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DISSIPATIVE EFFECTS IN FRIEDMANN UNIVERSES

by

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ABSTRACT

In this paper the relationship between the different temperatures present in a radiative plasma is examined. In particular, the physical and the operational meaning of Eckart's temperature is discussed. An entropy density formula for the radiative component and its fractional variation rate are derived. We have also suggested a reformulation of Weinberg's conditions for maximum entropy production. The effect of radiative bulk viscosity in diluting monopoles in the very early universe is estimated.

Key words: Radiative Plasma; Early Universe; Cosmology; Bulk Viscosity.

I. INTRODUCTION

An important physical system in cosmological and astrophysical problems is the so-called radiative plasma; a two component fluid consisting of some material medium in equilibrium with itself (very short mean free time) plus radiation quanta with a finite mean free time τ. It is well-known that this kind of mixture behaves like an imperfect relativistic simple fluid in the framework of the hydrodynamical formulation for dissipative processes developed by Eckart[1].

Weinberg[2], by using the solution of the relativistic transport equation first derived by Thomas[3], obtained an expression for the radiative bulk viscosity coefficient and used it to evaluate the entropy production in the early universe. Although successful in revealing that the radiative bulk viscosity is not so efficient in generating much entropy, some physical aspects not discussed at length by Weinberg deserve special attention.

The first one concerns the several temperature concepts related to a radiative plasma. For such a system there are in the collisional limit three different temperatures, namely: the matter temperature (T_m), the radiation kinetic effective temperature (T_r) and the Eckart temperature (T). The matter and the radiation effective temperature concepts are very simple, but the last one is rather subtle. It was introduced by Weinberg in order to obtain the bulk viscosity coefficient. Schweizer[4], generalising Weinberg's paper referred to it as the actually observed temperature. So, it seems desirable, from a conceptual as well as from a pedagogical point of view, to make the connection explicit between the Eckart temperature and a possible experimental device used to measure it. As suggested by Yourgran et al.[5], a device that seems consistent with the local equilibrium hypothesis, even in the case of mixture, is the following: "in order to measure the temperature at an arbitrary point of a fluid out of thermal equilibrium, isolate a small volume element surrounding the point and allow the matter in it to reach equilibrium. The temperature measured by the conventional method for this equilibrium state defines the temperature at that point". By considering an argument from elementary calorimetry we will show that, for a radiative plasma, the temperature measured as described above is just the Eckart temperature.

On the other hand, Eckart's phenomenological approach gives us only a general expression for the total entropy production of the mixture. However, sometimes (in cosmology, for instance), one might be more interested in the entropy variation rate of the radiative component. Qualitatively, the heat exchanges between matter and radiation and consequently the radiation entropy variation rate is closely related to the relative magnitudes of T_m and T_r. But, if one intends to establish the conditions under which the radiation entropy grows or diminishes, the natural way is to compute an expression for the radiation entropy itself. As will be seen further down, for a radiative plasma this is possible since all the transport properties are due exclusively to the radiation quanta.

In general viscous effects are expected to be more significant when $\tau\theta \sim 1$, where θ is the expansion parameter.

However, in the case of a radiative plasma, additional data about its equation of state must be supplied since, as shown by Weinberg, if the radiation is strongly dominant the bulk viscosity can be neglected. Taking into account these facts, we have suggested a reformulation of one of the conditions established by Weinberg for a maximum total entropy production. We also show by assuming $\tau\theta \sim 1$, that the maximum increasing rate of the specific radiation entropy occurs in a "dust-like stage". This is expected since, in this case, a considerable quantity of matter thermal energy can be transfered to the radiative component.

This paper is organized as follows. In section II the energy momentum tensor of the system under consideration is derived. We also clarify the concept of Eckart temperature by connecting it with its possible experimental meaning. In section III we obtain an expression for the entropy density of the radiative component. Its fractional variation rate is also calculated and a modification of Weinberg's condition is suggested. In section IV we apply the results obtained in the preceding sections to estimate the effect of radiative bulk viscosity in diluting monopoles in the very early universe.

