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The radiation era in scalar-tensor cosmology

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Abstract

An action inspired on the low-energy effective action for heterotic string and Weyl integrable space-time theory is used to describe the radiation era in scalar-tensor cosmology. The resulting field equations for a Friedmann-Robertson-Walker geometry are written and the general solution for this system is obtained and its main properties are discussed.

Key-Words: String Cosmology, Background Radiation.

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I. INTRODUCTION

Despite the outstanding success of General Relativity(GR), under extreme physical conditions of very large curvature, the standard description given by GR breaks down since it predicts that it cannot predict [1]. We expect GR to be a low-energy and low space-time curvature limit for some quantum theory of gravitation that has yet to be found. In this vein, string theory is been investigated [2,3], seeking for the consequences of these quantum gravitational corrections to GR in cosmology as well as in compact configurations, such as collapsed stars and black holes.

String-inspired cosmology has currently been largely investigated by Veneziano and collaborators.They have emphasized the possible importance of the duality symmetry which characterizes the equations of string cosmology [4] and allows a mechanism to evade the singularity. In this context, the big-bang no longer corresponds to a singularity, but to an instant of maximal curvature marking the transition from a string-driven growing-curvature regime to the decelerated evolution of the standard scenario.

The low-energy effective string field theory has also been investigated in other configurations. Dilaton fields appear naturally, coupled with Einstein-Maxwell fields, when the low-energy limit of the heterotic string theory is taken [10,7,6]. Besides this, dilaton fields also appear as a result of a dimensional reduction of the Kaluza-Klein Lagrangian [8,9]. These theories have revived the interest in dilaton fields coupled with matter, since they are of importance for the understanding of the more general theories from which the effective action is obtained. The same effective lagrangean, obtained in low-energy effective string field theory, can also be obtained by following a different approach according to which the space-time is represented by an integrable Weyl space-time (WIST). This structure, as showed by Ehlers et al [12], is well founded and is obtained by means of an axiomatic formulation that uses light rays and freely falling particles as basic concepts and where the axioms have a direct constructive and operational meaning. The detailed development of this model has been carried out before [13–17]

Following this reasoning, the well known problems of GR involving electromagnetic fields coupled with gravitation, in the context of the primordial universe [18,20–25,32,33] and in compact configurations such as collapsed stars and black holes, have now been largely investigated using the effective action of Einstein-Maxwell-dilaton theory. For example, the configurations representing charged black-holes [10,11] and other kinds of stationary dilatons with arbitrary electromagnetic field configurations [19]. In cosmology, it has been used to investigate the problem of formation of large scale magnetic fields [34–36].

The motivation for this paper is to find cosmological solutions to a model consisting of a massless scalar field (dilaton) interacting with an electromagnetic field coupled with gravity in a homogeneous and isotropic geometry. We deal with a simplified model since the Kalb-Ramond field, and the dilaton potential are setting equal to zero and so the graceful exit problem in string cosmology is not contemplated here [5]. We introduce a parameter λ in the scalar kinetic term in the action in order to describe also others scalar-tensor theories such as Weyl integrable sapace-time (WIST). The simplifications are justified by the simplicity of the general analytical solutions obtained and by the possibility to investigate the modifications produced by the non-constant dilaton field in the radiation era. We claim that the solutions exibited in this paper can be useful to describe the interaction of the dilaton field with radiation during the period before and after recombination.

II. FIELD EQUATIONS

The action we use is the one used in WIST [14]. In the case of $\lambda = 1$ is the same obtained in the low-energy limit of the heterotic string theory. It has the form

$$S = \int d^4x\sqrt{-g}\,(R + \lambda g^{\mu\nu}\omega_{|\mu}\omega_{|\nu} + \frac{1}{2}\mathrm{e}^{-2\omega}F_{\alpha\beta}F^{\alpha\beta}), \tag{1}$$

where R is the curvature scalar, ω is the dilaton field and $F_{\mu\nu}$ is the Maxwell field. We have introduced the coupling constant λ in order to describe more general lagrangeans where the kinetic term is not "a priori" fixed.

