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Local Quantum Theory Beyond Quantization

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Recent progress on a constructive approach to QFT which is based on modular theory is reviewed and compared with the standard quantization approaches.

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1 Local Quantum Physics beyond Quantization

The slow but steadily increasing impact of the Tomita-Takesaki modular theory on QFT over the last 20 years has presently reached a stage of maturity where it promises to reshape the conceptual basis of local quantum physics[1]. Originally linked to Kubo-Martin-Schwinger (KMS) thermal properties which characterize quantum statistical systems in the thermodynamic limit [2], it became soon an important mathematical concept in the pivotal[1] issue of localization [4] and there exists by now an impressive body of structural results obtained by modular methods, most of them have been reviewed in [1].

The main goal in QFT is however not the structural results for their own sake, but rather the classification of families of nontrivial field theories and their constructive mathematical control; the structural theorems are basically intuition- and confidence-creating intermediate steps in that conquest. In this area we are in my view presently arriving at an important change of paradigm in the development of QFT. The new message is that not only are wedge-localized algebras important structural building blocks in QFT (a fact which is not totally surprising in view of the importance of the Rindler wedge in Unruh's discussion of the thermal Hawking-like aspect of the light-front Horizon), but there are even concepts which allow to classify and construct "interacting" wedge algebras as subalgebras of all operators in the (incoming) Fock space of massive particles [5]. Having a covariant net of wedge algebras (by acting with the Poincaré-group on a standard wedge algebra), the nontriviality in the setting of algebraic QFT just amounts to show that suitable intersections (representing geometrically double cones) are not exhausted by multiples of the identity. In particular one notices the existence problem approached in this way is not directly threatened by short distance problems [6].

Before explaining three basic new concepts which allow to pursue such a program (and to analytically control it for nontrivial factorizing massive models in two dimensions), it may be helpful to review the standard approach and highlight some of its weak points. The QFT of almost all of the textbooks uses a parallelism to classical field theory usually referred to as "quantization" [2]. This is most evident in the canonical approach in the early days of QFT (Dirac, Heisenberg, Pauli and Fermi), but it also remained visible in the renormalization theory of Tomonaga, Feynman, Schwinger and Dyson as well as in the subsequent functional integral approach based on euclidean actions. Even the so called causal approach of Stueckelberg, Bogoliubov and Shirkov is not entirely independent of classical ideas (although it goes a long way in this direction) because the implementation of interactions by coupling free fields in a Fock space is still in analogy to classical field theory.

Such quantization approaches usually involve some amount of "artistry" in the sense that not all of the key requirements from which one starts are reflected in the physical results. *The physical* (renormalized)

[1]Different from QM which has to rely on an added interpretive setting, most of its physical interpretation, thanks to causality of observables and localization of states [3], is already contained within QFT.

[2]This word covers a variety of different meanings ranging from the rigorous functorial definition in connection with CCR and CAR algebras over classical test function spaces for free fields, to the present more artistic use for the implementation of interactions via Lagrangians. The use in the title of the talk is the latter.

operators simply do not fulfill (unless the model happens to be superrenormalizable, which is certainly not the case for the more interesting models) *canonical commutation relations or functional integral representations.* The only commutation structure which survives is spacelike (anti)commutativity and Einstein causality. In QM the situation is much better since e.g. path integral formulation has a solid mathematical basis. But even there it is not advisable to use it outside the quasiclassical approximation; it is extremely impractical to present a course on QM with rich illustrations in such a setting. Artistry as contained in the functional integral approach beyond QM is helpful as long as the people who use it take it as a temporary recipe, and not for the ultimate definition of what constitutes QFT.

The safest framework as far as avoiding such pitfalls is the so called algebraic approach. It places the causality and localization structure into the center of the stage and links it inexorably with the notions of commutants in the theory of von Neumann algebras. It does not ban the good old quantum fields completely, but attributes to them an auxiliary status analogous to coordinates in differential geometry. One may use fields for the generation of algebras, but there is nothing intrinsic or unavoidable about them [7][8]. It may be helpful to remind oneself that the use of coordinates preceded the elegant intrinsic style of modern differential geometry and even nowadays problem-adapted coordinates are frequently used.

The problem with AQFT up to recent times was of course that in trying to find a framework which avoids the above mentioned artistry[3], it suffered from an often criticized lack of practicality and constructiveness even in such cases where its proponents used the word "constructive" instead of "axiomatic" in the titles of their papers. In the following I would like to convince the reader that we are in the middle of a process of change: the use of modular theory is rendering AQFT more computable and less esoteric.