II. THE ENERGY-MOMENTUM TENSOR AND THE LOCAL EQUILIBRIUM TEMPERATURE

In order to make explicit the several temperature concepts coexisting in a radiative plasma and also to simplify some results to be derived in the next section, we shall now obtain the energy-momentum tensor of this system. Having in mind an application to the standard cosmological model (FRW), only the homogeneous and isotropic case will be considered. However, we remark that the main physical features relating the temperature concepts are independent (at least to first order in τ) of the shear viscosity and heat conduction effects which appear in a more complete treatment.

The energy-momentum tensor of the radiative component is defined by

$$T^{\alpha\beta}_{(r)} = g \int_{K^0 \geq 0} K^\alpha K^\beta F(x,K) \frac{d^3K}{K^0} \quad , \tag{2.1}$$

where the null 4-vector K^α is the radiation 4-momentum, $F(x,K)$ stands for the Lorentz-invariant distribution function so that gFd^3xd^3K is the number of quanta in the respective phase volume. The factor g is equal to $(2\pi)^{-3}$ times the number of spin states of the radiation quanta.

By hypothesis matter and radiation are out of but close to thermal equilibrium and so one may expand F as

$$F = F_0 + F_1 + F_2 + \ldots \quad , \tag{2.2}$$

where $\left|\frac{F_i}{F_{i-1}}\right| << 1$ and F_0 is the equilibrium distribution function given by (in our units $\hbar = k_B = c = 1$)

$$F_0 = \left[\exp\left(-\frac{u^\lambda K_\lambda}{T_m}\right) - \varepsilon\right]^{-1} \quad , \tag{2.3}$$

where u^λ is the 4-velocity of the material medium ($u^\lambda u_\lambda = -1$), T_m its temperature and the number ε is +1 for bosons and -1 for fermions.

By successive aproximations starting from F_0, one could in principle determine F to any order in the mean free time τ, by using the relativistic transport equation[6]. For our purpose it is sufficient to go to first order only. In this case, after a suitable linearization of the collisional term, the Boltzmann equation can be written as[7]

$$K^\mu \frac{\partial F_0}{\partial X^\mu} = -\omega\tau^{-1}(\omega, X^\mu) F_1 \quad , \tag{2.4}$$

where $\omega = -u^\lambda K_\lambda$ stands for the radiation energy in the rest frame of the material medium.

Now, considering the homogeneous and isotropic case, we have $\tau = \tau(\omega,t)$. Furthermore taking a moment of (2.4) one obtains that F_1 reduces to[7]

$$F_1 = \tau(\omega,t)\left(\frac{\omega}{T_m}\right) \frac{\partial F_0}{\partial \omega} \left[\frac{u^\alpha}{T_m} \frac{\partial T_m}{\partial X^\alpha} + \frac{1}{3} \frac{\partial u^\alpha}{\partial X^\alpha}\right] \quad . \tag{2.5}$$

Assuming that τ is frequency-independent and inserting eqs. (2.2), (2.3) and (2.5) into (2.1), a straightforward integration yields

$$T^{\alpha\beta}_{(r)} = N_* T^4_m \left[1 - 4\tau\left(\frac{u^\lambda}{T_m}\frac{\partial T_m}{\partial x^\lambda} + \frac{1}{3}\frac{\partial u^\lambda}{\partial x^\lambda}\right)\right] u^\alpha u^\beta +$$

$$\frac{1}{3} N_* T^4_m \left[1 - 4\tau\left(\frac{u^\lambda}{T_m}\frac{\partial T_m}{\partial x^\lambda} + \frac{1}{3}\frac{\partial u^\lambda}{\partial x^\lambda}\right)\right] h^{\alpha\beta} , \qquad (2.6)$$

where $h^{\alpha\beta} \equiv \eta^{\alpha\beta}+u^\alpha u^\beta$ is the projector onto the local rest-space of u^λ, the constant N_* is given by $N_* = (\frac{15+\varepsilon}{8})\,\frac{\pi^2}{30}$ so that, for $\varepsilon = +1$, N_* is two times the photon constant or 7/8 of it for $\varepsilon = -1$. Of course, if the radiative component itself is a mixture of bosons and fermions, eq. (2.6) would be modified in the factor N_*, its new value being

$$N_* = \frac{\pi^2}{30}\left(\sum_{\text{bosons}} g_b + \frac{7}{8}\sum_{\text{fermions}} g_F\right) , \qquad (2.7)$$

where g_b and g_F are the total number of boson and fermion degrees of freedom and the sum runs over all boson and fermion states. Moreover, if τ is also frequency-dependent, the net effect in (2.6) would be to introduce the Rosseland mean[7]

$$\langle\tau(t)\rangle = \int_0^\infty \tau(\omega,t)\,\omega^4 \left(\frac{\partial F_0}{\partial\omega}\right) d\omega \Big/ \int_0^\infty \omega^4 \left(\frac{\partial F_0}{\partial\omega}\right) d\omega \; .$$

