

# Rotating Detectors and Mach's Principle

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In this work we consider a quantum version of Newton's bucket experiment in a flat spacetime: we take an Unruh-DeWitt detector in interaction with a real massless scalar field. We calculate the detector's excitation rate when it is uniformly rotating around some fixed point and the field is prepared in the Minkowski vacuum and also when the detector is inertial and the field is in the Trocherries-Takeno vacuum state. These results are compared and the relations with Mach's principle are discussed.

**Key-words:**

**Pacs numbers:** 04.20.Cv, 04.62.+v

## I Introduction

Using the fact that it is possible to define a rotating quantum vacuum [1], in this paper we study an apparatus device interacting with a scalar field producing distinct situations that raise the same philosophical questions

discussed by Mach in the Newton's bucket experiment. We conclude that in a flat spacetime scenario Mach's principle does not work in the quantum level. For a historical background of the classic problem of the rotating disc, see ref. [2], and for the problem of the definition of a rotating quantum vacuum state, see ref. [3].

Put a bucket with water inside to rotate around its axis; because of the rotation, after some time the water will acquire a parabolic shape. It is therefore possible for an observer to tell whether or not the bucket is rotating: if the water is level, it is not; if it is parabolic, it is rotating. In this sense rotation is an absolute concept. Mach states that inertia is relative to all other masses in the universe, implying that one could equally well maintain the bucket fixed and rotate all the universe around the bucket axis [4], obtaining the same result: water with parabolic shape.

As pointed out above, the possibility of defining a rotating quantum vacuum state allows us to shed some

light on this problem. We will consider the interaction of an Unruh-DeWitt detector [5] with a massless hermitian Klein-Gordon field, this interaction being responsible for possible transitions between internal states of the detector. Two main independent results are used as basis for the subsequent discussion: the first one, as we stressed, is that it is possible to define a rotating quantum vacuum [1], and the second one, well known in the literature, is that the response function of a detector travelling in a generic world-line is given by the Fourier transform of the positive frequency Wightman two-point function.

The idea is therefore to compare the following physical situations, always assuming that initially the detector is in its ground state: the rotating detector interacting with the field in the Minkowski vacuum and the inertial detector interacting with the field in the Trocherries-Takeno vacuum state. In a certain sense the Trocherries-Takeno vacuum state is the analog of putting the whole universe to rotate. We calculate then the detector's excitation rate, that is, the probability per unit detector time that it ends up in the excited state due to the interaction with the field, in these two situations. The fact that the detector gets excited in both situations clearly indicates that, also in the quantum level, an observer can tell whether or not there is a relative rotation, but the question that concerns us

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here is if the rates in both settings are equal or not. It can be noted that all the above discussion touches upon the same philosophical problems raised by Mach regarding Newton's bucket experiment, but in this case in a quantum level and in a flat spacetime, that is, in the absence of matter.

Newton defined a family of reference frames, the so called inertial frames. But what then determines which frames are inertial? This question lead Newton to introduce the *absolute space* and the inertial frames were those in state of uniform motion with respect to this absolute space. The natural consequence of this assumption is that inertial forces like centrifugal forces must arise when the proper frame of the body is accelerated with respect to this absolute space. Mach could not accept Newton's absolute space and believed that inertial forces arise when the proper frame of the body is accelerated with respect to the *fixed stars*.

"For me only relative motion exists... When a body rotates relatively to the fixed stars, centrifugal forces are produced, when rotates relatively to some other different body not relatively to the fixed stars, no centrifugal forces are produced."

As was pointed out by Weinberg [6], Mach had replaced Newton's absolute acceleration with respect to the absolute space by acceleration relatively to all other masses in the universe. In this way, the *centrifugal forces*, which for Newton are caused by the rotation with respect to the *absolute space*, is regarded by Mach as truly *gravitational forces* because they arise from the (relative) motion of all other masses in the universe. The natural consequence of this is that inertia is determined by the surrounding masses, i.e. "relativity of inertia" (see ref. [7] and the vast literature cited therein). If the proper frame of a body does not rotate with respect to the distant stars then no Coriolis forces arise.

