Sigma_{ext} = \int d^3x \left\{ \overline{\Omega}_+ s\psi_+ - \overline{\Omega}_- s\psi_- - s\overline{\psi}_+\Omega_+ + s\overline{\psi}_-\Omega_- + \rho^* s\varphi + s\varphi^*\rho \right\} \quad . \tag{6}$$

The gauge condition, the ghost equation and the antighost equation [12] for (4) read

$$\frac{\delta\Sigma}{\delta B} = \partial^m A_m + \xi B \quad , \tag{7.a}$$

$$\frac{\delta\Sigma}{\delta \overline{c}} = \Box c \quad , \tag{7.b}$$

$$-i\frac{\delta\Sigma}{\delta c} = \Delta_{\rm class} \quad , \qquad \text{with:} \tag{7.c}$$

$$\Delta_{\rm class} = i\Box\overline{c} + q\overline{\Omega}_+\psi_+ - q\overline{\Omega}_-\psi_- + q\overline{\psi}_+\Omega_+ - q\overline{\psi}_-\Omega_- - Q\rho^*\varphi - Q\varphi^*\rho \quad .$$

Note that the right-hand sides being linear in the quantum fields, will not be submitted to renormalization. The QED$_3$-action[2] (4) is invariant under the reflexion symmetry P, whose action on the fields and external sources is fixed as below:

$$\begin{array}{lcll}
x_m & \xrightarrow{P} & x_m^P = (x_0, -x_1, x_2) \quad , & \\
\psi_\pm & \xrightarrow{P} & \psi_\pm^P = -i\gamma^1\psi_\mp \quad , & \overline{\psi}_\pm \xrightarrow{P} \overline{\psi}_\pm^P = i\overline{\psi}_\mp\gamma^1 \quad , \\
A_m & \xrightarrow{P} & A_m^P = (A_0, -A_1, A_2) \quad , & \\
\phi & \xrightarrow{P} & \phi^P = \phi \ , & \phi = \varphi,\, c,\, \bar{c},\, B \quad , \\
\Omega_\pm & \xrightarrow{P} & \Omega_\pm^P = -i\gamma^1\Omega_\mp \quad , & \overline{\Omega}_\pm \xrightarrow{P} \overline{\Omega}_\pm^P = i\overline{\Omega}_\mp\gamma^1 \quad , \\
\rho & \xrightarrow{P} & \rho^P = \rho \quad . &
\end{array} \tag{8}$$

The ultraviolet and infrared dimensions[3], d and r respectively, as well as the ghost numbers, $\Phi\Pi$, and the Grassmann parity, GP, of all fields and sources are collected in Table 1.

	A_m	φ	$\psi_\pm$	c	$\overline{c}$	B	ρ	$\Omega_\pm$
d	1/2	1/2	1	0	1	3/2	5/2	2
r	1/2	3/2	3/2	0	1	3/2	5/2	2
$\Phi\Pi$	0	0	0	1	−1	0	−1	−1
GP	0	0	1	1	1	0	1	0

Table 1: UV and IR dimensions, d and r, ghost numbers, $\Phi\Pi$, and Grassmann parity, GP.

The BRS transformations are defined by:

$$s\varphi = iQc\varphi \quad , \quad s\varphi^* = -iQc\varphi^* \quad ,$$

[2] For more details about QED$_3$ and τ_3QED$_3$ as well as their applications, and some peculiarities of parity and time-reversal in D=1+2, see refs. [1, 4, 5].

[3] We have to use a subtraction scheme which takes care of the presence of both massive and massless fields, subtracting off the UV divergences without introducing spurious IR singularities. Such a scheme is the one of Lowenstein and Zimmermann [13, 14]. The UV and IR dimensions mentioned here are those which are involved in this formalism. The terms in the action, as well as all counterterms, are constrained to have UV dimension ≤ 3 and IR dimension ≥ 3.

$$
\begin{aligned}
& s\psi_\pm = iqc\psi_\pm \ , \quad s\overline{\psi}_\pm = -iqc\overline{\psi}_\pm \ , \\
& sA_m = -\frac{1}{g}\partial_m c \ , \quad sc = 0 \ , \\
& s\overline{c} = \frac{1}{g}B \ , \quad sB = 0 \ ,
\end{aligned} \tag{9}
$$

where c is the ghost, $\overline{c}$ is the antighost and B is the Lagrange multiplier field.