If one defines the radiation effective kinetic temperature by the expression

$$T_r = T_m \left[1 - \tau\left(\frac{u^\lambda}{T_m}\frac{\partial T_m}{\partial x^\lambda} + \frac{1}{3}\frac{\partial u^\lambda}{\partial x^\lambda}\right)\right] , \qquad (2.8)$$

the energy-momentum tensor given in (2.6) assumes, to first order in τ, the usual perfect fluid form

$$T^{\alpha\beta}_{(r)} = N_* T_r^4 u^\alpha u^\beta + \frac{1}{3} N_* T^4 h^{\alpha\beta} \quad . \tag{2.9}$$

The above equation shows that the radiation quanta behave as if they were in thermal equilibrium at temperature T_r. In addition, since the material medium was supposed in thermal equilibrium with itself at temperature T_m, the total (matter plus radiation) energy-momentum tensor can be cast in the form

$$T^{\alpha\beta} = T^{\alpha\beta}_{(m)} + T^{\alpha\beta}_{(r)} = \left[\rho_m(T_m,n) + N_* T_r^4\right] u^\alpha u^\beta + \left[p_m(T_m,n) + \frac{1}{3} N_* T_r^4\right] h^{\alpha\beta} \quad , \tag{2.10}$$

where ρ_m and p_m are respectively the energy density and pressure of the material component. Notice that eq. (2.10) has only two terms which is a characteristic of the perfect fluid form. However, the difference in temperatures between the components is responsible for an irreversible heat exchange between them. This is the mechanism which accounts for the total entropy production within this mixture.

To gain some physical insight into the heat exchanges between matter and radiation, consider now the Gibbs law applied for each component:

$$nT_m d\sigma_m = d\rho_m - \left(\frac{\rho_m + p_m}{n}\right) dn \quad , \tag{2.11}$$

$$nT_r d\sigma_r = d\rho_r - \left(\frac{\rho_r + p_r}{n}\right) dn \quad , \tag{2.12}$$

and the equations of motion contained in the conservation laws:

$$\frac{\partial T^{\alpha\beta}}{\partial X^{\beta}} = 0 \quad , \tag{2.13}$$

$$\frac{\partial}{\partial X^{\alpha}} (n u^{\alpha}) = 0 \quad . \tag{2.14}$$

In eqs. (2.11) and (2.12), σ_m and σ_r stand for the dimensionless specific entropy (per particle) of matter and radiation respectively and n is the particle number density.

By taking T_m and n as independent thermodynamic variables, since $d\sigma_m$ and $d\sigma_r$ are exact differentials one may deduce that

$$T_m \left(\frac{\partial p_m}{\partial T_m}\right)_n = \rho_m + p_m - n \left(\frac{\partial \rho_m}{\partial n}\right)_{T_m} \quad , \tag{2.15}$$

and

$$T_m \left(\frac{\partial p_r}{\partial T_m}\right)_n = \rho_r + p_r - n \left(\frac{\partial \rho_r}{\partial n}\right)_{T_m} + O(\tau) \quad , \tag{2.16}$$

so that

$$T_m \left(\frac{\partial p}{\partial T_m}\right)_n = \rho + p - n \left(\frac{\partial \rho}{\partial n}\right)_{T_m} + O(\tau) \quad , \tag{2.17}$$

where $\rho = \rho_m + \rho_r$ and $p = p_m + p_r$.

