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FIELD THEORETICAL APPROACH TO PROTON-NUCLEUS REACTIONS. I - ONE STEP INELASTIC SCATTERING

by

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ABSTRACT

In this work we obtain a closed form expression to the double differential cross section for one step proton-nucleus reaction within a field theoretical framework. Energy and momentum conservation as well as nuclear structure effects are consistently taken into account within the field theoretical eikonal approximation. In our formulation the kinematics of such reaction is not dominated by the free nucleon-nucleon cross section but a new factor which we call relativistic differential cross section in a Born Approximation.

Key-words: Proton-nucleus reaction; Field theory; Glauber approximation.

INTRODUCTION

Recent experiments in nuclear physics, most of them performed with heavy ion or high precision electron beams at relativistic energies revealed the inadequace of traditional nuclear theories to describe many aspects of nature (ref.1). Specially, the important role played by subnuclear degrees of freedom has become most conspicuous. Inspite of this fact theoretical description of nuclei based on QCD are still in their infancy. However simpler field theoretical models which treat nucleons as Dirac particles coupled to various mesonic fields (ref.2) have been developed and are quite successful in explaining data as well as in given strong theoretical basis to some of the important characteristics of the traditional nuclear forces as the spin-orbit interaction.

It is our belief that field theoretical description of nuclear collisions is a very important topic in nuclear theory and the aim of the present paper is to develop a method for inelatic proton-nucleus scattering where the proton is treated as a Dirac particle and the nucleus can be treated as a composite particle. The formulation of such reaction processes of field theory is a necessary first step for the future description of nuclear processes with explicit inclusion of quarks and gluons. This work remains at the level of nucleons and mesons as elementary nuclear constituents even though it serves the purpose of proving a Glauber's multiple diffraction model (ref.3). One important advantage of our approach is that energy and momentum conservation as well as nuclear structure effects are correctly included. Recently relativistic Coulomb excitation in heavy ion collisions has been described in the context of QED (ref.4) and the total cross sections obtained (ref.5). In the case of high energy inelastic proton-nucleus reaction (about 1 GeV), the experimental observations (ref.6) cannot be accounted for in the framework of Glauber's theory. In this case, spectra cannot be obtained since the theory only allows for a description of angular distributions. Therefore Bertsch and Scholten (ref.7) developed a phenomenological model to describe spectra in such reaction. This model for the double differential cross section contains ingredients of Glauber's theory , the experimental free nucleon-nucleon cross section as the basic kinematical factor and a nuclear response function, should describes the dynamics of the process. As shown in ref.(7), the model seems to be fairly successful for some kinematical regions. Nevertheless, their hypothesis of factorizing the nucleon-nucleon scattering cross section at very low energy and momentum transfers seems to us to be inadequate. At excitation energies about 10 MeV, nuclear structure effects, specifically nuclear mean field effects cannot be disentangled from the kinematics of nucleon-nucleon scattering. In fact the work of Ref.(7) in this kinematical region shows discrepancies with the data where they are attributed to Pauli blocking effects in the nuclear response function which is introduced in a phenomenological way. We show for low momentum and energy tranfered that the kinematics of the reaction is strongly influenced by the mean field of the nucleus and that an adequate treatmet of such kinematical effects already reproduces the qualitative behaviour of the data. An analysis of the dynamical part of the cross section (the response function) together with the correct kinematical factor constitutes an interesting next step in describing the data quantitatively.

In our scheme, the proton is treated as a Dirac particle which interacts with the nucleus via exchange of scalar and vector mesons. When the eikonal approximation is introduced, we are able to sum up Feynman graph contributing to the process to all orders. The generalization of this scheme to multiple scattering is tecnically involved but straighforward (ref.3). The necessary approximations to retrieve Glauber's multiple scattering formula (ref.8) become apparent. We shall here present the method and focus our attention in the double differential cross section for one step inelastic scattering.

In section I we derive the eikonal Feynman amplitude and in the section II we are presenting the static limit and the connection with potential scattering theory. The relativistic features of the theory remain even in this limit. In section III we give the double differential cross section formula and discuss the main differences with available models. Finally, in Section IV we collect some concluding remarks.

I. EIKONAL FEYNMAN AMPLITUDE

We consider the scattering process of a high energy proton on a target nucleus interacting via the exchange of scalar and vector meson according to Walecka's model (Ref.2), whose interaction Lagrangian density is given by

$$\mathcal{L} = g_s \bar{\Psi}\Psi\phi - g_v \bar{\Psi}\gamma_\mu \Psi V^\mu \tag{I.1}$$

where Ψ is the nuclear spinor operator, ϕ is the scalar meson field and V_μ is the vector boson field. The quantities g_s and g_v are the coupling constants of these interactions, respectively.

When the four momentum transfer associated to each meson exchanged is small, it is a good approximation to express the scattering amplitude as a sum of Feynman diagrams in which meson fields are exchanged between the world line of the incident proton and the target nucleus without any bubble diagrams. We define one step excitation as those diagrams where the nuclear excitation is caused at one of the vertices. A typical diagram of the one step excitation process is illustrated in Fig.(1). There we are representing a process where the incident proton and the target nucleus exchange n intermediate scalar mesons and m vector bosons in addition to the one (scalar or vector) which causes the nuclear excitation, indicated in the figure from the r-th vertex to s-th vertex by a heavy line.

For the scattering processes involving small momentum transfer, it is always possible to choose a reference frame where the target nucleus can be treated nonrelativistically during the whole process (ref.4). In such a system, one can separate the nuclear center of mass motion from its intrinsic degree of freedom and all nuclear matrix elements can be treated with the standard nonrelativistic nuclear physics.

The scattering amplitute $\mathcal{M}$ can be expressed as

$$\mathcal{M} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \mathcal{M}_{n,m} \tag{I.2}$$

where $\mathcal{M}_{n,m}$ refers to the amplitude corresponding to the sum over distinct diagrams of the type indicated in Fig.(1). We write it as

$$i\mathcal{M}_{n,m} = \left[\frac{1}{(2\pi)^4}\right]^{N=n+m+1} (2\pi)^4 \int \prod_{i=1}^{N} d^4k_j \, \mathcal{P} \, \delta\left(q - \sum_{j=1}^{N} k_j\right) \tag{I.3}$$

where q is the four-momentum transfer of the process

$$q = \left(p_a - p_a'\right) = -\left(P_b - P_b'\right) \tag{I.4}$$

and k_j's stand for the four momenta of exchanged mesons. The term $\mathcal{P}$ represents a matrix element of a sum of products of propagators and vertex operators in an appropriate order taken between the final and initial states of the system.

The δ-function in eq.(I-3) can be used to eliminate one of the k-integrals. We choose the r-th meson momentum to induce the nuclear excitation. This arbitrariness will be eliminated through a symmetrization process (ref.9).