The BRS invariance of the action is expressed in a functional way by the Slavnov-Taylor identity

$$
\mathcal{S}(\Sigma) = 0 \ , \tag{10}
$$

where the Slavnov-Taylor operator $\mathcal{S}$ is defined, acting on an arbitrary functional $\mathcal{F}$, by

$$
\begin{aligned}
\mathcal{S}(\mathcal{F}) = \int d^3x \Big\{ & -\frac{1}{g}\partial^m c \frac{\delta \mathcal{F}}{\delta A^m} + \frac{1}{g} B \frac{\delta \mathcal{F}}{\delta \overline{c}} + \frac{\delta \mathcal{F}}{\delta \overline{\Omega}_+}\frac{\delta \mathcal{F}}{\delta \psi_+} - \frac{\delta \mathcal{F}}{\delta \overline{\Omega}_-}\frac{\delta \mathcal{F}}{\delta \psi_-} - \frac{\delta \mathcal{F}}{\delta \Omega_+}\frac{\delta \mathcal{F}}{\delta \overline{\psi}_+} + \frac{\delta \mathcal{F}}{\delta \Omega_-}\frac{\delta \mathcal{F}}{\delta \overline{\psi}_-} + \\
& + \frac{\delta \mathcal{F}}{\delta \rho^*}\frac{\delta \mathcal{F}}{\delta \varphi} - \frac{\delta \mathcal{F}}{\delta \rho}\frac{\delta \mathcal{F}}{\delta \varphi^*} \Big\} \ .
\end{aligned} \tag{11}
$$

The corresponding linearized Slavnov-Taylor operator reads

$$
\begin{aligned}
\mathcal{S}_\mathcal{F} = \int d^3x \Big\{ & -\frac{1}{g}\partial^m c \frac{\delta}{\delta A^m} + \frac{1}{g} B \frac{\delta}{\delta \overline{c}} + \frac{\delta \mathcal{F}}{\delta \overline{\Omega}_+}\frac{\delta}{\delta \psi_+} - \frac{\delta \mathcal{F}}{\delta \overline{\Omega}_-}\frac{\delta}{\delta \psi_-} + \frac{\delta \mathcal{F}}{\delta \psi_+}\frac{\delta}{\delta \overline{\Omega}_+} - \frac{\delta \mathcal{F}}{\delta \psi_-}\frac{\delta}{\delta \overline{\Omega}_-} + \\
& - \frac{\delta \mathcal{F}}{\delta \Omega_+}\frac{\delta}{\delta \overline{\psi}_+} + \frac{\delta \mathcal{F}}{\delta \Omega_-}\frac{\delta}{\delta \overline{\psi}_-} - \frac{\delta \mathcal{F}}{\delta \overline{\psi}_+}\frac{\delta}{\delta \Omega_+} + \frac{\delta \mathcal{F}}{\delta \overline{\psi}_-}\frac{\delta}{\delta \Omega_-} + \frac{\delta \mathcal{F}}{\delta \rho^*}\frac{\delta}{\delta \varphi} + \frac{\delta \mathcal{F}}{\delta \varphi}\frac{\delta}{\delta \rho^*} + \\
& - \frac{\delta \mathcal{F}}{\delta \rho}\frac{\delta}{\delta \varphi^*} - \frac{\delta \mathcal{F}}{\delta \varphi^*}\frac{\delta}{\delta \rho} \Big\} \ .
\end{aligned} \tag{12}
$$