On the other hand, from eqs. (2.13) and (2.14) it is readily seen that

$$u^{\alpha} \frac{\partial T_m}{\partial X^{\alpha}} = \left(\frac{\partial \rho}{\partial T_m}\right)^{-1} \left[n \left(\frac{\partial \rho}{\partial n}\right)_{T_m} - \rho - p \right] \frac{\partial u^{\alpha}}{\partial X^{\alpha}} \quad , \tag{2.18}$$

and using (2.17) the above equation can be rewritten as

$$\frac{u^{\alpha}}{T_m} \frac{\partial T_m}{\partial X^{\alpha}} + \frac{1}{3}\,\theta = \Omega\theta + O(\tau) \quad , \tag{2.19}$$

where $\theta = \frac{\partial u^\alpha}{\partial x^\alpha}$ is the expansion parameter of the fluid and Ω is defined by

$$\Omega = \frac{1}{3} - \left(\frac{\partial p}{\partial \rho}\right)_n \quad . \tag{2.20}$$

Finally, substituting (2.19) into (2.8) one finds that

$$T_r - T_m = -T_m \, \tau\Omega\theta \ . \tag{2.21}$$

The conditions for a maximum variation of the radiation entropy can be qualitatively understood analyzing eq. (2.21). Since the amount of heat flow from one component to the other depends on the difference $T_r - T_m$, it is easily seen from the last equation above that the temperature difference is larger for higher values of T_m. In cosmology, for a fixed value of Ω different from zero, one would expect a greater entropy variation of each component and also a maximum total entropy production at the beginning of the cosmic evolution. Moreover, for an expanding plasma, if $\Omega > 0$ $\left(\left(\frac{\partial p}{\partial \rho}\right)_n < \frac{1}{3}\right)$, $T_r < T_m$ and one should expect that the radiation will gain entropy from the material medium. Conversely if $\Omega < 0$, $T_r > T_m$ and it will lose entropy. In both cases the maximum variation rate will occur during the transition from a collision-dominated to collisionless regime for which $\tau\theta \sim 1$. Note also that if $p \to \frac{1}{3}\rho$ then $\Omega \to 0$ and $T_r \to T_m$ so that, as expected, in the limiting case there is no radiation entropy variation. If one assumes $\tau\theta \sim 1$, the maximum increasing rate of the radiation entropy will occur in a "dust-like" stage ($\Omega \sim \frac{1}{3}$), while the maximum decreasing one would occur in a "stiff matter-like" stage ($\Omega \sim -\frac{2}{3}$).

As shown by Weinberg, the energy momentum tensor (2.10) can be recast in the canonical form of imperfect fluid as defined by Eckart. In the homogeneous and isotropic case considered it reduces to

$$T^{\alpha\beta} = \rho u^{\alpha} u^{\beta} + (p+\pi) h^{\alpha\beta} \quad , \tag{2.22}$$

where

$$\rho(T,n) = \rho_m(T,n) + N_* T^4 \quad , \tag{2.23}$$

$$p(T,n) = p_m(T,n) + \frac{1}{3} N_* T^4 \quad , \tag{2.24}$$

$$\pi = -\xi\theta \quad ; \quad \xi = 4N_* T^4 \tau \Omega^2 \quad . \tag{2.25}$$

In expressions (2.22)-(2.25), π is the viscous pressure, ξ is the bulk viscosity coefficient and T is the Eckart Temperature defined in such a way that the comoving energy density $u_{\alpha} u_{\beta} T^{\alpha\beta}$ is equal to the energy density $\rho(T,n)$ for thermal equilibrium at temperature T. It is related to the matter temperature by[2]

$$T_m = T\left[1 + \frac{4N_* T^3 \tau}{(\frac{\partial\rho}{\partial T})_n}\left(\frac{u^{\alpha}}{T}\frac{\partial T}{\partial X^{\alpha}} + \frac{1}{3}\theta\right)\right] \quad . \tag{2.26}$$

It is worth mentioning that in this simple-fluid description direct information about the entropy variation rate of each component is lost. On the other hand, the total entropy production is explicitly given as a function of the viscous pressure:

$$\frac{\partial S^{\alpha}}{\partial X^{\alpha}} = \frac{\pi^2}{\xi T} = 4N_* T^3 \tau \Omega^2 \theta^2 \quad , \tag{2.27}$$

where $S^{\alpha} = n\sigma u^{\alpha}$ is the total entropy current 4-vector and σ is the total specific entropy.

In order to establish more clearly the relation between T_m, T_r and T notice that the factor $\frac{4N_* T^3}{(\frac{\partial\rho}{\partial T})_n}$ in (2.26) can be rewritten as

$$\frac{4N_* T^3}{(\frac{\partial\rho}{\partial T})_n} = 1 - \frac{\Omega}{\Omega_m} \quad , \qquad (2.28)$$

where $\Omega_m = \frac{1}{3} - (\partial p_m/\partial\rho_m)_n$. The factor Ω/Ω_m is always restricted upon the interval [0,1].

