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On Massive Vector Bosons and Abelian Magnetic Monopoles in $D = (3+1)$: a Possible Way to Quantise the Topological Mass Parameter

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Abstract

An Abelian gauge model, with vector and 2-form potential fields linked by a topological mass term mixing the two Abelian factors, is shown to exhibit Dirac-like magnetic monopoles in the presence of a matter background. In addition, considering a 'non-minimal coupling' between the fermions and the tensor fields, we obtain a generalised quantisation condition that involves, among others, the topological mass parameter. Also, it is explicitly shown that 1_loop (finite) corrections do not shift the value of such a mass parameter.

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[*] Dedicated to our Master Prof. José Leite Lopes on the occasion of his 80^{th} birthday. Earlier versions of this manuscript appeared in Ref.[18,19].

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Introduction

Magnetic monopoles were firstly proposed by Dirac [1] in the framework of Classical Electrodynamics with the main aim to provide a physical explanation of why the electric charges appear only as integer multiples of the elementary one (electron or proton charge, denoted by e). Indeed, Dirac obtained that *" if there exists one quantum magnetic pole in Nature, g_o, interacting with electric charges, then Quantum Mechanics demands the quantisation of the latter according to*:[1]

$$qg_o = 2\pi\hbar c \quad ,\text{with} \quad q = ne \quad , n \quad \text{integer.}'' \tag{1}$$

Among other features, his work pointed out the relation between gauge invariance and the singular structure of gauge potentials, the non-physical string (see also Ref.[2], section 2.5 and Ref.[3]).

In general, these objects are '*put in by hand*' in Electrodynamics-type models (Maxwell, Proca, etc.) by breaking the Bianchi's identity of the A_μ-sector (so, circumventing the Poincaré's lemma on differential forms). Their presence restore the *duality* between the electric and magnetic sectors, lost after the introduction of the electric current. Therefore, Dirac's monopoles render Electrodynamics more symmetric, and the $U(1)$-gauge group *compact*: the Abelian and unitary operator S which implements the gauge transformations becomes *single-valued*. In particular, this aspect is crucial for non-Abelian theories which have their vacuum symmetry broken by scalar fields (Higgs' mechanism). In these cases, if the original non-Abelian gauge group of the vacuum is broken to $U(1)$-compact group, then the classical dynamical equations yield *static* solutions carrying (Abelian) magnetic charge (at large distances, looking as Dirac's monopoles). This was firstly shown by 't Hooft [4] and Polyakov [5], dealing with the Georgi-Glashow's [6] model; see Ref.[2], Sections 5 and 6, for the extension to arbitrary *simply-connected* gauge groups (see also the references listed in [7]). Recently, it was shown that $N = 2$-supersymmetric Yang-Mills theories present *monopole condensation*, which seems to be essential for the understanding of quark confinement [8]. Eventually, if such non-Abelian gauge theories (supersymmetric or not) are correct, then their magnetic monopoles *must exist.*[2]

There are some similarities and differences between 't Hooft-Polyakov' and Dirac's monopoles. Here, we wish to pay attention to one of these differences: while the first type coexists with massive vector boson (the masses of both being given by the scalar fields, after the spontaneous symmetry breaking) the same does not happen to the second one. In fact, it seems that for Abelian theories (defined on Minkowski's flat space-time), Dirac's monopoles *can appear only if* the vector boson is *massless* [10]-[13]. This has been shown in several works to be true for the Proca's model (the simplest finite-range extension of Maxwell's theory, where the boson mass stems from explicity breaking the gauge

[1]We are using Lorentz-Heaviside's units for Electrodynamics. In his original paper Dirac [1] used Gaussian ones.