The following nilpotency identities hold:

$$
\mathcal{S}_\mathcal{F}\mathcal{S}(\mathcal{F}) = 0 \ , \quad \forall\, \mathcal{F} \ , \tag{13.a}
$$

$$
\mathcal{S}_\mathcal{F}\mathcal{S}_\mathcal{F} = 0 \quad \text{if} \quad \mathcal{S}(\mathcal{F}) = 0 \ . \tag{13.b}
$$

In particular:

$$
(\mathcal{S}_\Sigma)^2 = 0 \ , \tag{14}
$$

since the action Σ obeys the Slavnov-Taylor identity. The operation of $\mathcal{S}_\Sigma$ over the fields and the external sources is given by

$$
\begin{aligned}
& \mathcal{S}_\Sigma \phi = s\phi \ , \quad \phi = \psi_\pm,\ \overline{\psi}_\pm,\ \varphi,\ \varphi^*,\ A_m,\ c,\ \overline{c} \text{ and } B \ , \\
& \mathcal{S}_\Sigma \overline{\Omega}_+ = \frac{\delta \Sigma}{\delta \psi_+} \ , \quad \mathcal{S}_\Sigma \overline{\Omega}_- = -\frac{\delta \Sigma}{\delta \psi_-} \ , \\
& \mathcal{S}_\Sigma \Omega_+ = -\frac{\delta \Sigma}{\delta \overline{\psi}_+} \ , \quad \mathcal{S}_\Sigma \Omega_- = \frac{\delta \Sigma}{\delta \overline{\psi}_-} \ , \\
& \mathcal{S}_\Sigma \rho^* = \frac{\delta \Sigma}{\delta \varphi} \ , \quad \mathcal{S}_\Sigma \rho = -\frac{\delta \Sigma}{\delta \varphi^*} \ .
\end{aligned} \tag{15}
$$

In order to study the stability [15] of the action (4) under the radiative corrections, one has to find the most general counterterm, Σ^c, satisfying the following condition of BRS invariance:

$$
\mathcal{S}_\Sigma \Sigma^c = 0 \ . \tag{16}
$$

Σ^c is an integrated local polynomial in the fields and its derivatives with UV dimension ≤ 3, IR dimension ≥ 3 and with vanishing ghost number. It has to be invariant under the P-symmetry given by Eqs.(8), and it has also to satisfy the conditions

$$
\frac{\delta \Sigma^c}{\delta B} = 0 \ , \quad \frac{\delta \Sigma^c}{\delta \overline{c}} = 0 \ , \quad \frac{\delta \Sigma^c}{\delta c} = 0 \ , \tag{17}
$$

which follow from the conditions (7.a – 7.c), and, moreover:

$$W_{\rm rigid}\Sigma^c = 0 \quad , \tag{18}$$

where $W_{\rm rigid}$ is the Ward operator of rigid symmetry defined by

$$\begin{aligned} W_{\rm rigid} = \int d^3x \bigg\{ & q\psi_+\frac{\delta}{\delta\psi_+} + q\psi_-\frac{\delta}{\delta\psi_-} - q\overline{\psi}_+\frac{\delta}{\delta\overline{\psi}_+} - q\overline{\psi}_-\frac{\delta}{\delta\overline{\psi}_-} + Q\varphi\frac{\delta}{\delta\varphi} - Q\varphi^*\frac{\delta}{\delta\varphi^*} + \\ & + q\Omega_+\frac{\delta}{\delta\Omega_+} + q\Omega_-\frac{\delta}{\delta\Omega_-} - q\overline{\Omega}_+\frac{\delta}{\delta\overline{\Omega}_+} - q\overline{\Omega}_-\frac{\delta}{\delta\overline{\Omega}_-} + Q\rho\frac{\delta}{\delta\rho} - Q\rho^*\frac{\delta}{\delta\rho^*}\bigg\} \quad . \end{aligned} \tag{19}$$