Taking the extreme of the action (1) with respect to the scalar field ω, the vector potential A_μ and the metric $g_{\mu\nu}$ respectively yield the following field equations:

$$G_{\mu\nu} - \lambda(\omega_{|\mu}\omega_{|\nu} - \frac{1}{2}g_{\mu\nu}\omega_{|\alpha}\omega^{|\alpha}) = -\mathrm{e}^{-2\omega}(F_{\mu\alpha}F^{\alpha}_{\nu} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}), \tag{2a}$$

$$\Box\omega = -\frac{e^{-2\omega}}{2\lambda}F_{\mu\nu}F^{\mu\nu}, \tag{2b}$$

$$(e^{-2\omega}F^{\mu\nu})_{||\nu} = 0. \tag{2c}$$

We are looking for cosmological models with a homogeneous and isotropic spacial tri-surface that are described by a Friedmann-Robertson-Walker(FRW) metric. In a convenient coordinate system the metric can be written as:

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2). \tag{3}$$

Since the natural spatial sections of FRW geometry are isotropic, electromagnetic fields can generate such universe only after a suitable spatial average be performed [26]. The standard procedure [27] is just to set[1] for the electric field E_i and the magnetic field H_i the following mean values:

$$< E_i > = 0, \tag{4a}$$

$$< H_i > = 0, \tag{4b}$$

$$< E_i\, E_j > = -\frac{1}{3}E^2\, g_{ij}, \tag{4c}$$

$$< H_i\, H_j > = -\frac{1}{3}H^2\, g_{ij}, \tag{4d}$$

$$< E_i\, H_j > = 0. \tag{4e}$$

[1]We make use of Gaussian Cartesian coordinates. Latin indices run in the spatial range $(x,\, y,\, z)$, while Greek indices run in the spacetime range $(t,\, x,\, y,\, z)$.

Here E^2 and H^2 are both non-negative functions of time[2], and we denote by using angular brackets the volume spatial average (*e.g.*, $< X >$ represents the volume average of the arbitrary quantity X) for a given instant of time t,

$$< X > \doteq \lim_{v \to v_o} \frac{1}{v} \int X \, d^3x, \tag{5}$$

where $v = \int d^3x$ (with $x \in R$) being spatial coordinates, and v_o stands for the time dependent volume of the whole space.

The canonical stress-energy tensor associated with Maxwell Lagrangian is given by

$$T_{\mu\nu} = F_{\mu\,\alpha} \, F^{\alpha}{}_{\nu} + \frac{1}{4} \, F \, g_{\mu\nu}, \tag{6}$$

in which $F \doteq F_{\mu\nu} \, F^{\mu\nu} = 2(H^2 - E^2)$. Equations (4) imply

$$< F_{\mu\alpha} \, F^{\alpha}{}_{\nu} > = \frac{2}{3}(E^2 + H^2) V_\mu \, V_\nu + \frac{1}{3}(E^2 - 2\,H^2) \, g_{\mu\nu}, \tag{7}$$

where V^μ represents the four velocity vector field $V^\mu = \delta^\mu_o$, which is orthogonal to the three-dimentional surface of homogeneity of the FRW geometry.

Using this result into the expression (6) of the stress-energy tensor, it follows that its average value $< T_{\mu\nu} >$ reduces to a perfect fluid configuration

$$< T_{\mu\nu} > = (\rho + p) \, V_\mu \, V_\nu - p \, g_{\mu\nu}, \tag{8a}$$

with energy density

$$\rho = \frac{1}{2} \, (E^2 + H^2), \tag{8b}$$

and pressure

$$p = \frac{1}{3} \, \rho. \tag{8c}$$

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[2]They are not scalars, however, but they depend on the set of coordinates, as far as expression (5) is not a tensor definition.