Einstein obtained an answer for these questions in some place between both, Newton and Mach. The equivalence principle lies somewhere between these authors. If someone gives the total renormalized stress-tensor of all non-gravitational fields of the universe, then it is possible to find the metric tensor via the Einstein equations. With the gravitational potentials we can find the connections and solve the geodesic equation, to find an *inertial frame* or a freely falling frame. In this frame the laws of physics are those of special relativity. The reader may wonder if Mach's principle is valid in general relativity. It is well known that general relativity admits non-Machian solutions such as for example the Gödel solution [8], but with the fundamental problem that it presents closed timelike curves. It is possible to show that there is rotation of the matter relative to the inertial frames. There is an improvement of Gödel's cosmological solution, the Ozsváth-Schücking model [9], which assumes also a non-zero cosmological constant. This model admits foliation by a sequence

of spacelike hypersurfaces but presents the same anti-Machian behavior i.e., there is a rotation of the matter relative to the local inertial reference frame. We conclude that the Mach's principle is not contained in general relativity.

Today it is still a matter of controversy how to give a precise meaning of Mach's principle and whether general relativity includes Mach's principle or must be modified in order to be consistent with the principle. Nevertheless there is a general agreement that the dragging of inertial frames by rotating masses is a Machian effect. The first author that did the calculations of such effects was Thirring [10]. Using a weak-limit to Einstein's equations this author found that a slowly rotating massive shell can drag the inertial frames within it. Still studying rotating shells, Brill and Cohen and also Orwig again found dragging effects [11]. Foucault's pendulum is useful to give an insight of how works the dragging of inertial frames [4]. It is known that exactly on the poles the precession of the plane of oscillation of the pendulum reaches its maximum value: to an observer situated on the pole, the plane of oscillation gives one turn every twenty four hours. Following Newton one says that the plane of oscillation is fixed relative to the absolute space while the earth gives one turn beneath it. But Mach's followers would sustain that the plane of oscillation is completely determined by all masses in the universe, including, of course, the earth. In this way, if the earth were alone in the universe, its mass would be the sole determiner of the inertial properties of the pendulum and therefore earth's rotation alone would "tell" the pendulum how to precess, whereas for Newton the pendulum would still oscillate relative to the absolute space and nothing would change. Therefore, according to Mach, the earth must be dragging along the inertial frames in its vicinity, however slight may be this dragging in comparison with the influence of all other masses in the universe.

In the experimental territory there are some attempts to shed some light on these problems. Does the presence of large nearby masses affect the laws of motion? Because if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right, then no such effect would occur. Cocconi and Salpeter [12] pointed out that since there are large masses near us it is possible to perform experiments to verify if these masses affect the inertia of small bodies in the earth. Hughes, Robinson and Beltran-Lopez [13] made an extensive series of measurements with the purpose to test Mach's principle. According to Mach's principle inertial effects are due to the distribution of matter in the universe; in this way there should be present a small anisotropy in these effects due to the distribution of masses in our galaxy relatively to us. The  $Li^7$  nucleus in the ground state has a spin 3/2, so it splits in a magnetic field into four energy levels, which should be

equally spaced *if the laws of nuclear physics* are rotation invariant. If inertia were anisotropic there would appear spaced resonant lines. The results of the experiments went in opposite direction to the Mach principle. Nevertheless some authors claim that this kind of experiment does not contradict Mach's principle since the nuclear forces should also exhibit an anisotropy and someone should expect a null result in the experiment. There are a lot of attempts to incorporate the Mach's principle in general relativity, as for example ref. [14]. This author claims that the Minkowski spacetime is Machian. In the present paper we will discuss some of these questions using some new results in the literature concerning the definition of a rotating quantum vacuum.