Eq.(18) follows from the rigid $U(1)$ invariance of the action[4]:

$$W_{\rm rigid}\Sigma = 0 \quad . \tag{20}$$

We find that the most general invariant counterterm Σ^c, i.e. the most general field polynomial of UV and IR dimensions bounded by $d{\leq}3$ and $r{\geq}3$, with ghost number zero, respecting P-symmetry, (8) and the conditions displayed in Eqs.(16), (17) and (18), is given by an arbitrary superposition of the following expressions:

$$\begin{aligned} \Big\{ & F^{mn}F_{mn} \ , \ \ i(\overline{\psi}_+ \not{D}\psi_+ + \overline{\psi}_- \not{D}\psi_-) \ , \ \ (\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-) \ , \\ & (\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi \ , \ \ D^m\varphi^* D_m\varphi \ , \ \ \varphi^*\varphi \ , \ \ (\varphi^*\varphi)^2 \ , \ \ (\varphi^*\varphi)^3 \Big\} \quad . \end{aligned} \tag{21}$$

The BRS consistency condition in the sector of ghost number zero, given by Eq.(16), constitutes a cohomology problem due to the nilpotency (14) of the linearized Slavnov-Taylor operator (12). Its solution can always be written as a sum of a trivial cocycle $\mathcal{S}_\Sigma\widehat{\Sigma}$, where $\widehat{\Sigma}$ has ghost number -1, and a nontrivial part $\Sigma_{\rm phys}$ belonging to the cohomology of $\mathcal{S}_\Sigma$ (12) in the sector of ghost number zero, i.e. which cannot be written as a $\mathcal{S}_\Sigma$-variation:

$$\Sigma^c = \Sigma_{\rm phys} + \mathcal{S}_\Sigma\widehat{\Sigma} \quad . \tag{22}$$

One checks indeed that the general invariant counterterm, expanded in the basis (21), admits the representation (22), with

$$\begin{aligned} \Sigma_{\rm phys} = & \ z_g\left(g\frac{\partial}{\partial g} - N_A + N_B - 2\xi\frac{\partial}{\partial\xi}\right)\Sigma + z_{m_0}\, m_0\frac{\partial\Sigma}{\partial m_0} + \\ & + z_y\, y\frac{\partial\Sigma}{\partial y} + z_{\mu^2}\,\mu^2\frac{\partial\Sigma}{\partial\mu^2} + z_\zeta\,\zeta\frac{\partial\Sigma}{\partial\zeta} + z_\lambda\,\lambda\frac{\partial\Sigma}{\partial\lambda} \quad , \end{aligned} \tag{23.a}$$

$$\begin{aligned} \mathcal{S}_\Sigma\widehat{\Sigma} = & \ \mathcal{S}_\Sigma\int d^3x\,\left[z_\psi\left(\overline{\psi}_+\Omega_+ - \overline{\Omega}_+\psi_+ - \overline{\psi}_-\Omega_- + \overline{\Omega}_-\psi_-\right) + z_\varphi\left(\rho^*\varphi - \varphi^*\rho\right)\right] \\ = & \ z_\psi\left(N_{\psi_+} + N_{\overline{\psi}_+} + N_{\psi_-} + N_{\overline{\psi}_-} - N_{\Omega_+} - N_{\overline{\Omega}_+} - N_{\Omega_-} - N_{\overline{\Omega}_-}\right)\Sigma \\ & + z_\varphi\left(N_\varphi + N_{\varphi^*} - N_\rho - N_{\rho^*}\right)\Sigma \quad , \end{aligned} \tag{23.b}$$

where the counting operators are defined by

$$N_\phi = \int d^3x\ \phi\,\frac{\delta}{\delta\phi} \quad , \quad \phi = \psi_\pm,\ \overline{\psi}_\pm,\ \Omega_\pm,\ \overline{\Omega}_\pm,\ \varphi,\ \varphi^*,\ \rho,\ \rho^*,\ A_m \ \text{and}\ B \quad . \tag{24}$$

This way of writing the counterterm makes explicit the separation between the physical counterterms, on the one hand, which amount to the renormalization of the physical masses and coupling constants m_0, μ, g, y, ζ, λ, and the trivial ones, on the other hand, which correspond to the unphysical renormalization of the amplitudes of the fields $\psi_\pm$ and φ – the other field renormalizations not being independent. The form of the classical action Σ (4), taken as a P-invariant solution of the functional identities expresssing

[4]Rigid invariance itself follows from the antighost equation (7.c) and from the validity of the Slavnov-Taylor identity (10).

the various symmetries of the theory, is thus stable under small perturbations, the general solution in a neighbourhood of Σ being obtained through an arbitrary variation of the parametrization.