Since the nuclear excitation may take place at any vertex, we have sum over all possibles contributions as

$$\mathcal{P} = \sum_{r=1}^{N} \mathcal{P}_r \tag{I.5}$$

The quantity $\wp_r$ is then written

$$\wp_r = \sum_D \prod_{j=1}^{N} \Delta_F(k_j) \Big[\bar{\mathcal{U}}(p'_a)\, \Gamma_N\, S(p'_a+k_N)\, \Gamma_{N-1}\, S(p'_a+k_N+k_{N-1}) \times \ldots$$

$$\times \Gamma_{r+1}\, S(p'_a+k_N+\ldots+k_{r+1})\, \Gamma_r\, S(p_a-k_1-k_2-\ldots-k_{r-1})\, \Gamma_{r-1} \times \ldots$$

$$\times \Gamma_2\, S(p_a-k_1)\, \Gamma_1\, \mathcal{U}(p_a) \Big] \times \langle P'_b;f|\hat{F}(k'_N)|P'_b;f\rangle \langle P'_b;f|\hat{G}(P'_b-k'_N)|P'_b;f\rangle$$

$$\times \langle P'_b;f|\hat{F}(k'_{N-1})|P'_b;f\rangle \times \langle P'_b;f|G(P'_b-k'_N-k'_{N-1})|P'_b;f\rangle \times \langle P'_b;f|\hat{F}(k'_{N-2})|P'_b;f\rangle \times$$

$$\times \ldots \times \langle P'_b;f|G(P'_b-k'_N-\ldots-k'_{s-1})|P'_b;f\rangle \times \langle P_b;f|\hat{F}\Big(q-\sum_{i\neq r}^{N} k_i\Big)|P_b;qs\rangle \times$$

$$\times \langle P_b;qs|G(P_b+k'_1+\ldots+k'_{s-1})|P_b;qs\rangle \times \ldots \times \langle P_b;qs|\hat{F}(k'_1)|P_b;qs\rangle \qquad \text{(I.6)}$$

where $\sum_D$ stands for the sum over all distinct diagrams, and k's are the momenta exchanged mesons in the chronological order of absorption in the nucleus world line. In other words, the ordered set$\{k'_1, k'_2, \ldots, k'_N\}$is a suitable permutation of$\{k_1, \ldots, k_N\}$ according to a particular diagram D. The operators $\hat{F}(k)$ are defined as

$$\hat{F}_s(k) = \int d^3\xi\, \exp\left[i\vec{k}.\vec{\xi}\right] \bar{\Psi}(\xi)\, \psi(\xi) \qquad \text{(I.7a)}$$

and

$$\hat{F}^{\mu}(k) = \int d^3k \, \exp[i\vec{k}.\vec{\xi}]\, \bar{\psi}(\xi)\, \gamma^{\mu}\, \psi(\xi) \tag{I.7b}$$

for the scalar and vector vertex interaction respectively. The coordinate ξ are the internal nucleons coordinate in the nucleus rest frame.

Let us now proceed to define all quantities which appear in eq.(I.6). In our model, the incident proton is treated as a structureless particle of mass m which satisfies the Dirac equation. So, its evolution is given by a fermionic Feynman propagator

$$S_F(p) = \frac{\gamma_{\mu}p^{\mu} + m}{p^2 - m^2 + i\epsilon}$$

while the nucleus is treated in its proper system, such that its relativistic Green's function can be expressed as

$$\hat{G}(P) = \frac{1}{\sqrt{P^2} - \hat{H}_N + i\epsilon}$$

Here, $\hat{H}_N$ is the nuclear Hamiltonian operator which includes the nuclear rest mass, M (ref.4). It is convenient to say that we are using the Bjorken-Drell convention (ref.10).

Now let us introduce the eikonal approximation which consist of dropping $\gamma^{\mu}k_{\mu}$ terms in the numerators of nucleon propagators. Moreover, we are also neglecting the quadratic terms in momentum transfer in the denominator of these same propagators, so that we can rewrite them as (ref.11)

$$S_F(p\pm k)\sim \frac{\gamma_\mu p^\mu + m}{\pm 2pk + i\epsilon}$$

Within this approximation the spinorial part of proton variables can be simply replaced by C-numbers in eq.(I.6) through the use of Dirac equation in the same spirit as in section C of the ref.(9). Then we will establish a practical rule as following:

$$\Gamma S_F(p'+k') \to \frac{1}{\frac{p'k'}{m}+i\epsilon} \qquad \text{and} \qquad S_F(p-k)\Gamma \to \frac{1}{\frac{-pk}{m}+i\epsilon} \tag{I.8}$$

where $\lambda = \lambda' = g_s \mathbb{1}$ when the Γ matrices originate from scalar mesons, and $\lambda(\lambda') = g_v \frac{p^\mu}{m}\left(\frac{p'^\mu}{m}\right)$ if from the vector mesons. Thus the spinor wavefunctions $\bar{u}(p')$ and $u(p)$ commute with all factors in expression (I.6) up to r-th vertex, where a factor $\bar{u}(p')\,\Gamma_r\,u(p)$ is generated.

On the nucleus world line, we also use the eikonal approximation for the nuclear propagator, so that they can be rewritten as

$$\langle P_0;qs|\hat{G}(P+K)|P_0;qs\rangle \equiv \langle P_0;qs|\frac{1}{\sqrt{(P+K)^2-\hat{H}_N+i\epsilon}}|P_0;qs\rangle \sim \frac{1}{\frac{PK}{M}+i\epsilon} \tag{I.9a}$$

and

$$\langle P_0';f|\hat{G}(P'-K)|P_0';f\rangle \equiv \langle P_0';f|\frac{1}{\sqrt{(P'-K)^2-\hat{H}_N+i\epsilon}}|P_0';f\rangle \sim \frac{1}{\frac{-P'K}{M'}+i\epsilon} \tag{I.9b}$$

where we have used $P^2 = M^2$ and $P'^2 = M'^2$. Inserting (I.8) and

(I.9) in (I.6), we get

$$\mathcal{P}_r = \sum_D \bar{u}(p_a')\,\Gamma_r\, u(p_a)\,\Delta_F(k_r)\langle P_b';f|\hat{F}(k_r)|P_b;qs\rangle \prod_{i=1}^{s-1}\Delta_F(\bar{k}_i)\times$$

$$\times\langle P_b;qs|\hat{F}(\bar{k}_i)|P_b;qs\rangle \prod_{j=s+1}^{N}\Delta_F(\bar{k}_j)\,\langle P_b';f|\hat{F}(\bar{k}_j)|P_b';f\rangle\times\frac{\lambda_N}{\frac{p_a'k_N}{m}+i\epsilon}\times$$

$$\times\frac{\lambda_{N-1}}{\frac{p_a'(k_N+k_{N-1})}{m}+i\epsilon}\times\cdots\times\frac{\lambda_1}{-\frac{p_ak_1}{m}+i\epsilon}\times\frac{1}{-\frac{P_b'k_N'}{M}+i\epsilon}\times\cdots\times\frac{1}{\frac{P_bk_1'}{M}+i\epsilon} \qquad (I.10)$$

Here we have neglected the nuclear recoil in the propagators of the form (I.9).