The paper is organized in the following way: in section 2 we discuss the response function of a Unruh-DeWitt detector traveling in inertial or rotating world-lines interacting with a massless scalar field prepared in the Minkowski or Trocherries-Takeno vacuum states. The outcomes of these different situations allow us to discuss the validity of the Mach's principle in a flat spacetime. Conclusions are given in section 3. (In this paper  $\hbar = c = 1$ .)

## II Inertial and rotating detector excitation rates

The fact that it is possible to define a rotating quantum vacuum different from the Minkowski vacuum was proved in [1]. There, the scalar field is quantized in both inertial and rotating frames, the coordinates of which are related by the Trocherries-Takeno coordinate transformation (see below). Using the fact that it is possible to find the mode solutions for the Klein-Gordon equation in terms of both sets of coordinates, one is capable to compare the two quantizations by means of the Bogolubov transformations. The computation of the Bogolubov coefficient  $\beta_{ij}$  between inertial and rotating

modes [1] gives a non-vanishing result, implying that for a rotating observer the Minkowski vacuum is seen as a many Trocherries-Takeno particles state.

Let us call  $R_M^{(r)}$  the response function per unit detector proper time of the monopole detector travelling in a rotating world-line and interacting with the field prepared in the Minkowski vacuum, and let us call  $\mathcal{R}_T^{(i)} = R_T^{(i)} - R_T^{(r)}$  the normalized response function per unit time of an inertial detector in interaction with the field prepared in the Trocherries-Takeno vacuum state, which is the vacuum state properly defined by a rotating observer. Are these two quantities equal or not? If they are equal, this is in accord with Mach's principle; otherwise, Mach's principle is not obeyed in a flat spacetime.

In the following we shall be using the results of ref. [1]. In those works it is assumed that the transformation of coordinates from an inertial reference frame to a uniformly rotating one is given by the Trocherries-Takeno transformations, for which three assumptions are made: (i) the transformation laws constitute a group; (ii) for small velocities we must recover the usual linear velocity law ( $v = \Omega r$ ); and (iii) the velocity composition law is also in agreement with special relativity. In fact, the above transformation predicts that the velocity of a point at distance  $r$  from the axis is given by  $v(r) = \tanh(\Omega r)$ . The Trocherries-Takeno coordinate transformations read:

$$t = t' \cosh \Omega r' - r' \theta' \sinh \Omega r', \quad (1)$$

$$r = r', \quad (2)$$

$$\theta = \theta' \cosh \Omega r' - \frac{t'}{r'} \sinh \Omega r', \quad (3)$$

$$z = z'. \quad (4)$$

It is then possible to write the line element and also the Klein-Gordon equation in the rotating frame in terms of Trocherries-Takeno coordinates. A complete set

of solutions of the Klein-Gordon equation was found, being given by  $\{u_{qmk}, u_{qmk}^*\}$ , where

$$u_{qmk}(t, r, \theta, z) = N_2 e^{ikz} \exp[i(m \cosh \Omega r + \omega r \sinh \Omega r) \theta] \times \exp\left[-i\left(\frac{m}{r} \sinh \Omega r + \omega \cosh \Omega r\right) t\right] J_m(qr), \quad (5)$$

where  $\omega^2 = q^2 + k^2$  and  $N_2$  is a normalization factor.  $m = 0, \pm 1, \pm 2, \pm 3, \dots$ ,  $0 \leq q < \infty$  and  $-\infty < k < \infty$ . One sees that these modes are well-behaved throughout the whole manifold. Making use of the transformations (1-4) one can show that these modes are of positive frequency by using the criterium of di Sessa [15], which

states that a given mode is of positive frequency if it vanishes in the limit  $(t') \rightarrow -i\infty$ , where  $t'$  is the inertial time coordinate, while  $u_j^*$  are modes of negative frequency. In this way, the field operator is expanded in terms of these modes as:

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$$\phi(t, r, \theta, z) = \sum_m \int dq dk \left( a_{qmk} u_{qmk}(t, r, \theta, z) + a_{qmk}^\dagger u_{qmk}^*(t, r, \theta, z) \right), \quad (6)$$

where the coefficients  $a_{qmk}$  and  $a_{qmk}^\dagger$  are, respectively, the annihilation and creation operators of the Trocherries-Takeno quanta of the field. The vacuum state defined by the rotating observer is thus the Trocherries-Takeno vacuum state  $|0, T\rangle$  and it is given by

$$a_{qmk} |0, T\rangle = 0, \quad \forall q, m, k. \quad (7)$$

The many-particle states, as defined by the rotating observer, can be obtained through successive applications of the creation operators on this vacuum state.