[2]Nevertheless, the observation of such objects is deeply jeopardized by their huge masses. For example, for $SU(5)$- gauge group these masses are of the order of 10^{16}Gev, increasing with the enlargement of the group because the energy breaking scales are shifted up. See, for example, Ref. [9].

symmetry) and, in addition, some attempts have been made to bypass this impossibility, by considering pairs of monopoles (with opposite charges) joined by a Dirac' string [12], or even the presence of a '*massive tachyon*' as being the *superluminal counterpartner* of the 'physical massive photon' [13].

It is precisally on this subject that lies our motivation for this work: are there any physical arguments that rule out the coexistence of both massive vector bosons and Dirac monopoles within an Abelian model defined on flat Minkowski space-time? Would such an impossibility arise from the structure of a particular theory or from the specific *mechanism* for gauge boson mass generation?
At the attempt of taking some glance on this question, we shall study a particular model, within which two Abelian sectors (a vector and a 2-form gauge potentials) are linked by a *topological mass* term, giving us a massive vector boson as its particle physical content, Ref.[14, 15].

We should stress that, even though we are not presenting here a general proof for the question raised above; our purpose is to provide one more explicit example of a theory in which Dirac-like monopoles do not show up while the intermediate gauge boson is massive. The particular mechanism for gauge-field mass generation does not seem to be relevant for the suppression of the monopoles: once the gauge-field propagator develops a non-trivial pole, Dirac monopoles are ruled out (we shall come to this matter throughout our paper).
To conclude the presentation of the arguments that motivate our work, we should draw the attention to a peculiar feature: the monopole appearing in the CSKR model is such that the gauge-field mass parameter enters the charge quantisation relation, as it will become clear at the end of our paper.
This paper is outlined as follows: in Section 1, we start by presenting the model as well as some of its basic characteristics. In Section 2, we show that the model under consideration does not admit, consistently, the coexistence of both Dirac's monopoles and massive vector boson, unless we take a special *ansatz* for the current, previously incorporated in the model interacting with A_μ gauge field. We start Section 3 by allowing an 'extra-coupling' between the fermionic current and the tensorial gauge sector, by means of a gauge and Lorentz invariant term. In addition, it is shown that if the current *ansatz* is implemented, we get a *generalised* quantisation condition, which contains, among others, the mass parameter. This section is closed with a discussion on the no-shift of the topological mass parameter by (finite) 1_loop contributions. The relevant Feynman's graph and its result are presented in the Appendix. Finally, we conclude the paper by making a brief discussion about the results and some possible consequences of them.

1 The model and some basic aspects

The Cremmer-Scherk-Kalb-Ramond (CSKR) model [14, 15] in the absence of matter fields reads:

$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{6}G_{\mu\nu\kappa}G^{\mu\nu\kappa} + \mu_0\epsilon_{\mu\nu\kappa\lambda}A^\mu\partial^\nu H^{\kappa\lambda}\,, \tag{2}$$

with the definitions for the field strengths:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \text{and} \qquad G_{\mu\nu\kappa} = \partial_\mu H_{\nu\kappa} + \partial_\nu H_{\kappa\mu} + \partial_\kappa H_{\mu\nu}\,, \tag{3}$$

$H_{\mu\nu} = -H_{\nu\mu}$. Here, we are using Minkowski metric $diag(\eta_{\mu\nu}) = (+,-,-,-)$ and $\epsilon^{0123} = +1 = -\epsilon_{0123}$ for the four-dimensional Levi-Civita symbol; greek indices run $0,\ldots 3$; latin characters go from 1 to 3.

As it can be easily checked, the action $S_1 = \int dx^4 \mathcal{L}_1$ is invariant under the independent local Abelian gauge transformations:

$$A_\mu(x) \overset{U(1)_{A\mu}}{\longmapsto} A'_\mu(x) = A_\mu(x) - \partial_\mu \Lambda(x)\,, \tag{4}$$

$$H_{\mu\nu}(x) \overset{U(1)_{H\mu\nu}}{\longmapsto} H'_{\mu\nu}(x) = H_{\mu\nu}(x) + \partial_\mu \xi_\nu(x) - \partial_\nu \xi_\mu(x)\,, \tag{5}$$

provided that we assume that the parameters Λ and ξ_μ vanish at infinity.