The sum over all Feynman diagrams will be organized in the following way. First we define a class of diagrams characterized by a set of numbers $\{\alpha 11', \alpha 12', \alpha 21', \alpha 22', \beta 11', \beta 12', \beta 21', \beta 22'\}$ which symbolically represent:

$\alpha 11'$: the total number of scalar mesons emitted before the r-th vertex and absorbed before the s-th vertex;

$\alpha 12'$: the total number of scalar mesons emitted before the r-th vertex and absorbed after the s-th vertex;

$\alpha 21'$: the total number of scalar mesons emitted after the r-th vertex and absorbed before the s-th vertex;

$\alpha 22'$: the total number of scalar mesons emitted after the r-th vertex and absorbed after the s-th vertex.

The numbers for the vector bosonic field are defined in a similar manner (Fig.2). These numbers defined above satisfy the following relations:

$$n = \alpha 11' + \alpha 12' + \alpha 21' + \alpha 22'$$

$$m = \beta 11' + \beta 12' + \beta 21' + \beta 22'$$

$$[(r-1)] = \alpha 11' + \alpha 12' + \beta 11' + \beta 12'$$

$$[n+m-(r-1)] = \alpha 21' + \alpha 22' + \beta 21' + \beta 22'$$

$$(s-1) = \alpha 11' + \alpha 21' + \beta 11' + \beta 21'$$

$$[n+m-(s-1)] = \alpha 12' + \alpha 22' + \beta 12' + \beta 22'$$

Let us denote the set of vertices on the world line of the nucleus before S by $\pi 1$ and the set of vertices after S by $\pi 2$. Analogously, $\pi 3$ and $\pi 4$ correspond to the sets of vertices along the world line of the incident nucleon before and after the r-th vertex, respectively.

For a given set of numbers ($\alpha 11', \alpha 12', \ldots$) any other distinct diagram in this class can be obtained by interchanging the emittion and absorption points of the meson lines in it of the sets $\pi 1$, $\pi 2$, $\pi 3$ and $\pi 4$. Therefore the sum of all diagrams belonging to this class can be obtained as a sum over all possible permutations $\pi 1$, $\pi 2$, $\pi 3$ and $\pi 4$ independently. In this process we will obtain some repeted diagrams which can be easily discounted through the factor

$$Z = \frac{1}{\alpha 11'!\,\alpha 12'!\,\alpha 21'!\,\alpha 22'!\,\beta 11'!\,\beta 12'!\,\beta 21'!\,\beta 22'!} \qquad (I.11)$$

Finally, the sum over all distinct Feynman diagrams can be obtained as a sum over different classes (C), that is, over differents numbers ($\alpha(11')$,....) which is denoted as $\sum_{\{C\}}$ followed by the sum with respect to the two distinct excitation mechanisms at the s-th vertex:

$$\sum_{D} = \sum_{s,v} \sum_{\{C\}} z \sum_{\pi 1} \sum_{\pi 2} \sum_{\pi 3} \sum_{\pi 4} \tag{I.12}$$

The last identity is applied only to the product of proton and nucleus propagators since all other factors are invariant under such permutations for a given class (C). Then, with the help of the identity (ref.9)

$$\sum_{\substack{\text{permutation} \\ (A_i, A_j)}} \frac{1}{A_1} \times \frac{1}{(A_1+A_2)} \times \cdots \frac{1}{(A_1+A_2+\cdots+A_n)} = \prod_{\ell=1}^{n} \left(\frac{1}{A_\ell} \right) \tag{I.13}$$

we have

$$\mathfrak{P}_r = \sum_{s,v} \sum_{\{c\}} z\, \bar{u}(p_a') \Gamma_r u(p_a) \Delta_F(k_r) \langle P_b'; f | \hat{F}(k_r) | P_b; qs \rangle \times$$

$$\prod_{i=1}^{s-1} \Delta_F(\bar{k}_i) \langle P_b; qs | \hat{F}(\bar{k}_i) | P_b; qs \rangle \prod_{j=s+1}^{N} \Delta_F(\bar{k}_j) \langle P_b'; f | \hat{F}(\bar{k}_j) | P_b'; f \rangle \times$$

$$\prod_{\alpha=1}^{r-1} \frac{1}{\frac{p_a' k_\alpha}{m} + i\epsilon} \prod_{\beta=r+1}^{N} \frac{1}{\frac{p_a' k_\beta}{m} + i\epsilon} \prod_{\gamma=1}^{s-1} \frac{1}{\frac{P_b k_\gamma}{M} + i\epsilon} \prod_{\theta=s+1}^{N} \frac{1}{\frac{-P_b' k_\theta}{M} + i\epsilon} \tag{I.14}$$

Defining the transition operator as

$$\hat{T}_s(x) = q_s^2\, \bar{u}(p_a')\, u(p_a) \int \frac{d^4k}{(2\pi)^4}\, \Delta_F^s(k)\, \hat{F}_s(k)\, \exp\left[ikx\right] \qquad (I.15)$$

and

$$\hat{T}_v(x) = q_v^2\, \bar{u}(p_a')\, \gamma_\mu\, u(p_a) \int \frac{d^4k}{(2\pi)^4}\, \Delta_F^v(k)\, \hat{F}_v^\mu(k)\, \exp\left[ikx\right] \qquad (I.16)$$

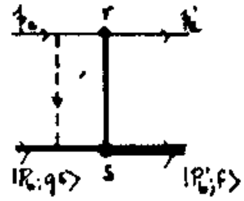
we can use the inverse Fourier transfor and the fact that $k_r = q - \sum_{i \neq r}^{N} k_i$ to obtain:

$$q_s^2\, \bar{u}(p_a')\, u(p_a)\, \Delta_F^s\Big(q - \sum_{i\neq r}^{N} k_i\Big)\, \hat{F}_s\Big(q - \sum_{i\neq r}^{N} k_i\Big) = \int d^4x\, \hat{T}_s(x)\, \exp\Big[-i\Big(q - \sum_{i\neq r}^{N} k_i\Big)x\Big] \qquad (I.17)$$

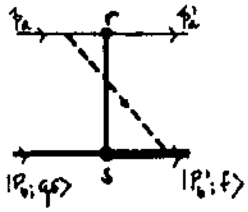
and

$$q_v^2\, \bar{u}(p_a')\, \gamma_\mu\, u(p_a)\, \Delta_F^v\Big(q - \sum_{i\neq r}^{N} k_i\Big)\, \hat{F}_v^\mu\Big(q - \sum_{i\neq r}^{N} k_i\Big) = \int d^4x\, \hat{T}_v(x)\, \exp\Big[-i\Big(q - \sum_{i\neq r}^{N} k_i\Big)x\Big] \qquad (I.18)$$

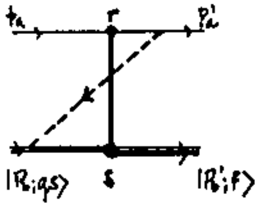
where these definitions correspond to scalar and vector mesonic fields respectively. Then, we can substitute these expressions in eq. (I.14) and find that all k- integrals are completely factorized. Moreover these k-integrals may be classified into eight different types according to the nature of the mesonic fields and the positions of emittion and absorption, which are expressed as:



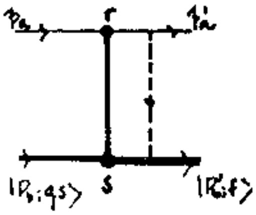
$$I_1^S = g_s^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^S(k) \langle P_b;qs|\hat{F}_S(k)|P_b;qs\rangle \frac{1}{\frac{-p_a k}{m}+i\epsilon} \times \frac{1}{\frac{P_b k}{M}+i\epsilon}$$



$$I_2^S = g_s^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^S(k) \langle P'_b;f|\hat{F}_S(k)|P'_b;f\rangle \frac{1}{\frac{-p_a k}{m}+i\epsilon} \times \frac{1}{\frac{-P'_b k}{M}+i\epsilon}$$

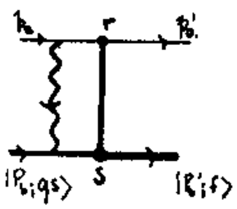


$$I_3^S = g_s^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^S(k) \langle P_b;qs|\hat{F}_S(k)|P_b;qs\rangle \frac{1}{\frac{p'_a k}{m}+i\epsilon} \times \frac{1}{\frac{P_b k}{M}+i\epsilon}$$

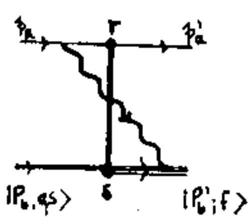


$$I_4^S = g_s^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^S(k) \langle P'_b;f|\hat{F}_S(k)|P'_b;f\rangle \frac{1}{\frac{p'_a k}{m}+i\epsilon} \times \frac{1}{\frac{-P'_b k}{M}+i\epsilon}$$

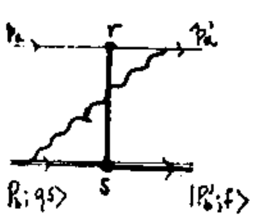
for the scalar mesons, while for the vector ones, we have:



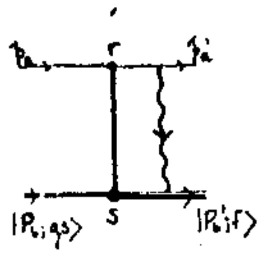
$$I_1^V = g_v^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^V(k) \left(\frac{p_a}{m}\right)_\mu \langle P_b;qs|\hat{F}_V^\mu(k)|P_b;qs\rangle \frac{1}{\frac{-p_a k}{m}+i\epsilon} \times \frac{1}{\frac{P_b k}{M}+i\epsilon}$$



$$I_2^V = g_v^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^V(k) \left(\frac{p_a}{m}\right)_\mu \langle P'_b;f|\hat{F}_V^\mu(k)|P'_b;f\rangle \frac{1}{\frac{-p_a k}{m}+i\epsilon} \times \frac{1}{\frac{-P'_b k}{M}+i\epsilon}$$

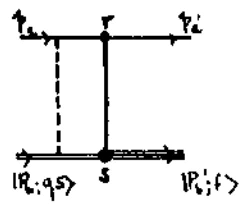


$$I_3^V = g_v^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^V(k) \left(\frac{p'_a}{m}\right)_\mu \langle P_b;qs|\hat{F}_V^\mu(k)|P_b;qs\rangle \frac{1}{\frac{p'_a k}{m}+i\epsilon} \times \frac{1}{\frac{P_b k}{M}+i\epsilon}$$

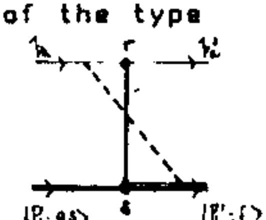


$$I_4^V = g_V^2 \int \frac{d^4k}{(2\pi)^4} \exp[ikx]\, \Delta_F^V(k) \left(\frac{P_a'}{m}\right)_\mu \langle P_b';f|\hat{F}_V^\mu(k)|P_b';f\rangle \frac{1}{\frac{P_a'k}{m}+i\epsilon} \times \frac{1}{\frac{-P_b'k}{M}+i\epsilon}$$

In this graphical representation we find that in eq.(I.14) there are $\alpha_{11'}$ integrals of the type



or a number $\alpha_{12'}$ of the type



and so on. Therefore, collecting all of them, we get

$$iM_{n,m} = \int d^4x \, \exp[-iqx] \langle P_b';f|\hat{T}_V(x)+\hat{T}_S(x)|P_b;qs\rangle \times$$

$$\times \sum_{\{C\}} Z \Big[(I_1^S)^{\alpha_{11'}} \times (I_2^S)^{\alpha_{12'}} \times (I_3^S)^{\alpha_{21'}} \times (I_4^S)^{\alpha_{22'}} \times$$

$$(I_1^V)^{\beta_{11'}} \times (I_2^V)^{\beta_{12'}} \times (I_3^V)^{\beta_{21'}} \times (I_4^V)^{\beta_{22'}} \Big] \tag{I.19}$$

We must to remember that we have chosen the r-th vertex as the one where the effective nuclear excitation occurs arbitrarilly. Then, since the nuclear excitation may be take place at any vertex, we can use eq.(I.5). Now, noting that the sum over $\{C\}$ can be written explicitely as

$$\sum_{\{C\}} = \sum_{\substack{n=n_1+n_2 \\ m=m_1+m_2}} \delta_{(n_1+m_1),(r-1)} \sum_{n_1=\alpha_{11'}+\alpha_{12'}} \sum_{n_2=\alpha_{21'}+\alpha_{22'}} \sum_{m_1=\beta_{11'}+\beta_{12'}} \sum_{m_2=\beta_{21'}+\beta_{22'}} \qquad (I.20)$$

where $\delta_{(n_1+m_1),(r-1)}$ is the kronecker δ, the successive use of Newton's binomial formula leads to

$$iM_{n,m} = \int d^4x \, \exp[-iqx] \langle P_b'; f | \hat{T}_v(x) + \hat{T}_s(x) | P_b; qs \rangle \times \frac{1}{n!} \times \frac{1}{m!} \times$$

$$\left[I_1^s + I_2^s + I_3^s + I_4^s \right]^n \times \left[I_1^v + I_2^v + I_3^v + I_4^v \right]^m \qquad (I.21)$$