Having sketched the canonical quantization of the scalar field in the rotating frame, we now pass to consider the probability of excitation of a detector which is moving in a circular path at constant angular velocity  $\Omega$ , interacting with the scalar field. The initial state of the detector is its ground state and for the initial state of the field we will consider the two distinct vacuum states: the usual Minkowski vacuum state and also the Trocherries-Takeno vacuum state. The interaction with the field may cause transitions between the energy levels of the detector and if it is found, after the interaction, in an excited state, one can say that it has detected a vacuum fluctuation of the field.

As a detector we shall be considering mainly the detector model of Unruh-De Witt [5], which is a system with two internal energy eigenstates with monopole matrix element between these two states different from zero. According to standard theory [16, 17, 18], the probability of excitation per unit proper time of such a system (modulo the selectivity of the detector, which does not interest us here), or simply, its excitation rate,

is given by:

$$R(E) = \int_{-\infty}^{\infty} d\Delta t e^{-iE\Delta t} G^+(x(t), x(t')), \quad (8)$$

where  $\Delta t = t - t'$ ,  $E > 0$  is the difference between the excited and ground state energies of the detector and  $G^+(x(t), x(t'))$  is the positive-frequency Wightman function calculated along the detector's trajectory. Let us note that the positive-frequency Wightman function is given by

$$G^+(x(t), x(t')) = \langle 0 | \phi(x(t)) \phi(x(t')) | 0 \rangle, \quad (9)$$

where  $|0\rangle$  is the vacuum state of the field, which can either be  $|0, M\rangle$  or  $|0, T\rangle$ . Let us consider first the second possibility.

If one splits the field operator in its positive and negative frequency parts with respect to the Trocherries-Takeno time coordinate  $t$ , as  $\phi(x) = \phi^+(x) + \phi^-(x)$  (we wrote  $\phi(x)$  for  $\phi(x(t))$ ), where  $\phi^+(x)$  contains only annihilation operators and  $\phi^-(x)$  contains only creation operators (see Eq.(6)), and also considers  $|0\rangle$  as the Trocherries-Takeno vacuum state, i.e.,  $|0\rangle = |0, T\rangle$  then, using Eq.(7), one finds that:

$$G_T^+(x(t), x(t')) = \sum_i u_i(x) u_i^*(x'), \quad (10)$$

where the subscript  $T$  stands for the Wightman function calculated in the Trocherries-Takeno vacuum state. Considering now the modes given by Eq.(5) and that we are interested in the situation where the detector is at rest in the Trocherries-Takeno frame, i.e.,  $\theta = \text{constant}$ ,  $z = \text{constant}$  and  $r = R_0 = \text{constant}$ , one finds:

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$$G_T^+(x(t), x(t')) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_2^2 e^{-i[\frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0] \Delta t} J_m^2(q R_0). \quad (11)$$

Putting the above expression in Eq.(8), we find:

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$$R_T^{(r)}(E, R_0) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_2^2 J_m^2(q R_0) \int_{-\infty}^{\infty} d\Delta t e^{-i[E + \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0] \Delta t}. \quad (12)$$

(In the above, the subscript  $T$  stands for the Trocherries-Takeno vacuum and the superscript  $(r)$  stands for the rotating world-line followed by the detector.) The last

integral gives us  $2\pi\delta\left(E + \frac{m}{R_0}\sinh\Omega R_0 + \omega\cosh\Omega R_0\right)$ , for which the argument is non-null only if  $m < 0$ ; we can take the summation index to run for  $m = 1, 2, 3, \dots$ , leaving us with

$$R_T^{(r)}(E, R_0) = 2\pi \sum_{m=1}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_2^2 J_m^2(qR_0) \delta\left(E - \frac{m}{R_0}\sinh\Omega R_0 + \omega\cosh\Omega R_0\right). \quad (13)$$