This turn of events was made possible because of three recent concepts which went beyond the above mentioned result of Bisognano and Wichmann and which are schematically explained in the sequel.

1. It is possible to convert the unique Wigner one particle representations (m, s) directly into the net of free field algebras, thus avoiding the intermediary use of field coordinatizations altogether [5][9]. The construction uses the fact that the Wigner theory of the positive energy representation of the connected part of the Poincaré-group augmented by spacetime reflections on the rim of a wedge allows to introduce an unbounded antilinear involutory pre-Tomita operator s

$$\begin{aligned} s &= kj\delta^{\frac{1}{2}} \\ H_R(W) &= real\ span\,\{\phi \in H; s\phi = \phi\} \end{aligned} \tag{1}$$

and to define a net of closed real subspaces $H_R(W)$ of the complex Wigner representation space H. In case of integer spin the requirement that $k = 1$ and $jH_R(W) = H_R(W^{opp})$ agrees with the geometric opposite real localization subspace, whereas for semiinteger spin k is a phase factor related to the so-called Klein twist operator which preempts the statistics on the level of the Wigner

[3]The prototype of a a balanced formulation of a quantum theory is of course the operator/Hilbert space formulation of QM. AQFT tries to achieve such a balance in the presence of relativistic causality/localization.

theory. For $d \geq 1+3$ the net of real Hilbert spaces generated by Poincaré transformations is then carried directly the net of wedge affiliated von Neumann algebras by the Weyl (CCR) resp. the CAR functor and the Hawking-Unruh thermal aspect of modular localization becomes manifest [5].

In the case of d=1+2 and non semiinteger (anyone=nonquantized) spin, the braid group statistics phase is already visible in the pre-Tomita theory in Wigner space. In this case there exists no functor, the multiparticle space does not have tensor product structure and the "would be" fields with braid group statistics applied to the vacuum need the vacuum polarization clouds to maintain the very braid group statistics. This net of localized algebras cannot be obtained in the above functorial way (i.e. by rigorous quantization) but has to be obtained by a method similar to that used in the subsequent interacting case.

2. For interacting bosonic/fermionic massive particles the Tomita involution J for the interacting wedge algebra has the following representation in terms of the incoming free J_{in}

$$J = S_{scat} J_{in} \tag{2}$$

This relation can be derived from the TCP transformation of the S-matrix S_{scat} and the close relation of J to the TCP operator. In d=1+1 theories J is equal to the TCP operator. The prerequisite is the validity and completeness of the scattering theory which requires the standard mass gap assumption (as in the LSZ scattering theory). The other modular object of the pair $(\mathcal{A}(W), \Omega)$ is the Lorentz boost This, as the entire connected part of the Poincaré group, is independent of interactions (at least in the bookkeeping based on scattering theory).

3. The interacting wedge algebra has polarization free generators (PFG's) $F(f)$ [5][6] which create massive one particle state vectors with a mass gap from the vacuum.

$$F(f)\Omega = one - particle\ vector \tag{3}$$

This property should be seen in the context of the following intrinsic characterization of the presence of interactions (private communication by D. Buchholz): operators associated to algebras localized in regions whose causally completion is smaller than a wedge (i.e. the smallest region which admits a L-boost automorphism) and which have a nonvanishing one-particle component if acting on the vacuum create in addition a nontrivial (interaction-determined) charge neutral polarization cloud (unless the net is interaction free i.e. permits a free field generation). The existence of PFG's is another illustration of the magic ability of QFT to maximally utilize the breakdown of a physical argument (in this case a proof) for the unfolding of a new unexpected phenomenon. The PFG-generators can be explicitely charaterized in terms of their vacuum expectations. Although they are determined in terms of the S-matrix (including the so-called "bound state" poles[4]), they enter the vacuum expectation values in a novel "nested" fashion which gives rise to "nondiagonal inclusive

[4] Strictly speaking the hierarchy of elementary versus bound particle states ceises to make sense in QFT where it must

processes" (referring to the diagonal inclusive processes in cross sections) [6]. The vacuum restricted to the wedge algebra generated by the PFG's is a thermal KMS state of the type investigated by Unruh. For factorizing d=1+1 models (examples: sine (sinh)-Gordon) the PFG's coalesce with a KMS representation of the Zamolodchikov-Faddeev algebra thus equipping the latter for the first time with a spacetime interpretation [5][6].