In the coordinate system we are using the field equations become:

$$-3\frac{\dot{a}^2}{a^2} = \frac{\lambda}{2}\dot{\omega}^2 - e^{-2\omega}\rho, \tag{9}$$

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = -\frac{\lambda}{2}\dot{\omega}^2 + e^{-2\omega}p, \tag{10}$$

$$\frac{(a^3\dot{\omega})^{\cdot}}{a^3} = -\frac{e^{-2\omega}}{\lambda}(H^2 - E^2). \tag{11}$$

We will study two cases: the period just before recombination, which we call "plasma period", and another after recombination, which we named radiation.

A. Plasma period

The matter content of the Universe in this period can be represented by a plasma that quickly relaxes into a state of thermal equilibrium. On time scales much longer and spatial scales much larger than those characteristic of collisional processes, the plasma behaves as a conducting fluid. The current density, $J^i = \sigma E^i$, where σ is the conductivity [25], dominates largely the other terms of Ampere's equation for the electric field so that E^i goes to zero in the plasma rest frame. As a consequence, during the period under consideration only the average of the squared magnetic field H^2 survives[3] [28–31].

In order to simplify the equations we perform the following coordinate transformation:

$$dt = a^3(\tau)\, d\tau \tag{12}$$

The equations in the new time coordinate τ are:

$$-3(\frac{a'}{a})^2 = \frac{\lambda}{2}(\omega')^2 - a^6 e^{-\omega/2}\rho \tag{13}$$

[3]This is strictly true for a viscosity free ionized plasma. When plasma viscosities are considered the resulting mean squared electric field may be non-zero, but it would still be much smaller than its magnetic counterpart.

$$-2\frac{a''}{a} + 5(\frac{a'}{a})^2 = -\frac{\lambda}{2}\omega'^2 + \frac{1}{3}a^6\rho e^{-\omega/2} \tag{14}$$

$$\omega'' = -\frac{2a^6}{\lambda}\rho e^{-\omega/2} \tag{15}$$

Substituting ρ from (15) and adding (13) and (14) we obtain:

$$-2(\frac{a'}{a})' = \frac{\lambda}{3}\omega'' \tag{16}$$

The general solution for this equation is:

$$a(\tau) = a_0\, e^{-\frac{\omega_0}{2}\tau} e^{-\frac{\lambda\omega}{6}} \tag{17}$$

Using the first integral of (16) and (15) in (14) we obtain the following equation for ω:

$$\frac{\lambda}{2}\omega'' + (r\omega' + \frac{s}{r})^2 + \Omega_1 = 0 \tag{18}$$

where

$$\Omega_1 = \frac{3}{4}\omega_0^2 - (\frac{s}{r})^2, \tag{19}$$

$$r^2 = \frac{3}{36}\lambda^2 + \frac{\lambda}{2}, \tag{20}$$

$$s = \frac{\lambda}{4}\omega_0. \tag{21}$$

Now we define a new variable x as:

$$x = r\omega' + \frac{s}{r}. \tag{22}$$

The equation (18) thus become:

$$\frac{\lambda}{2r}x' + x^2 + \Omega_1 = 0 \tag{23}$$

This equation can be integrated as

$$\tau = -\frac{\lambda}{2r}\int \frac{dx}{x^2 + \Omega_1} \tag{24}$$

We can distinguish three different solution for this integral:

1. $\lambda > 0$ $\Omega_1 = Q^2$. With solution:

$$\omega = -\frac{s}{r^2}\tau + \omega_1 + \frac{\lambda}{2r^2}\ln\left(\cos\frac{2rQ}{\lambda}\tau\right) \tag{25}$$

With this result the scalar factor becomes:

$$a(\tau) = A_0 \mathrm{e}^{(\frac{s\lambda}{6r^2}-\frac{\omega_0}{2})\tau}\left(\cos\frac{2rQ}{\lambda}\tau\right)^{\frac{-\lambda^2}{12r^2}} \tag{26}$$

2. $\lambda < 0$ $\Omega_1 = -Q^2$ and $x^2 > Q^2$. With solution:

$$\omega = \omega_1 - \frac{s}{r^2}\tau + \frac{\lambda}{2r}\ln\sinh\left(\frac{2rQ}{\lambda}\tau\right) \tag{27}$$

$$a(\tau) = A_0 \mathrm{e}^{(\frac{s\lambda}{6r^2}-\frac{\omega_0}{2})\tau}\left(\sinh(\frac{2rQ}{\lambda})\tau\right)^{\frac{-\lambda^2}{12r^2}} \tag{28}$$