Finally, doing the limit $n \to \infty$ and $m \to \infty$ we can perform the sum over the differents mesonic fields, so that the eikonal Feynmam amplitude becomes defined by the eq.(I.2). Thus, we have

$$iM^{eik} = \int d^4x \, \exp[-iqx] \langle P_b'; f | \hat{T}_s(x) + \hat{T}_v(x) | P_b; qs \rangle \exp[i\chi(x)] \qquad (I.22)$$

where $\chi(x)$ is the relativistic eikonal phase factor, and defined as

$$\chi(x) = -i\left[U_1 + U_2 + U_3 + U_4\right] \qquad \text{(I.23)}$$

with the functions U given by

$$U_1(x;p_a,P_b) = \int \frac{d^4k}{(2\pi)^4} \exp[ikx] \langle gs| \left[g_v^2 \Delta_F^v(k) \left(\frac{p_a}{m}\right)_\mu \hat{F}_v^\mu(k) - g_s^2 \Delta_F^s(k) \hat{F}_s(k) \right] |gs\rangle \frac{1}{-\frac{p_a k}{m} + i\epsilon} \times \frac{1}{\frac{P_b k}{M} + i\epsilon}$$

$$U_2(x;p_a,P_b') = \int \frac{d^4k}{(2\pi)^4} \exp[ikx] \langle f| \left[g_v^2 \Delta_F^v(k) \left(\frac{p_a}{m}\right)_\mu \hat{F}_v^\mu(k) - g_s^2 \Delta_F^s(k) \hat{F}_s(k) \right] |f\rangle \frac{1}{-\frac{p_a k}{m} + i\epsilon} \times \frac{1}{-\frac{P_b' k}{M} + i\epsilon}$$

$$U_3(x;p_a',P_b) = \int \frac{d^4k}{(2\pi)^4} \exp[ikx] \langle gs| \left[g_v^2 \Delta_F^v(k) \left(\frac{p_a'}{m}\right)_\mu \hat{F}_v^\mu(k) - g_s^2 \Delta_F^s(k) \hat{F}_s(k) \right] |gs\rangle \frac{1}{\frac{p_a' k}{m} + i\epsilon} \times \frac{1}{\frac{P_b k}{M} + i\epsilon}$$

$$U_4(x;p_a',P_b') = \int \frac{d^4k}{(2\pi)^4} \exp[ikx] \langle f| \left[g_v^2 \Delta_F^v(k) \left(\frac{p_a'}{m}\right)_\mu \hat{F}_v^\mu(k) - g_s^2 \Delta_F^s(k) \hat{F}_s(k) \right] |f\rangle \frac{1}{\frac{p_a' k}{m} + i\epsilon} \times \frac{1}{-\frac{P_b' k}{M} + i\epsilon}$$

II. STATIC LIMIT AND CONNECTION TO THE POTENTIAL SCATTERING

In the scattering process of an incident proton on a relatively heavy nucleus, it is a good approximation to neglect the recoil kinetic energy of the target in the eikonal phase factor $\chi(x)$. We will assume that the nucleon motion into the target nucleus is nonrelativistic, such that we may neglect terms of order (v/c) compared to unity. This nonrelativistic reduction leave us with

$$F_{\mu}^{(v)}(k) \sim \delta_{\mu 0} F^{(s)}(k) \tag{II.1}$$

Another simplifying assumption is the following. If we consider that the nuclear deformation is not too large, it is a good approximation to take matrix elements of the vertex functions as

$$\langle f|\hat{F}_s(k)|f\rangle \sim \langle gs|\hat{F}_s(k)|gs\rangle \equiv F_{gs}(k) \tag{II.2}$$

where $F_{gs}(k)$ is the form factor of the nucleus in its ground state, as was shown in eq.(I.7a).

Under these approximations and following the scheme of proposed by M.Lévy and J.Sucher (ref.9), we finally get:

$$\underset{\substack{\text{static}\\ \text{limit}}}{\chi(\vec{r})} = -\int_0^\infty d\xi \left[V\left(\vec{r}+\frac{\vec{p}_a'}{m}\xi\right) + V\left(\vec{r}-\frac{\vec{p}_a}{m}\xi\right)\right] \tag{II.3}$$

where

$$V\left(\vec{r}\pm\frac{\vec{p}}{m}\xi\right) = \int d^3r'\,\rho(r')\left[g_v^2\,\frac{E_a}{m}\,\frac{e^{-m_v\left|\vec{r}'-\left(\vec{r}\pm\frac{\vec{p}}{m}\xi\right)\right|}}{\left|\vec{r}'-\left(\vec{r}\pm\frac{\vec{p}}{m}\xi\right)\right|} - g_s^2\,\frac{e^{-m_s\left|\vec{r}'-\left(\vec{r}\pm\frac{\vec{p}}{m}\xi\right)\right|}}{\left|\vec{r}'-\left(\vec{r}\pm\frac{\vec{p}}{m}\xi\right)\right|}\right] \tag{II.4}$$

Here, we used the target laboratory system, $P = (M,\vec{0})$, and the energy approximation $E_a \approx E_a'$.

The last equation shows the sum of a folded repulsive and attractive potentials results. They are characterized by vector and scalar parameters, respectively. We have also defined the nuclear density distribution

$$\rho(r) = \int \frac{d^3k}{(2\pi)^3} F_{qs}(k) \exp[i\vec{k}.\vec{r}] \qquad \text{(II.5)}$$

Moreover, we may note that the coupling constant of the repulsive contribution is redefined by a Lorentz factor

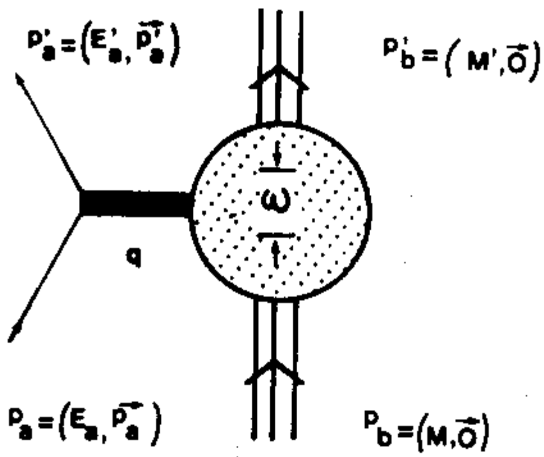
$$\gamma = \frac{E_a}{m} = \left[1 - \left(\frac{v_a}{c}\right)^2\right]^{-\frac{1}{2}}$$

which related to the incident proton energy. So when the incident proton energy is increasing, most of the contribution to the eikonal phase factor comes from the repulsive vector field, as shown in the Fig.(3). It is interesting to note that the behaviour of the static nuclear potential shown in that figure is essentially connected to the parameters of the nucleon-nucleon potencial which are relativelly well known such as the ranges of the attractive and repulsive parts. It is quite independent of the choice of the nuclear density.

Now if we consider small variations on the eikonal phase factor, it is easy to verify that there will be a shift in the exchanged momentum magnitude. However, this shift is very small compared to $|\vec{p}|$, such that we can estimate it about $20 MeV/c$. From this, we assume that the eikonal phase factor has a very small contribution, and it will be neglect in the calculations that follows.