Furthermore, the procedure carried out with the temperature T_m in eqs. (2.11)-(2.19) can be repeated for T. In this case, instead of eq. (2.19) one would have

$$\frac{u^{\alpha}}{T}\frac{\partial T}{\partial x^{\alpha}} + \frac{1}{3}\theta = \Omega\theta + \frac{\pi\theta}{T(\frac{\partial\rho}{\partial T})_n} \quad . \qquad (2.29)$$

Now, using eqs. (2.28) and (2.29), the expression (2.26) leads to

$$T_m = T\left[1 + (1 - \frac{\Omega}{\Omega_m})\ \tau\Omega\theta\right] \quad , \qquad (2.30)$$

and from (2.8)

$$T_r = T\left[1 - \frac{\Omega}{\Omega_m}\ \tau\Omega\theta\right] \quad . \qquad (2.31)$$

Hence, for an expanding plasma, it follows from eqs. (2.21), (2.30) and (2.31) that if $\Omega > 0$ the temperatures satisfy $T_m > T > T_r$. Alternatively, if $\Omega < 0$ $T_r > T > T_m$. Then, the Eckart temperature has always an intermediate value between the matter and radiation temperatures. Of course, this is not sufficient to prove that it coincides with the one that could be measured by the experimental device outlined in the introduction. In order to show this, let us calculate the temperature difference of each component with respect to the Eckart temperature. The specific thermal capacities of matter and radiation can be defined by

$$C_r = \frac{1}{n}\left(\frac{\partial\rho_r}{\partial T_r}\right)_n \quad , \tag{2.32}$$

and

$$C_m = \frac{1}{n}\left(\frac{\partial\rho_m}{\partial T_m}\right)_n \quad . \tag{2.33}$$

Thus, to first order in τ one obtains

$$\frac{C_r}{C_r+C_m} = 1 - \frac{\Omega}{\Omega_m} + O(\tau) \quad , \tag{2.34}$$

and substituting this result into eqs. (2.30) and (2.31) it follows that

$$T_m = T\left(1 + \frac{C_m}{C_m+C_r}\,\tau\Omega\theta\right) \quad , \tag{2.35}$$

and

$$T_r = T\left(1 - \frac{C_m}{C_m+C_r}\,\tau\Omega\theta\right) \quad . \tag{2.36}$$

Then, the difference between the matter or radiation temperature and the Eckart one are respectively given by

$$\Delta T_m = T_m - T = \frac{C_r}{C_m+C_r}\,T\tau\Omega\theta \quad , \tag{2.37}$$

and

$$\Delta T_r = T_r - T = -\frac{C_m}{C_m+C_r}\, T\tau\Omega\theta \quad . \tag{2.38}$$

therefore

$$\left|\frac{\Delta T_m}{\Delta T_r}\right| = \frac{C_r}{C_m} \quad . \tag{2.39}$$

Thus, for each component, the temperature difference with respect to the Eckart temperature and its thermal capacity are inversely proportional, a well known result from elementary calorimetry for two homogeneous substances that have been put in thermal contact at differents temperatures. Such a result means that, in fact, the Eckart temperature is the one measured by means of the device described in the introduction. Thus, it seems reasonable to identify the Eckart temperature as being physically relevant in the mixture. It is worth mentioning that eq. (2.39) also provides an alternative method for introducing the Eckart temperature which is clearly consistent with the local equilibrium hypothesis.

III. THE RADIATION ENTROPY AND ITS VARIATION RATE

The radiation entropy current 4-vector can be defined as[8]

$$S^{\mu}_{(r)} = -g \int_{K^0 \geq 0} K^{\mu} \, [F\ln F - (1+F)\ln(1+F)] \, \frac{d^3K}{K^0} \, . \quad (3.1)$$

By using $F = F_0 + F_1$, the term inside the bracket in the right--hand side of (3.1) can be written, to first order in τ , as

$$F\ln F - (1+F)\ln(1+F) \approx F_0 \ln F_0 - (1+F_0)\ln(1+F_0) + F_1 \ln \left(\frac{F_0}{1+F_0}\right) \, . \quad (3.2)$$