At the quantum level the vertex functional Γ, which coincides with the classical action (4) at order 0 in $\hbar$:

$$\Gamma = \Sigma + \mathcal{O}(\hbar) \quad , \tag{25}$$

has to satisfy the constraints

$$\frac{\delta\Gamma}{\delta B} = \partial^m A_m + \xi B \quad , \tag{26.a}$$

$$\frac{\delta\Gamma}{\delta \overline{c}} = \Box c \quad , \tag{26.b}$$

$$-i\frac{\delta\Gamma}{\delta c} = \Delta_{\rm class} \quad , \tag{26.c}$$

$$W_{\rm rigid}\Gamma = 0 \quad , \tag{26.d}$$

where $W_{\rm rigid}$ has already been defined by equation (19) and Eqs.(26.a – 26.c) are the quantum extension of Eqs.(7.a – 7.c).

According to the Quantum Action Principle [16, 17] the Slavnov-Taylor identity (10) gets a quantum breaking

$$\mathcal{S}(\Gamma) = \Delta\cdot\Gamma = \Delta + \mathcal{O}(\hbar\Delta) \quad , \tag{27}$$

where Δ is an integrated local functional with ghost number 1 and dimension 3.

The nilpotency identity (13.a) together with

$$\mathcal{S}_\Gamma = \mathcal{S}_\Sigma + \mathcal{O}(\hbar) \tag{28}$$

implies the following consistency condition for the breaking Δ:

$$\mathcal{S}_\Sigma\Delta = 0 \quad . \tag{29}$$

Other constraints on Δ follow from the constraints (26.a – 26.d) and from the algebra

$$\frac{\delta\mathcal{S}(\mathcal{F})}{\delta B} - \mathcal{S}_\mathcal{F}\left(\frac{\delta\mathcal{F}}{\delta B} - \partial^m A_m - \xi B\right) = \frac{1}{g}\left(\frac{\delta\mathcal{F}}{\delta\overline{c}} - \Box c\right) \quad , \tag{30.a}$$

$$\frac{\delta\mathcal{S}(\mathcal{F})}{\delta\overline{c}} + \mathcal{S}_\mathcal{F}\frac{\delta\mathcal{F}}{\delta\overline{c}} = 0 \quad , \tag{30.b}$$

$$-i\int d^3x\frac{\delta}{\delta c}\mathcal{S}(\mathcal{F}) + \mathcal{S}_\mathcal{F}\int d^3x\left(-i\frac{\delta}{\delta c}\mathcal{F} - \Delta_{\rm class}\right) = W_{\rm rigid}\mathcal{F} \quad , \tag{30.c}$$

$$W_{\rm rigid}\mathcal{S}(\mathcal{F}) - \mathcal{S}_\mathcal{F}W_{\rm rigid}\mathcal{F} = 0 \quad , \tag{30.d}$$

($\mathcal{F}$ arbitrary functional of ghost number zero) .