III.DOUBLE DIFFERENTIAL CROSS SECTIONS AND THE NUCLEAR RESPONSE FUNCTION

First of all, the process that we are considering could be represented as



where ω is the excitation energy, whose magnitude is much smaller than E_a and E'_a. Then, defining an effective mass as $M' = M + \omega$ and considering the laboratory system, where $P = (M,\vec{0})$, the double differential cross section is given by

$$\frac{d^2\sigma}{d\Omega\, dE'_a} = \frac{1}{4\pi^2}\left(\frac{m}{M}\right)^2 \frac{|\vec{p}'_a|}{|\vec{p}_a|}\left|\mathcal{M}^{eik}\right|^2 \delta\left[E'_a-(E_a-\omega)\right] \tag{III.1}$$

Here we are using $\chi = 0$ from that was explained above. Furthermore, the $\mathcal{M}^{eik}$ in this case is

$$i\mathcal{M}^{cik} = \int d^4x \ \exp\left[-iqx\right] \langle P_b';f|\hat{T}_{eff}|P_b;qs\rangle \qquad \text{(III.2)}$$

where

$$\hat{T}_{eff} = \hat{T}_v(x) + \hat{T}_s(x) \qquad \text{(III.3)}$$

Now if we assume that $\omega \ll M$, equation (III.1) becomes

$$\frac{d^2\sigma}{d\Omega\, dE_a'} = \frac{1}{4\pi^2}\left(\frac{m}{M}\right)^2 \frac{|\vec{p}_a'|}{|\vec{p}_a|} \left|\bar{u}(p_a')\left[q_v^2 \frac{E_a}{m}\frac{1}{q^2-m_v^2} - \frac{1}{q^2-m_s^2}\right]u(p)\right|^2 S(q,\omega) \qquad \text{(III.4)}$$

where the last term is expressed as

$$S(q,\omega) = \sum_f \left|\langle P_b';f|\hat{F}_s(q)|P_b;qs\rangle\right|^2 \delta\left[E_a' - (E_a-\omega)\right] \qquad \text{(III.4)}$$

and is called Nuclear Response Function.

The double differential cross section can be rewritten as

$$\frac{d^2\sigma}{d\Omega\, dE_a'} = \left.\frac{d\sigma}{d\Omega}\right)^{Born} S(q,\omega) \qquad \text{(III.6)}$$

where

$$\left.\frac{d\sigma}{d\Omega}\right)^{Born} = \frac{1}{4\pi^2}\left(\frac{m}{M}\right)^2 \frac{|\vec{p}_a'|}{|\vec{p}_a|} \left|\bar{u}(p_a')\left[q_v^2 \frac{E_a}{m}\frac{1}{q^2-m_v^2} - q_s^2 \frac{1}{q^2-m_s^2}\right]u(p_a)\right|^2 \qquad \text{(III.7)}$$

Notice that eq.(III.7) does not correspond exactly to the usual nucleon-nucleon differential cross section in the Born approximation. It is shown in the Appendix that the Eq.(III.7) corresponds to the scattering of a nucleon by a very heavy spinless particle. This has consequence of changing the angular distribution qualitatively. In order to see this effect, in Fig.4-a, we compare the result of Eq.(III.7) with the nucleon-nucleon cross section in the same approximation (Born approximation). The parameters of the model are those of Ref.10. Note that there appears a peak around $\theta_{Lab} \sim 20^{\circ}$,although the free nucleon-nucleon cross section with the same parameters is simply forward peaked. This shows that the influence of the nuclear mean field in the kinematics can be substantial.

In Fig.4-b, we compare the free nucleon-nucleon cross section calculated with the same parameters as used in Fig.4-a to the experimental nucleon-nucleon cross section used in Ref.7. We note that although the calculated free nucleon-nucleon cross section is also forward peaked, it is substantially different. This can be either due to the quality of the Born approximation or to the inadequacy of parameters used which are determined from the static properties of nuclei in the mean-field approximation (ref.12). On the other hand, by varying the meson masses by a factor of approximately two in such a way that the ground state properties of nuclear matter are preserved, we can also fit the free nucleon-nucleon scattering cross section in the Born approximation (Fig.5-a). It is extremely interesting that in this case, the peak position in the experimental data in Ref.7 can be almost perfectly reproduced (Fig.5-b).

IV. CONCLUSIONS AND FINAL REMARKS

In this work we have constructed a Field Theoretical Framework to treat the inelastic scattering of composite particles. It can be viewed as a generalization of M.Levy and J.Sucher's work which studied a simpler process, namely the elastic scattering of two structureless and spinless particles. In this way, we succeeded in including the energy-momentum conservation as well as nuclear structure effects in the proton-nucleus scattering process. As discussed in text, for proton-nucleus reactions we find that the relativistic structure of the theory survives in the static limit and is reflected by an energy dependence in the folded nuclear potential. Also the contribution of the elastically scattered mesons is estimated to be small and a simple closed form expression for the double differential cross section is given. This cross section differs from the one based on the Glauber's theory in an essential point: the energy-momentum transfer may be absorbed by the nucleus as a whole and not by a single nucleon. In the kinematical region where this happens the nuclear mean field plays an important role. The kinematical consequences of this fact are displayed in figures 4 and 5. We come to the conclusion that in this kinematical region, proton spectra may bear strong influence of the relativistic kinematics of the elastic nucleon-nucleus scattering.

Finally a word about the perspectives that we have is in order. It would be very interesting to generalize this work to multiple scattering and to derive the cross section for particle production in the same scheme. Work along these lines are in progress.

APPENDIX A

We could see from eq. (II.1) that the term $\bar{u}(p')\gamma_0 u(p)$ will appear in the double differential cross section. Then, our intention is to get an more appropriate form. We start from the identities

$$\bar{u}(p')\gamma_\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)_\mu}{2m} + i\,\frac{\sigma_{\mu\nu}(p'-p)^\nu}{2m}\right]u(p)$$

and

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$$

so that

$$\bar{u}(p')\gamma_0 u(p) = \bar{u}(p')\left[\frac{E'+E}{2m} + \frac{1}{4m}\left(2\gamma_0\gamma_\nu q^\nu - q_0\right)\right]u(p)$$

Here we assume that the excitation energy, $q_0 = \omega$, is too smaller than the nucleon rest mass energy. Therefore, we have

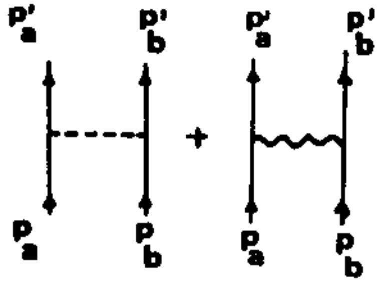
$$\bar{u}(p')\gamma_0 u(p) \sim \frac{E'+E}{2m}\,\bar{u}(p')u(p)$$

Now using the energy conservation and the fact that $E' = E + \omega$, we finally get

$$\bar{u}(p')\gamma_0 u(p) \sim \frac{E}{m}\,\bar{u}(p')u(p)$$

where the term (E/m) will appear multiplying the vector coupling constant .

Now, let us consider the nucleon-nucleon scattering amplitude in the one-boson exchange limit, which is represented by the following figure



We then have

$$\mathcal{M}_{fi} = g_s^2\, \bar{u}(p'_a)\, u(p_a)\, \frac{1}{q^2 - m_s^2}\, \bar{u}(p'_b)\, u(p_b) + g_v^2\, \bar{u}(p'_a)\, \gamma_\mu u(p_a)\, \frac{1}{q^2 - m_v^2}\, \bar{u}(p'_b)\, \gamma^\mu u(p_b)$$

Note that, in the limit where $\gamma_\mu \to \gamma_0$. the second term on the right side becomes

$$\bar{u}(p'_a)\left(g_v \frac{E_a}{m}\right) u(p_a)\, \frac{1}{q^2 - m_v^2}\, \bar{u}(p'_b \left(g_v \frac{E_b}{m}\right) u(p_b)$$

which is well different from that obtained in the equation (III.7). Here, we can see that if we have the particle b as a heavy particle, the last expression becomes

$$g_v^2 \left(\frac{E_a}{m}\right) \bar{u}(p'_a)\, \frac{1}{q^2 - m_v^2}\, u(p_a)\, \langle P' | \hat{F}(q) | P_b \rangle$$

where $\hat{F}(\ q\ )$ is the vertex operator on the line of the particle b.