Substituting the above expression into (3.1), with F_0 and F_1 given by eqs. (2.3) and (2.5) a straightforward integration of (3.1) yields

$$S^{\mu}_{(r)} = \frac{4}{3} N_* T_m^3 \left[1 - 3\tau\left(\frac{u^{\lambda}}{T_m}\frac{\partial T_m}{\partial x^{\lambda}} + \frac{1}{3}\frac{\partial u^{\lambda}}{\partial x^{\lambda}}\right)\right] u^{\mu} \, . \quad (3.3)$$

Using eq. (2.8) relating T_r and T_m, it follows that

$$S^{\mu}_{(r)} = \frac{4}{3} N_* T_r^3 \, u^{\mu} \, . \quad (3.4)$$

Therefore, as expected, the radiation entropy current 4-vector has the same form as in equilibrium at temperature T_r. By using eq. (2.30) one can rewrite (3.4) in the Eckart frame as

$$S^{\mu}_{(r)} = \frac{4}{3} N_* T^3 \left(1 - 3\,\frac{\Omega^2}{\Omega_m}\,\tau\theta\right) u^{\mu} \, , \quad (3.5)$$

with the dimensionless radiation entropy per particle in the comoving frame given by

$$\sigma_r = \frac{4}{3}\,\frac{N_* T^3}{n}\,(1 - \frac{3\Omega^2}{\Omega_m}\,\tau\theta) \quad . \tag{3.6}$$

Since the homogeneous and isotropic cosmological model is usually described in the comoving coordinate system, for the sake of simplicity we will addopt it from now on. In this case, the radiation entropy variation rate can be readily derived from eq. (3.6). In fact, differentiating (3.6) with respect to time and using (2.14), (2.25) and (2.29) one may deduce that

$$\frac{\dot{\sigma}_r}{\sigma_r} = 3\Omega\theta + \frac{12N_* T^3\Omega^2\tau\theta^2}{(\frac{\partial\rho}{\partial T})_n} - 3\,\frac{d}{dt}\left[(\frac{\Omega^2}{\Omega_m}\,\tau\theta)(1 + 3\,\frac{\Omega^2}{\Omega_m}\,\tau\theta)\right] \quad . \tag{3.7}$$

Now, to go further on, it is necessary to formulate some hypothesis regarding the time derivative of the mean free time τ . We shall assume in the following that $\dot{\tau}$ is of the same order as τ . This is a reasonable assumption in the framework of the "quase-stationary" regime for which Eckart's theory is valid. In strongly transient regimes, $\dot{\tau}$ can be of the same order as the equilibrium variables. In this case, a causal thermodynamic theory as developed by Israel[9], Stewart[10] and Pavõn et al.[11] must be evoked. Hence, to first order in τ, eq. (3.8) reduces to

$$\frac{\dot{\sigma}_r}{\sigma_r} = 3\Omega\theta + \frac{12N_* T^3\Omega^2\tau\theta^2}{(\frac{\partial\rho}{\partial T})_n} - 3\,\frac{d}{dt}\,(\frac{\Omega^2}{\Omega_m}\,\tau\theta) \tag{3.8}$$

Or equivalently using eq. (2.28)

$$\frac{\dot{\sigma}_r}{\sigma_r} = 3\Omega\theta + 3(1 - \frac{\Omega}{\Omega_m})\Omega^2\tau\theta^2 - 3\frac{d}{dt}(\frac{\Omega^2}{\Omega_m}\tau\theta) \quad . \tag{3.9}$$

In Eckart's system the total entropy production is given by (see eq. (2.27))

$$n\dot{\sigma} = \frac{\pi^2}{\xi T} = 4N_* T^3\Omega^2\tau\theta^2 \quad . \tag{3.10}$$

Substituting (3.10) in (3.8) yields

$$\frac{\dot{\sigma}_r}{\sigma_r} = 3\Omega\theta + \frac{3n\dot{\sigma}}{(\frac{\partial\rho}{\partial T})_n} - 3\frac{d}{dt}(\frac{\Omega^2}{\Omega_m}\tau\theta) \quad . \tag{3.11}$$

Next consider these results in the framework of the Friedmann-Robertson-Walker Universes.