From (2), there follow the field equations:

$$\partial_\mu F^{\mu\nu} = -\mu_0 \epsilon^{\nu\kappa\alpha\beta} \partial_\kappa H_{\alpha\beta} = -\frac{\mu_0}{3} \epsilon^{\nu\kappa\alpha\beta} G_{\kappa\alpha\beta}\,, \tag{6}$$

$$\partial_\mu G^{\mu\nu\kappa} = +\mu_0 \epsilon^{\nu\kappa\alpha\beta} \partial_\alpha A_\beta = +\frac{\mu_0}{2} \epsilon^{\nu\kappa\alpha\beta} F_{\alpha\beta}\,, \tag{7}$$

and, from the antisymmetric property of the field strenghts, we get the Bianchi's identities (geometrical equations):

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \text{and} \qquad \partial_\mu \tilde{G}^\mu = 0\,, \tag{8}$$

with: $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\lambda} F^{\kappa\lambda}$ and $\tilde{G}_\mu = \frac{1}{6}\epsilon_{\mu\nu\kappa\lambda} G^{\nu\kappa\lambda}$ defining the dual tensors.

The linking term between the gauge fields is *topological* because it does not contribute to the gauge invariant energy-momentum tensor (and so, carrying no energy and propagating no physical degrees of freedom), what is obvious since it requires no metric for its definition (like the Chern-Simons term in 3 dimensions). On the other hand, one sees that one gauge field (or more precisally, its field strength) provides a *current* for another, and vice-versa, having these currents came about from the topological term.

The spectrum of the model is the following: if we take $\mu_0 = 0$ (free Lagrangean), A_μ describes a massless vectorial boson and $H_{\mu\nu}$ behaves as a massless scalar field. Therefore, we have 3 degrees of freedom (on-shell). In the other case ($\mu_0 \neq 0$), we have a massive vector boson (with mass $M^2 = +2\mu_0^2$). Here, this particle can be described by A_μ as well as by $H_{\mu\nu}$. Thus, in both cases, the model has 3 on-shell degrees of freedom, what is physically convincent, because the topological term introduces no additional ones, as we said earlier. In fact, it provides a mass generating mechanism, that was called *topological dynamic symmetry breaking*, by Cremmer and Scherk [14]. Kalb and Ramond [15] studied it in the context of classical interaction of strings in dual models.

Moreover, it has been shown that the model is unitary and renormalisable (in the presence of fermions interacting with the A_μ gauge field; the model presented in section 2, equation (17)), and also that its mass generating mechanism is different (at quantum level) from the Higgs when this is added to the Maxwell theory [16]. Among others features, the vacua funtional for the model was obtained by Amorim and Barcelos-Neto [17].

2 The matter background and the Dirac-type monopole configuration in the model

Here, we shall show that, at a naïve step, Dirac's monopoles cannot appear within the CSKR-model. Nevertheless, situation can be changed (at low momentum limit) if we introduce matter current in the model behaving in a particular way. We start by introducing classical configurations of Dirac's magnetic monopole in the CSKR-model. This is done by 'breaking' the Bianchi's identity for the A_μ-sector[1, 2], say:[3]

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \stackrel{monopole}{\longmapsto} \partial_\mu \tilde{F}^{\mu\nu} = \chi^\nu \,, \tag{9}$$

where the conserved magnetic 4-current is given by: $\chi^\mu = (\chi^0, \vec{\chi})$.