This collection of obtained results requires some further comments. The net of wedge algebras defined in terms of the previous PFG construction is not sufficient in order to extract the physics. It is well known in algebraic QFT that the full physical information is contained in the net of double cone algebras and not yet in the wedge algebras. The calculation of the former by intersections of wedge algebras is an unusual step, for which presently efficient techniques are missing. It is the step from real particle (on-shell) creation contained in the S-matrix to the virtual polarization clouds which constitute the characteristic feature of QFT. It is amazing that the wedge idea allows to divide the original problem into two parts: the on-shell wedge algebra structure which is reminiscent of some relativistic QM (with channel coupling between channels of different particle numbers) whereas the off-shell double cone problem is QFT par excellence. The wedge situation gives some justification to the (otherwise quite meaningless) particle/field duality which sometimes is invoked in analogy to the quantum mechanical particle/wave duality. The wedge localization is the only one in which the two notions coexist: the particle notion in the on-shell property of the wedge algebra and the field aspect in the fact that the algebra has no annihilators and upon forming intersections leads to the full off-shell virtual particle structure in a natural way (i.e. without any further input). Everything is already preempted by the net of wedge algebras: either the double cone algebras are void (only multiples of the identity) in which case there is no local QFT compatible with the wedge data, or the net of local observables exists and is unique.

It turns out that to decide which case appears in a given situation is quite hard, even in the d=1+1 factorizing situation where the S-matrix commutes with the incoming particle number. The point is that the intersection condition which can be formulated as a vanishing commutator

$$[A, F_a(f)] = 0 \tag{4}$$
$$A \in \mathcal{A}(W),\ F_a(f) \in \mathcal{A}(W_a)$$

where $F(f)$ are the wedge generators (see [6] for the notation) and a denotes a spatial translation of W into itself $W_a \subset W$. The coefficient functions of A contained in the double cone relative commutant in terms of the Z-F algebra generators can be computed and one finds the so-called kinematical pole relation which describes the structure of the multiparticle polarization clouds in terms of matrix elements of A in the "Zamolodchikov basis". With other word the present formalism for the calculation of the algebra of

be replaced by the hierarchy of charges. It has no intrinsic meaning to say e.g. that a particular particle is a soliton independent of the chosen description (there are in general various descriptions) unless the soliton charge explains all the other charges by fusion and is not itself the result of more fundamental charge fusion.

operators in the relative commutant $\mathcal{A}(W)' \cap \mathcal{A}(W_a)$ only determines them as bilinear forms, which is not enough to calculate e.g. their correlation functions. To specialists this problem is well known from the Karowski-Weisz-Smirnov bootstrap-formfactor approach, where the control of convergence properties in the construction of correlation function has (apart from some too simple cases as the Ising order/disorder fields) remains as an unresolved issue. In our formulation with nonpointlike finite localization regions the problem looks somewhat simpler since (according to the Payley-Wiener theorem) the momentum-space fall-off properties are better. There is however another quite fundamental idea which one expects to lead to significant simplifications. This is the idea to study instead of the wedge algebras for massive d=1+1 models their *chiral conformal holographic projections.* Intuitively this is related to light cone physics, but the standard methods in this area are not good enough for the present purpose. The conceptually as well as mathematically satisfactory method to handle such holographic problems is the method of modular inclusions of subfactors [10]. These are subfactors (in the language of von Neumann algebras) inside a larger factor whose modular group fulfills a certain consistency condition with respect to the action of the larger modular automorphism on the subfactor. The main theorem is that such situations are isomorphic to chiral conformal theories in the sense of AQFT. It turns out that if one takes in the above construction of relative commutants instead of a spacelike a one of the two lightlike translations $a_\pm$ one is precisely in that situation of a modular inclusion. The associated chiral theory is localized on the light cone. For the main theorem and its prove the reader is referred to the literature [12][13]. It needs to be stressed that this holographic association of a chiral conformal QFT to a massive theory is quite different from its short distance universality class. Whereas the fields of the latter generally decompose into tensor products belonging to the two light rays, the holographic chiral theory remembers its origin because it admits in addition to the 3 geometric Moebius transformations another "hidden" (nongeometric) automorphism. This corresponds to the on W locally acting opposite (-) light ray translation which if transferred to the (+) ray becomes spread out i.e. totally "fuzzy". Here we used the equality

$$\mathcal{A}(\mathbb{R}_+) = \mathcal{A}(W) \tag{5}$$

between (half the) the light ray algebra and the wedge algebra has been made [6] as well as the fact that the natural nets for both algebras are nonlocal relative to each other. The mass operator is neither given by P_+ nor by P_- (which separately have a gapless spectrum), but rather by the product of the geometric with the hidden generator P_+P_-.