To zeroth order (adiabatic regime) we have from the above equations that

$$\frac{\dot{\sigma}_r}{\sigma_r} = 3\Omega\theta + O(\tau) \quad . \tag{3.12}$$

Inserting (3.12) in (3.10) and using eq. (2.29) with $\theta = 3\dot{R}/R$, where R is the scale factor, we obtain

$$\dot{\sigma} = \dot{\sigma}_r \, \tau\Omega\theta + O(\tau^2) \tag{3.13}$$

or still

$$\dot{\sigma} = \dot{\sigma}_r \frac{\tau}{RT}\frac{d}{dt}(RT) + O(\tau^2) \quad . \tag{3.14}$$

The above equation is exactly eq. (3.14) of Weinberg's paper[2]. He used this equation together with the condition $\frac{\tau}{RT}\left|\frac{d}{dt}(RT)\right| << 1$ to conclude that the maximum total entropy variation rate is the radiation variation rate in adiabatic regime. Since $\dot{\sigma}_r$ is proportional to σ_r, in order for it be maximum, σ_r must contribute appreciably to the total entropy σ. This is contained in the first condition for a maximum total entropy growth rate established by Weinberg. His second condition states that the mean free time τ, must be comparable to the inverse of the fractional rate of change of RT, namely

$$\tau \sim \frac{RT}{\left|\frac{d}{dt}\ (RT)\right|} \qquad .$$

Weinberg used this last quantity as the macroscopic time scale. However, as remarked before, in an expanding homogeneous and isotropic fluid, the natural macroscopic time scale is the inverse of the expansion parameter θ. Observe also that the above condition can be rewritten as $\tau\theta \sim |\Omega|^{-1}$. Thus, in certain sense, the Weinberg condition is misleading since if $|\Omega| << 1$ it implies that the radiative component is fairly decoupled ($\tau\theta >> 1$) and one could guess that such an approach is valid in the collisionless limit. In fact this is not the case. Taking into account the arguments presented in the section 2, it seems for us that a more appropriate condition would be $\tau\theta \sim 1$ provided that Ω approaches its extremum values i.e., $\Omega \sim \frac{1}{3}$ for a radiation increasing entropy or $|\Omega| \sim \frac{2}{3}$ for a decreasing one. From eq. (3.13) one finds for the maximum entropy production rates, $\dot{\sigma} \leq \frac{1}{3}\ \dot{\sigma}_r$ if $0 < \Omega \leq \frac{1}{3}$ and $\dot{\sigma} \leq \frac{2}{3}|\dot{\sigma}_r|$ if $-\frac{2}{3} \leq \Omega < 0$. Such rates are even more restrictive than the upper bound $\dot{\sigma} \leq |\dot{\sigma}_r|$ established by Weinberg.

IV. MONOPOLES AND DISSIPATION

In this section we apply some results derived in the preceding sections to estimate the effect of the radiative bulk viscosity in diluting monopoles in the very early universe.

A known problem of Grand Unified Theories, when confronted with standard Cosmology, is the monopole problem. Monopoles are expected whenever a semi-simple group-G, breaks down to a group which contains a U(1) factor[12].

The number density of such particles is given by

$$n_M \sim \frac{1}{\xi^3} \quad , \tag{4.1}$$

where ξ is some characteristic correlation lenght of the Higgs field[13]. However, causality implies that ξ must be less than the horizon distance d_h, and a lower bound to n_M is (we shall use standard values)

$$n_M \gtrsim \frac{1}{{d_h}^3} \sim 5\ N_*^{3/2}\ \left(\frac{{T_c}^2}{m_{p\ell}}\right)^3 \quad , \tag{4.2}$$

where $T_c \sim 10^{14}$ Gev is the critical temperature and $m_{p\ell} \sim 10^{19}$ GeV is the Planck mass. Using eq. (4.2) we obtain an upper bound to the equilibrium radiation entropy per monopole,

$$\sigma_M \lesssim 10^{-2}\ \left(\frac{m_{p\ell}}{T_c}\right)^3 \sim 10^{13} \quad . \tag{4.3}$$

Preskill[14] has shown that, in the adiabatic regime, if σ_M is greater than 10^{10}, monopole-antimonopole annihilations will not be efficient enough to reduce the initial monopole abundance. On the other hand, he also showed that if σ_M is less than 10^{10}, it will increase very quickly to $\sim 10^{10}$ by annihilations.