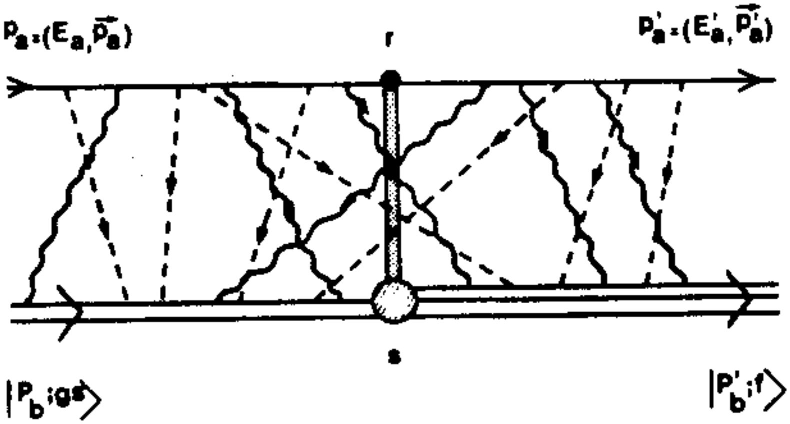


FIGURE 1

A typical exchange-type Feynman diagram of the one step excitation process. The scalar mesons are represented by crossed lines.

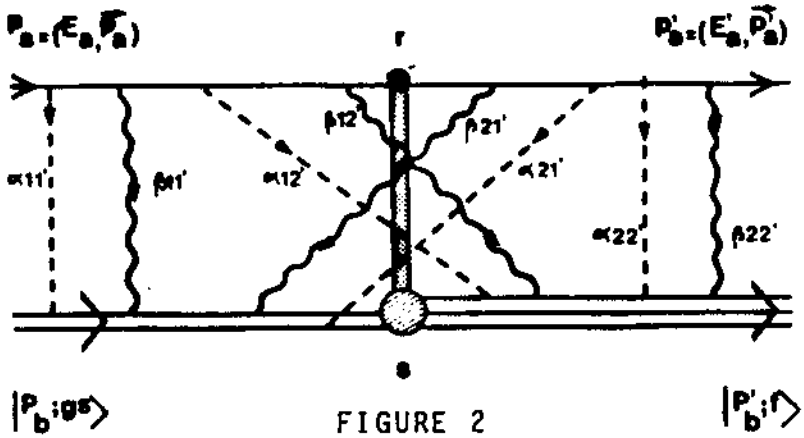


FIGURE 2

A suitable class (C) from general Feynman diagrams.

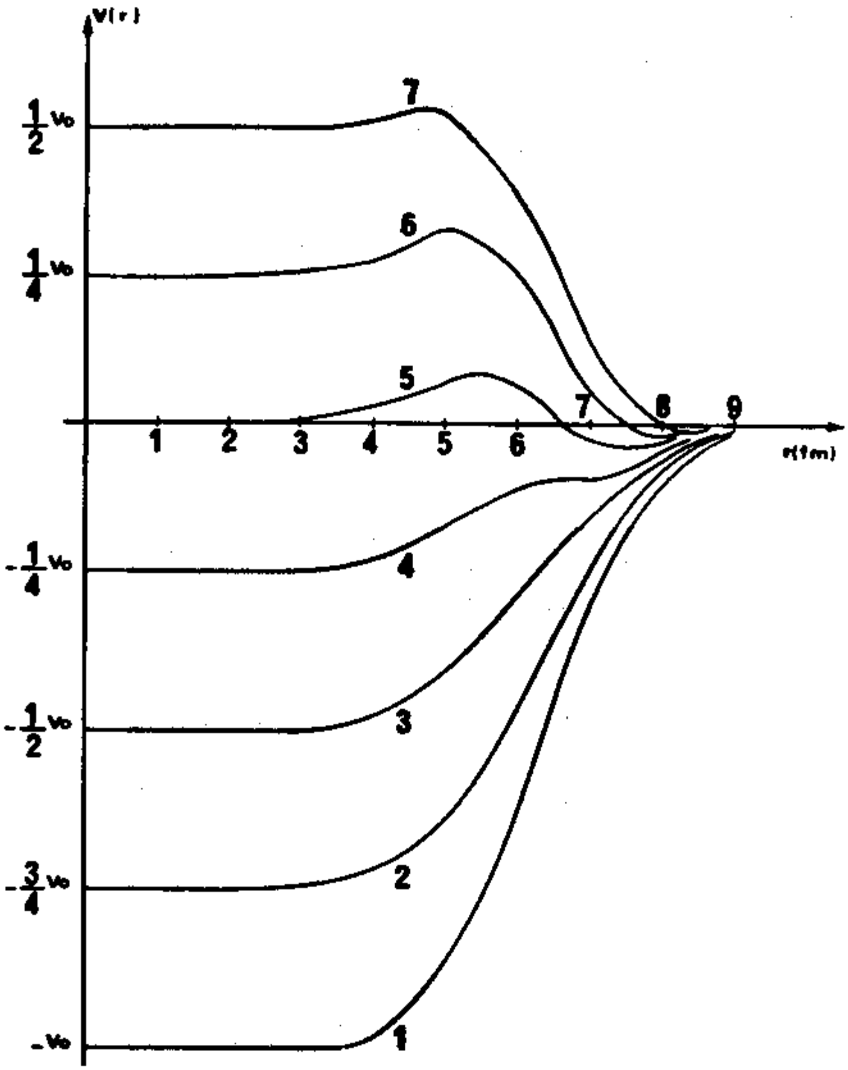


FIGURE 3

Behaviour of the static potential when the proton's incident energy is incresing, according as following:

graphics	1	2	3	4	5	6	7
Lorentz factor	1.0	1.1	1.2	1.3	1.4	1.5	1.6

It was used a Fermi nuclear distribution to plot Eq.(II.4), in agreement with Eisenberg and Greiner, Nuclear Physics (Vol.II).