These constraints on the breaking Δ read:

$$\frac{\delta\Delta}{\delta B} = 0 \quad , \tag{31.a}$$

$$\frac{\delta\Delta}{\delta\overline{c}} = 0 \quad , \tag{31.b}$$

$$\int d^3x\frac{\delta}{\delta c}\Delta = 0 \quad , \tag{31.c}$$

$$W_{\rm rigid}\Delta = 0 \quad . \tag{31.d}$$

The Wess-Zumino consistency condition (29) constitutes a cohomology problem like in the zero ghost number case (16). Its solution can always be written as a sum of a trivial cocycle $\mathcal{S}_\Sigma\widehat{\Delta}^{(0)}$, where $\widehat{\Delta}^{(0)}$ has ghost number 0, and of nontrivial elements belonging to the cohomology of $\mathcal{S}_\Sigma$ (12) in the sector of ghost number one:

$$\Delta^{(1)} = \widehat{\Delta}^{(1)} + \mathcal{S}_\Sigma\widehat{\Delta}^{(0)} \quad , \tag{32}$$

where $\Delta^{(1)}$ must be even under P-symmetry and obey the conditions imposed by Eqs. (31.a – 31.d). The trivial cocycle $\mathcal{S}_\Sigma\widehat{\Delta}^{(0)}$ can be absorbed into the vertex functional Γ as a noninvariant integrated local couterterm $-\widehat{\Delta}^{(0)}$. On the other hand, a nonzero $\Delta^{(1)}$ would represent an anomaly.

Considering the condition (31.c), to be satisfied by (32), it can be concluded that

$$\Delta^{(1)} = \int d^3x \; K_m^{(0)} \, \partial^m c \quad . \tag{33}$$

By analyzing the Slavnov-Taylor operator $\mathcal{S}_\Sigma$ (12) and the Wess-Zumino consistency condition (29), one sees that the breaking $\Delta^{(1)}$ has UV and IR dimensions bounded by $d\leq\frac{7}{2}$ and $r\geq 2$. Therefore, the dimensions of $K_m^{(0)}$ must be bounded by $d\leq\frac{5}{2}$ and $r\geq 1$, it has ghost number 0, and due to Eq.(29) and Eqs. (31.a – 31.b), it must respect the conditions

$$\frac{\delta K_m^{(0)}}{\delta B} = 0 \quad \text{and} \quad \frac{\delta K_m^{(0)}}{\delta \overline{c}} = 0 \quad . \tag{34}$$

Now, rewriting $K_m^{(0)}$ as a linear combination

$$K_m^{(0)} = \sum_{i=1}^{7} a_i \; K_m^{(0)i} \quad , \tag{35}$$

where

$$\begin{aligned} &K_m^{(0)1} = A_m A^n A_n \;\; , \;\; K_m^{(0)2} = A_m (A^n A_n)^2 \;\; , \;\; K_m^{(0)3} = A_m (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) \;\; , \\ &K_m^{(0)4} = A_m A^n A_n \varphi^* \varphi \;\; , \;\; K_m^{(0)5} = A_m (\varphi^* \varphi)^2 \;\; , \;\; K_m^{(0)6} = A_m \varphi^* \varphi \;\; , \\ &K_m^{(0)7} = \overline{\psi}_+ \gamma_m \psi_+ + \overline{\psi}_- \gamma_m \psi_- \quad , \end{aligned} \tag{36}$$

and solving all the conditions it has to fulfil, we can easily show, with the help of Eqs.(15), that there exist local functionals $\widehat{\Delta}^{(0)i}$ such that

$$\int d^3x \; K_m^{(0)i} \, \partial^m c = \mathcal{S}_\Sigma \widehat{\Delta}^{(0)i} \;\; , \quad i = 1, \cdots, 7 \quad . \tag{37}$$

This means $\widehat{\Delta}^{(1)}$=0 in (32), which implies the implementability of the Slavnov-Taylor identity to every order through the absorbtion of the noninvariant counterterm $-\sum_i a_i \widehat{\Delta}^{(0)i}$.

Of course, invariant counterterms may still be arbitrarily added at each order. However the result of the discussion on the stability of the classical theory shows that these counterterms correspond to a renormalization of the parameters of the theory. Their coefficients have to be fixed by suitable normalization conditions.

In conclusion, we have shown the renormalizability and absence of gauge anomaly for the parity-preserving QED_3 coupled to scalar matter in the symmetric phase.

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