Since the present specific radiation entropy per barion is $\sigma_b \sim 10^{10}$, admiting adiabatic expansion, one would have

$$\frac{n_M}{n_b} \sim 1 \quad . \tag{4.4}$$

However, since the monopole mass M_M is $\sim 10^{16}$ greater than a baryon mass, it is easy to show that the age of the Universe would be less than 10^5 years[15].

Several authors have suggested some schemes to reduce the monopole density. The inflationary universe scenarios are the most accepted ones, but we shall not consider them in this paper.

As remarked by Preskill[15], another way to enhance σ_M further is through nonadiabatic processes which would increase the radiation entropy. However, at the same time, the entropy per baryon will also be increased by the same factor and the monopole baryon ratio would remain unaltered. In principle, the problem could be solved if entropy was generated before bariogenesis. This restricts the occurence of these processes to temperatures between $T \sim 10^{14}$ GeV ($t \sim 10^{-35}$ sec) just after the phase transition and $T \sim 10^{10}$ GeV, when the Higgs bosons decay generating the baryon asymmetry.

The monopoles begin to dominate the energy density of the Universe at the temperature $T \sim \frac{n_M}{n_r} M_n$, where n_r is the number density of the relativistic particles. The initial density of monopoles depends on the duration and on the details of the phase transition. Here we shall assume that the initial concentration is very high, $\frac{n_M}{n_r} \sim 10^{-2}$, in such a way that immediately after the phase transition the monopole dominance era begins give raising an effective dust-like stage[15]. As we have seen before, this is one of the conditions for a maximum specific radiation entropy enhancement. The other condition is $\tau\theta \sim 1$ and in order to overstimate the role of the dissipative effects we shall assume the validity of this condition during the whole the process.

In a relativistic plasma, with temperature $T < m$, the main mechanism for monopole interaction with the surrounding medium is multiple scattering of plasma particles by the monopole, with effective cross section[16]

$$C_S \sim \frac{1}{TM} \quad . \tag{4.5}$$

The collision mean free time is given by

$$\tau \sim \frac{1}{n_M C_S} \quad . \tag{4.6}$$

Differentiating (4.6) with respect to time, using (2.29) and ignoring monopole-antimonopole annihilations we obtain

$$\dot{\tau} \sim \tau\theta \quad . \tag{4.7}$$

Substituting (4.7) in (3.9), using that $\Omega \sim 1/3$, $\frac{\Omega}{\Omega_m} \sim 1$ and assuming $\tau\dot{\theta} \sim 1$, we have

$$\frac{\dot{\sigma}_M}{\sigma_M} \sim \frac{\theta}{2} \tag{4.8}$$

Integrating the above equation we obtain

$$\sigma_M \propto R^{3/2} \quad . \tag{4.9}$$

Even assuming a high initial monopole density it is easy to show that the system is in fact diluted, that is, $\frac{\lambda}{L} >> 1$, where λ is the mean free time and L is the mean interparticle distance. Thus, the kinetic approach is valid and the maximum bulk viscosity effect is to reduce the effective pressure to zero. Therefore, it is reasonable to assume

$$R \propto t^{2/3} \quad . \tag{4.10}$$

Substituting (4.10) in (4.9) we obtain that the specific entropy per monopole increases linearly with time, that is,

$$\sigma_M \propto t \quad . \tag{4.11}$$

Observing that the condition $\tau\theta \sim 1$ is only valid between $T \sim 10^{14}$ GeV ($t \sim 10^{-35}$ sec), when $\tau\theta \sim 10^{-1}$, and $t \sim 10^{-33}$ sec when $\tau\theta \sim 10$ (note that $\tau\theta$ increases linearly with time). So, after $t \sim 10^{-33}$ sec we can assume that the radiation completely decouples from the monopoles and the entropy transfer can be neglected. In this case we have

$$\sigma_{Mf} \sim 10^2 \, \sigma_{Mi} \sim 10^5 \quad . \tag{4.13}$$

Had we considered annihilations the monopoles density would have been diluted faster. Consequently the mean free time would have increased more rapidly and the dissipative effects would have been much less important. So, what we have shown is that radiative bulk viscosity is not an efficient mechanism to dilute monopoles. Further, from Preskill's results[14], the effects of annihilations are much more important and we conclude that, in general these non adiabaticities can be neglected.

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