This kind of holography has a generalization to higher dimensions, but instead of a $(d-1)$ dimensional light front theory (as one would have naively expected), the formalism suggests to take $(d-1)$ identical chiral theories carefully placed in a suitable relative position in a common Hilbert space. A more appropriate terminology would be to speak of a "chiral scanning" of a higher dimensional theory. All this seems to be extremely powerful in a constructive approach, but it has not yet been explored. The modular method applied to d=1+1 factorizing systems in conjunction with its chiral holographic projection, with the aim to obtain complete mathematical constructive control over this interesting class of nontrivial 2-dimensional models is what I am presently working on [6].

2 Future Prospects

The only way of giving some more hints in a page-limited conference report is to mention some of the dreams which this new approach lends itself to.

- **A purely field-theoretic classification and construction of chiral theories based on modular theory**

  The present constructions are using special (affine) algebras which did not originate in QFT and which have no known higher dimensional generalizations. On the other hand the exchange algebra which is of pure field theoretical origin is (unlike its special case of CCR and CAR algebra) incomplete in that its distribution theoretical character at colliding points remains unspecified. With some hindsight and artistry on monodromy, one is able to compute 4-point functions for the family of minimal models [14][15], but this is a far shot from. On the other hand a modular construction would start as the exchange algebras from the statistics data (R-matrices obtained by classifying Markov traces[5] on the universal braid group $B_\infty, \simeq$ topological field theory) and use them (instead of the) for the modular construction of the halfline algebra. By Moebius transformation (instead of intersections) one should be able to obtain the charge-carrying (which carry the plectonic charges) operators for arbitrary small regions. In this way an old dream 74/75 of Swieca and myself could still come true which consisted in the hope to classify the conformal block decompositions by purely field theoretic methods (unlike the later use of representation theory of special (affine) algebras) and in this way convert chiral QFT into a bona fide theoretical laboratory of general QFT. This dream would also include the understanding of the very peculiar and special Friedan-Qiu-Shenker quantization in the energy-momentum tensor as a consequence of the more universal (DHR-Jones-Wenzl) statistics quantization.

- **A theory of "free plektons"**

  It has been known for some time, that the d=1+2 carriers of braid group statistics must have a semiinfinite spacelike string localization and that they obey a spin-statistics theorem. Unlike chiral theories, 3-dim. theories do allow for deformation parameters (coupling strength whose continuous change have no effect on the superselection rules). It is an educated guess that there exist "free plektons" in the sense that they have vanishing cross sections although their S-matrix is not one but rather piecewise constant and equal to one of the R-matrices (with all the caveats related to the very definition of scattering matrices in the absence of a Fock space tensor structure!). Such an

[5] This method (in the special case for which the braid group degenerates into the permutation group P_∞) appears already in the 70/71 DHR work [7]

S would still commute with the incoming particle number (as in the d=1+1 factorizing case) and would be used in a modular approach in order to construct PFG wedge algebra generators and, via intersections, to obtain spacelike cone localized (string-like) plektonic operators. Although without real particle creation, such a free plektonic theory would (unlike free Bosons and Fermions) have a rich virtual vacuum polarization structure [16]. The nonrelativistic limit should be carried out in such a way that the spin-statistics connection remains intact. The nonrelativistic limit would maintain its field theoretic vacuum polarization structure (necessary to upheld the spin-statistics relation) and the result would therefore not be consistent with the well-known Leinaas-Myrrheim or Aharonov-Bohm (in Wilczek‘s approach) quantum mechanical description. Plektons are also too noncommutative in order to permit a well-defined Lagrangian quantization description.

- **Holographic or scanning methods for higher dimensional constructions**

  Difficult problems as the classification and constructive determination of higher dimensional QFT‘s often render themselves more susceptible to solution if one succeeds to chop them into several easier pieces. The "smallest" and best understood kind of QFT is certainly chiral QFT. According to previous comments, one may hope that a finite collection of chiral theories in carefully arranged relative positions (modular intersections and inclusions) in a common Hilbert space may contain all the necessary information for the construction of a higher dimensional QFT.

Such a construction, if feasible, would sharpen a paradigm which underlies AQFT[6]: the new physical reality (of AQFT or Local Quantum Physics) is not that of a manifold with a material content (the Newtonian reality which extends into relativity and QM), but is rather thought of as coming about through relations between objects which themselves have no individuality (like the monades of Leibnitz). In technical-mathematical language the physical reality of local nets comes about by modular inclusions/intersections (and more general kinds of relative positioning) of the unique hyperfinite type III_1 von Neumann factors which (apart from being able to be contained in each other and form intersections) behave in many respects as points in geometry.

[6] This has been formulated on different occasions in past articles and conference reports by R. Haag

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