For our purposes, should be more convenient to work with the field equations in vector notation. So, we define:

$$A^\mu \equiv (\Phi, +\vec{A}) \qquad H_{\mu\nu} = \begin{cases} H_{0i} \equiv (+\vec{a})_i \\ H_{ij} \equiv -\epsilon_{ijk}(\vec{\varphi})_k, \end{cases} \,, \tag{10}$$

and the field strengths as:

$$F_{\mu\nu} = \begin{cases} F_{0i} \equiv +(\vec{E})_i \\ F_{ij} \equiv -\epsilon_{ijk}(\vec{B})_k \end{cases} \qquad G_{\mu\nu\kappa} = \begin{cases} G_{0ij} \equiv -\epsilon_{ijk}(\vec{\mathcal{E}})_k \\ G_{ijk} \equiv +\epsilon_{ijk}\mathcal{B} \end{cases} \,, \tag{11}$$

which give us: $\tilde{G}^\mu = (\mathcal{B}, +\vec{\mathcal{E}})$.

Now, the set of equations (6,7,9) and the identity $\partial_\mu \tilde{G}^\mu = 0$, describing a static and point-like magnetic monopole ($\chi^0 = +g\delta^3(x)$; $\vec{\chi} = 0$ and the static limit for the fields) take the forms:

$$\nabla \wedge \vec{B}(\vec{r}) = -2\mu_0 \vec{\mathcal{E}}(\vec{r}) \quad , \quad \nabla \cdot \vec{E}(\vec{r}) = -2\mu_0 \mathcal{B}(\vec{r}) \tag{12}$$

$$\nabla \wedge \vec{\mathcal{E}}(\vec{r}) = +\mu_0 \vec{B}(\vec{r}) \quad , \quad \nabla \cdot \vec{\mathcal{E}}(\vec{r}) = 0 \tag{13}$$

$$\nabla \cdot \vec{B}(\vec{r}) = \chi^0(\vec{r}) = g\delta^3(\vec{r}) \quad \text{and} \quad \nabla \wedge \vec{E}(\vec{r}) = 0 \tag{14}$$

$$\nabla \mathcal{B}(\vec{r}) = -\mu_0 \vec{E}(\vec{r}) \tag{15}$$

Now, to study the self-consistency of the above equations, we split them in two sets: one involving the $\mathcal{B}$ and $\vec{E}$ fields, and the another with $\vec{B}$ and $\vec{\mathcal{E}}$ vectors . For the first set, it is easy to find good solutions [10, 18]:

$$\vec{E}(\vec{r}) = \frac{\vec{E}_0}{4\pi} exp(-\sqrt{2}\mu_0|\vec{r}|) \quad \text{and} \quad \mathcal{B}(\vec{r}) = \frac{\mathcal{B}_0}{4\pi} exp(-\sqrt{2}\mu_0|\vec{r}|), \tag{16}$$

with $\sqrt{2}\mu_0\mathcal{B}_0\hat{r} = +\vec{E}_0$. Nevertheless, for the other set we have troubles: the monopole-like solution that comes from: $\vec{B}(\vec{r}) = +g\vec{r}/4\pi r^3 \equiv \vec{B}^D(\vec{r})$ is inconsistent with $\nabla \wedge \vec{B}(\vec{r}) =$

[3] We shall use the expressions *electric* and *magnetic* for the A_μ sector, by its analogy with the Maxwell's Electrodynamics.

$-2\mu_0\vec{\mathcal{E}}(\vec{r})$ ($\neq 0$, a priori). Even here, we may search for a more general solution for $\vec{B}$: $\vec{B}(\vec{r}) = \vec{B}^D(\vec{r}) + \vec{B}'(\vec{r})$ (and similar forms to $\vec{A}$ and $\vec{\mathcal{E}}$ [10, 18]) with $\vec{B}'$ given by:

$$\vec{B}'(\vec{r}) = \nabla \wedge \frac{\vec{\mathcal{E}}'(\vec{r})}{\mu_0} = +\frac{2\mu_0}{4\pi}\int d^3\vec{r}'(1+\sqrt{2}\mu_0 R)\,\frac{exp(-\sqrt{2}\mu_0 R)}{R^3}\,(\vec{\mathcal{E}}^D(\vec{r}')\wedge\vec{R}),$$