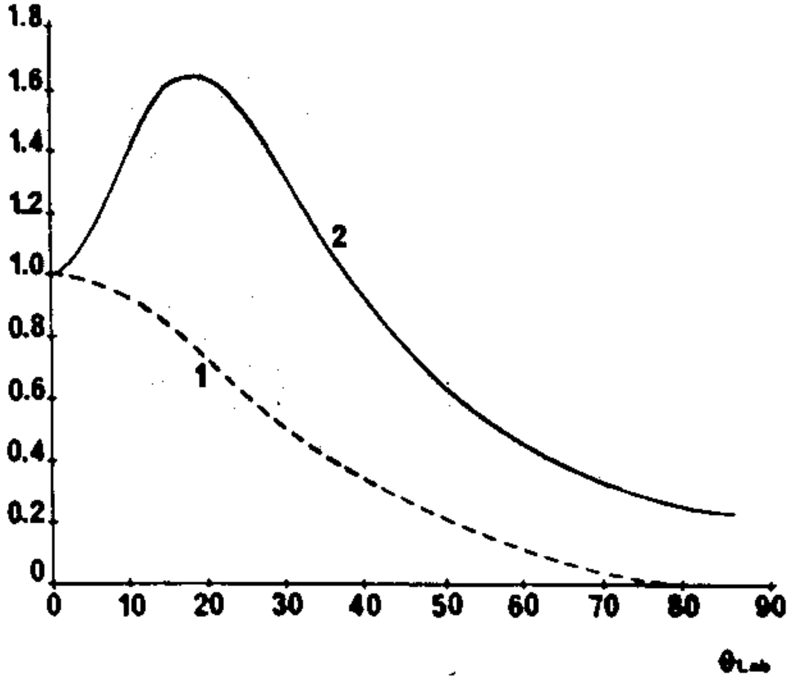


FIGURE 4a

Curve (1) represents the experimental data of the elatic nucleon nucleon cross section from Ref.[7], which was normalized to one. The curve (2) represents the Eq.(III.7) where Walecka's free parameters were used in agreement with Ref.[2].

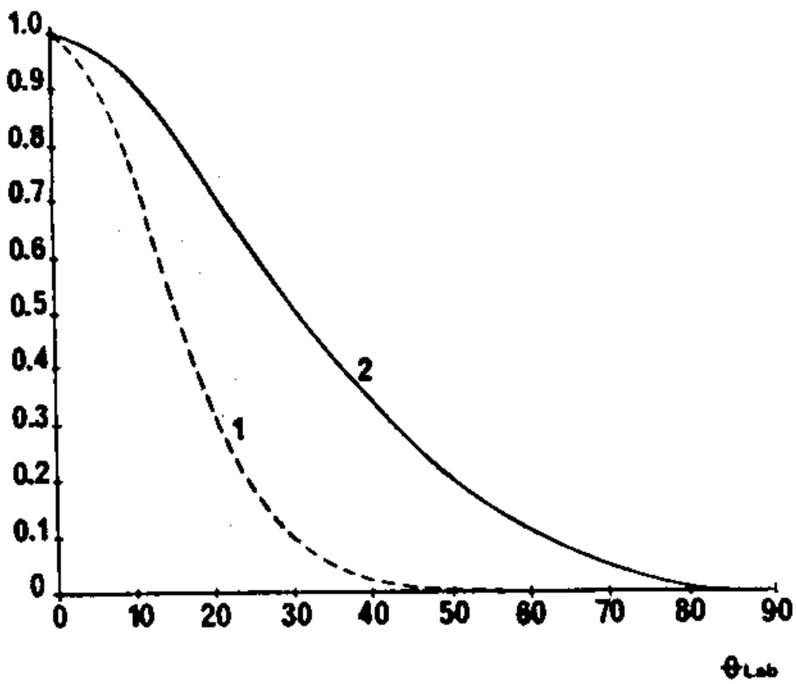


FIGURE 4b

Curve (1) represents the experimental data of the elastic nucleon-nucleon cross section from Ref.[7] and the curve (2) is the nucleon-nucleon cross section from the Born Approximation using the Walecka's parameters.

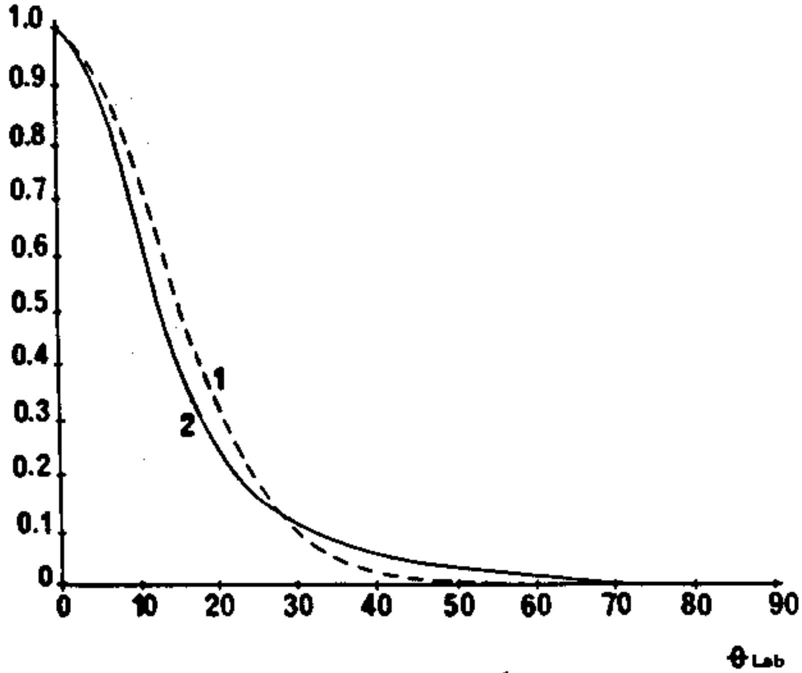


FIGURE 5a

Fit between the experimental (curve 1) and the Born Approximation (curve 2) elatic nucleon-nucleon cross section, where the last one was calculated with new values to the free parameters.

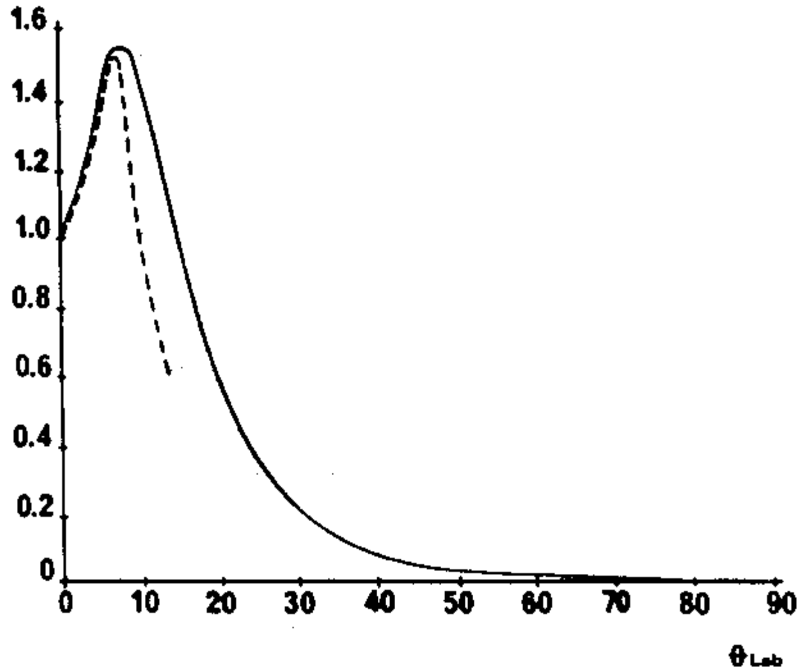


FIGURE 5b

Angular distribution in accordance with Eq.(III.7) with the parameters which fit the nucleon-nucleon elastic scattering in Born Approximation (Appendix A).

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