3. $\lambda < 0$ $\Omega_1 = -Q^2$ and $x^2 < Q^2$. With solution:

$$\omega = \omega_1 - \frac{s}{r^2}\tau + \frac{\lambda}{2r^2}\ln\cosh\left(\frac{2rQ}{\lambda}\tau\right) \tag{29}$$

$$a(\tau) = A_0 \mathrm{e}^{(\frac{s\lambda}{6r^2}-\frac{\omega_0}{2})\tau}\left(\cosh(\frac{2rQ}{\lambda}\tau)\right)^{\frac{-\lambda^2}{12r^2}} \tag{30}$$

B. Radiation

After the recombination matter decouples from radiation, in this case we have $E^2 = H^2$. The interaction of radiation and the scalar field amounts for a local non-conservation of radiation energy density as can be seen from the following balance equation:

$$\dot{\rho} + \theta(\rho + p) = 2e^{-2\omega}\rho\dot{\omega} \tag{31}$$

Using the same coordinate transformation for the time coordinate the field equations become:

$$-3\frac{a'^2}{a^2} = \frac{\lambda}{2}\omega'^2 - e^{-2\omega}\rho a^6 \tag{32}$$

$$-2\frac{a''}{a} + 5\frac{a'^2}{a^2} = -\frac{\lambda}{2}\omega'^2 + e^{-2\omega}\frac{\rho}{3}a^6. \tag{33}$$

$$\omega'' = 0. \tag{34}$$

The solution for equation (34) is simple

$$\omega = \omega_0\tau + \omega_1. \tag{35}$$

Using the previous solution and the other two equations we obtain:

$$-2\frac{a''}{a} + 4(\frac{a'}{a})^2 = -\frac{\lambda}{3}\omega_0^2. \tag{36}$$

Making the following variable transformation

$$x = \frac{a'}{a}, \tag{37}$$

the equation (36) becomes:

$$x' = x^2 + \Omega_1 \tag{38}$$

where $\Omega_1 = \frac{\lambda}{6}\omega_0^2$. This equation can be integrated as follows:

$$\tau = \int \frac{dx}{x^2 + \Omega_1} \tag{39}$$

The integral has different classes of solutions depending on the signal of λ and the constant Ω_1.

1. If $\lambda > 0$, then $\Omega_1 = a_0^2 = \frac{\lambda}{6}\omega_0^2$ and the solution will be non-singular:

$$a(\tau) = \frac{a_1}{(\cos a_0\tau)^{a_0^2}}, \quad -\pi < 2a_0\tau < \pi. \tag{40}$$

2. In the case $\lambda < 0$, $\Omega_1 = -a_0^2 = -\frac{|\lambda|}{6}\omega_0^2$ and we have to distinguish between two cases:

(a) Case $x^2 < a_0^2$:

$$a(\tau) = a_1 \exp(\sinh a_0 \tau) \tag{41}$$

(b) Case $x^2 > a_0^2$:

$$a(\tau) = a_1 (\cosh a_0 \tau)^{-1} \tag{42}$$

It is important to note that the first case is non-singular, while the second case is singular as expected from the Raychaudhuri equation.

III. CONCLUSION

A model inspired on the effective action for heterotic string and WIST theory was constructed in order to study the radiation era in scalar-tensor cosmology in the presence of a nonconstant dilaton scalar field. The very well-known average procedure introduced by Tolman and Ehrenfest was used to make the electromagnetic field compatible with the FRW geometry. The general solution obtained has a non-constant scalar field and can be used to describe the radiation era in scalar-tensor cosmology. The positive value of the constant λ, possible in the WIST version of the action used here allows for a non-singular solution. The consequences of the dilaton field in the dynamics of perturbations and on the background radiation deserve further investigation.

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