The above expression predicts excitation for the detector, and depends in a non-trivial way on the position  $R_0$  where it is put. Note that we arrive at the same confrontation between canonical quantum field theory and the detector formalism, which was settled by Letaw and Pfautsch and also Padmanabhan and Singh [3]: how is it possible for the *rotating* detector to be excited in the *rotating* vacuum? However a crucial distinction exists between our present analysis and the above-mentioned works: we state, as proved in ref. [1], that the rotating vacuum **is not** the Minkowski vacuum. The non-null excitation rate, Eq.(13), is attributed independently to the non-staticity of the Trocherries-Takeno metric and also to the detector model considered by us, which is capable to be excited through emission processes, and these two independent origins were carefully analysed in [1].

We now discuss the other case of putting the detector in a rotating trajectory and preparing the scalar field in the usual inertial vacuum  $|0, M\rangle$ . Writing  $|0, M\rangle$  for  $|0\rangle$  in Eq.(9), it is easy to show that the positive frequency Wightman function is given by:

$$G_M^+(x(t), x(t')) = \sum_j v_j(x)v_j^*(x'), \quad (14)$$

where  $M$  stands for the Minkowski vacuum state. As the rate of excitation Eq.(8) is given in terms of the detector's proper time, we shall express Eq.(14) in terms of the rotating coordinates, using the inverse of Takeno's transformations. Let us begin with  $G_M^+(x(t'_1), x(t'_2))$  written in inertial coordinates, with identifications  $r'_1 = r'_2 = R_0$  and  $z'_1 = z'_2$ , as demanded for this case:

$$G_M^+(x(t'_1), x(t'_2)) = \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_1^2 e^{-i\omega(t'_1-t'_2)+im(\theta'_1-\theta'_2)} J_m^2(qR_0), \quad (15)$$

in which  $N_1$  is the normalization of the inertial modes [1]. The inverse of Takeno's transformations read

$$t' = t \cosh \Omega r + r \theta \sinh \Omega r, \quad (16)$$

$$r' = r, \quad (17)$$

$$\theta' = \theta \cosh \Omega r + \frac{t}{r} \sinh \Omega r, \quad (18)$$

$$z' = z. \quad (19)$$

Using the above in Eq.(15) and taking note of the fact that the detector is at rest in the rotating frame, i.e.,  $\theta_1 = \theta_2$ , we see that in this manner the Minkowski Wightman function is a function of the difference in proper time  $\Delta t = t_1 - t_2$ , which allows us to calculate the rate of excitation of the orbiting detector when the field is in the Minkowski vacuum:

$$R_M^{(r)}(E, R_0) = 2\pi \sum_{m=1}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_1^2 J_m^2(qR_0) \delta\left(E - \frac{m}{R_0}\sinh\Omega R_0 + \omega\cosh\Omega R_0\right). \quad (20)$$

The result above is very much like Eq.(13), with the exception that in the above it appears the normalization of the inertial modes  $N_1$  instead of  $N_2$ . In the context of Newton's bucket experiment, the situation above is the analog of putting the bucket to rotate relative to the *fixed stars* and notice that the fact that  $R_M^{(r)}(E, R_0) \neq 0$  is translated as *water with parabolic shape*.