whith $\vec{R} \equiv (\vec{r} - \vec{r}')$. Unfortunately, these new solutions prevent us from obtaining a *conserved angular momentum operator*, $\mathcal{J}$, and so from quantise the system of an electrically charged particle placed into this magnetic field[4] (at the non-relativistic limit), whose Lagrangean is $L_p = \frac{1}{2}m\dot{r}^2 + q\vec{A}\cdot\dot{\vec{r}}$, with $\vec{A} = \vec{A}^D + \vec{A}'$.
Alternatively, based on the Wu-Yang's approach [20], one can demonstrate the non-existence of a Abelian and unitary operator S which would relate two functions A^a_μ and A^b_μ, in a overlapping region around the monopole, by a gauge transformation (this is worked out in Ref.[18]). Consequently, at this first stage, the CSKR-model *is not* compatible with Dirac's monopoles and this comes about due the massive character of the vectorial boson. In other words, the mass parameter prevents the magnetic field created by the monopole from being spherically symmetric and this, in turn, leads us to the troubles discussed above.

Let us carry on our work and take the CSKR-model with matter fields (say, fermionic). The Lagrangean reads:

$$\mathcal{L}_1 \overset{matter}{\longmapsto} \mathcal{L}_2 = \mathcal{L}_1 + \overline{\psi}(x)\,(\imath D_\mu\gamma^\mu - m_f)\,\psi(x)\,, \tag{17}$$

with $\mathcal{L}_1$ already defined in (2) and $D_\mu\psi(x) \equiv (\partial_\mu + \imath e A_\mu)\psi(x)$.

It is easy to see that S_2 is $U(1)_{A\mu}\otimes U(1)_{H\mu\nu}$-invariant, provided that the fermionic fields transform in the usual way: $\psi(x) \mapsto \psi'(x) = e^{+\imath e\Lambda(x)}\psi(x)$ and $\overline{\psi}(x) \mapsto \overline{\psi'}(x) = e^{-\imath e\Lambda(x)}\overline{\psi}(x)$.

From $\mathcal{L}_2$, the dynamical equations for the fermions follow:

$$(\imath D_\mu\gamma^\mu - m_f)\psi(x) = 0 \qquad \text{and} \qquad \overline{\psi}(x)(\imath\overleftarrow{\partial}_\mu\gamma^\mu + eA_\mu\gamma^\mu + m_f) = 0\,. \tag{18}$$

Analogously, for the gauge fields, we obtain their dynamical equations:

$$\partial_\mu F^{\mu\nu} = -\mu_0\epsilon^{\nu\kappa\lambda\rho}\partial_\kappa H_{\lambda\rho} + eJ^\nu = -2\mu_0\tilde{G}^\nu + eJ^\nu\,, \tag{19}$$

$$\partial_\mu G^{\mu\nu\kappa} = +\mu_0\epsilon^{\nu\kappa\lambda\rho}\partial_\lambda A_\rho = +\mu_0\tilde{F}^{\nu\kappa}\,, \tag{20}$$

and also the Bianchi's identities (8). Here, the conserved fermionic 4-current is defined by: $J^\mu \equiv \overline{\psi}\gamma^\mu\psi = (\rho, \vec{J})$.