Finally, let us suppose that it is possible to prepare the field in the rotating vacuum and the detector is in an inertial world-line and let us calculate the excitation

rate in this situation:

$$R_T^{(i)}(E, R_0) = \int_{-\infty}^{\infty} d\Delta t' e^{-iE\Delta t'} G_T^+(x(t_1), x(t_2)), \quad (21)$$

where the superscript  $(i)$  stands for the inertial world-line followed by the detector,  $\Delta t'$  is the difference in proper time in the inertial frame, and  $G_T^+(x(t), x(t'))$  is given by Eq.(10). It is not difficult to write  $G_T^+(x(t), x(t'))$  in terms of the inertial coordinates, recalling that now the detector is not at rest in the rotating frame. We have therefore the result that:

$$R_T^{(i)}(E, R_0) = 2\pi \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_2^2 J_m^2(qR_0) \times \delta \left( E - \left( \omega\Omega R_0 - \frac{m}{R_0} \right) \sinh(2\Omega R_0) - (m\Omega - \omega) \cosh(2\Omega R_0) \right). \quad (22)$$

In order to study the activity of the Trocheries-Takeno vacuum, we calculated the rate of excitation of an Unruh-De Witt detector in two different situations: when it is put in the orbiting and in the inertial world-lines (respectively eqs. (13) and (22)). Since in the first case we found a non-null rate, contrary to the idea that the orbiting detector co-rotating with the rotating vacuum should not perceive anything, it can be considered

as a noise of the rotating vacuum, being it perceived regardless of the state of motion of the detector. This amounts to say that the inertial detector will also measure this noise, and we normalize the rate in this situation by subtracting from it the value Eq.(13), resulting in a normalized excitation rate for the inertial detector in interaction with the field in the rotating vacuum:

$$\mathcal{R}_T^{(i)}(E, R_0) = 2\pi \sum_{m=-\infty}^{\infty} \int_0^{\infty} dq \int_{-\infty}^{\infty} dk N_2^2 J_m^2(qR_0) \times \left[ \delta \left( E - \left( \omega\Omega R_0 - \frac{m}{R_0} \right) \sinh(2\Omega R_0) - (m\Omega - \omega) \cosh(2\Omega R_0) \right) - \delta \left( E - \frac{m}{R_0} \sinh \Omega R_0 + \omega \cosh \Omega R_0 \right) \right]. \quad (23)$$

In the context of Newton's bucket experiment, this situation is the analog of putting the *fixed stars* to rotate while keeping the bucket still. The fact that  $\mathcal{R}_T^{(i)}(E, R_0) \neq 0$  is translated again as *water with parabolic shape*, but note that the fact that  $\mathcal{R}_T^{(i)}(E, R_0) \neq R_M^{(r)}(E, R_0)$  means that in the quantum level these two are distinct physical situations, although in the classical level it may be that Mach's conjecture

concerning Newton's experiment is the right one.

As eqs.(20) and (23) are not equal we conclude that putting the detector in a rotating world-line in interaction with the field in the inertial vacuum is a situation not equivalent to putting it in an inertial world-line interacting with the field in the Trocheries-Takeno vacuum state. In this way we demonstrated that, regarding the Trocheries-Takeno coordinate transformations

between an inertial and a rotating frame, Mach's principle is not valid in a flat spacetime scenario.

### III Conclusions

In this paper we study an Unruh-DeWitt detector travelling in different world-lines interacting with a scalar field ensuing distinct situations that raise the same philosophical problems discussed by Mach in the Newton's bucket experiment. The calculations of the last section show us that:  $R_M^{(r)}$  the response function per unit detector proper time of the monopole detector travelling in a rotating world-line and interacting with the field prepared in the Minkowski vacuum, and  $\mathcal{R}_T^{(i)} = R_T^{(i)} - R_T^{(r)}$ , the normalized response function per unit time of an inertial detector in interaction with the field prepared in the Trocherries-Takeno vacuum state, which is the vacuum state properly defined by a rotating observer, are not equal. We are forced to conclude that Mach's principle does not work in a flat spacetime.

A natural extension of this work is to repeat the calculations for the spin-half field. Another direction is to repeat the calculations for the scalar field using the Heisenberg equations of motion [19] instead of using first order perturbation theory, as was done for the case of a uniformly accelerated observer. We expect to present these calculations in a next future.

### IV Acknowledgement

We would like to thank V.A. De Lorenci for valuable discussions. This paper was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil.

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