As we did earlier, introducing static and point-like monopole and taking the equations describing it, with fermionic 4-current, we get:

$$\nabla\wedge\vec{B}(\vec{r}) = +e\vec{J}(\vec{r}) - 2\mu_0\vec{\mathcal{E}}(\vec{r}) \quad , \qquad \nabla\cdot\vec{E}(\vec{r}) = e\rho - 2\mu_0\mathcal{B}(\vec{r}) \tag{21}$$

$$\nabla\wedge\vec{\mathcal{E}}(\vec{r}) = +\mu_0\vec{B}(\vec{r}) \quad , \qquad \nabla\cdot\vec{\mathcal{E}}(\vec{r}) = 0 \tag{22}$$

$$\nabla\cdot\vec{B}(\vec{r}) = +\chi^0 = +g\delta^3(\vec{r}) \quad , \qquad \nabla\wedge\vec{E}(\vec{r}) = 0 \tag{23}$$

$$\nabla\mathcal{B}(\vec{r}) = -\mu_0\vec{E}(\vec{r}) \quad . \tag{24}$$

[4]This point is not so obvious. The arguments which lead us to this result are presented in Ref.[10], and are based upon $SU(2)$ algebra analysis.

It is clear that, the presence of this current in the above equations leads us to describe *another type* of magnetic monopoles, different of those Dirac's ones. This difference will be later clarified.

Let us study the self-consistency of these equations: again, for the set of $\vec{E}$ and $\mathcal{B}$ fields it is easy to obtain well-behaved solutions:

$$\mathcal{B}(\vec{r}) = -\frac{e\mu_0}{4\pi}\frac{exp(-\sqrt{2}\mu_0|\vec{r}|)}{|\vec{r}|} \quad , \quad \vec{E}(\vec{r}) = \frac{e}{4\pi}(r - \sqrt{2}\mu_0 r^2)\frac{exp(-\sqrt{2}\mu_0|\vec{r}|)}{|\vec{r}|^3}\hat{r}$$

Now, to solve the former problem presented by the another set, we look for $\mathcal{L}_2$ at low momentum[5]

$$\mathcal{L}_2 \stackrel{p\to 0}{\longrightarrow} \mathcal{L}_{p\to 0} \approx +\mu_0\epsilon^{\mu\nu\kappa\lambda}A_\mu\partial_\nu H_{\kappa\lambda} - eJ^\mu A_\mu + (\text{fermionic mass term})\,, \tag{25}$$

(' $\approx$ ' stands for *approximately to*). Taking the field equation for A_μ, we get:

$$eJ_\mu = +2\mu_0\tilde{G}_\mu\,, \tag{26}$$

Here, we are dealing essentially with the non-relativistic limit (low momentum) of a physical system (particle into a external magnetic field); therefore, it is physically acceptable to take the following *ansatz*:[6]

$$e\vec{J}(\vec{r}) = +2\mu_0\vec{\mathcal{E}}(\vec{r})\,. \tag{27}$$

Employing this relation in the first equation of (21), the sectors of $\vec{B}$ and $\vec{\mathcal{E}}$ fields become consistent. In other words, the *ansatz* (27) damps the $\vec{B}'$ part of $\vec{B}$. Physically, what seems to happen (at low energy level) is that the $\vec{\mathcal{E}}$ (or more precisaly, the matter background current, $\vec{J}$) field cuts the effect of $\vec{B}'$, at least as the total field felt by the electric charge.

Returning to the presence of the fermionic current in eqs. (21-24), we shall interpret this current as a material background onto which the magnetic monopole configurations are placed. It is just in this sense that we distinguish between them and those of Dirac's types: these latter are classical configurations in the vacuum (Classical Electrodynamics in vacuum, to be more precise), and so, they need no material media for their *'existence'*. Even though, our monopoles *cannot appear* in vacua, they would configurate, for exemple, in a superconductor medium, inside which the Cooper's pairs of electrons would be this background, at any stationary limit, because $e\nabla\cdot\vec{J} = 2\mu_0\nabla\cdot\vec{\mathcal{E}} = 0$). In addition, notice a similarity: both, the CSKR-model and a superconductor medium appear to have massive 'photon'.

Another point that should be stressed concerns the background: we suppose, and this seems reasonable, that the charges acting as the sources for the electric and magnetic fields that yield the monopole configuration weakly affect the background, so that the

[5]Noticing the correspondence: $\imath\partial_\mu \leftrightarrow p_\mu$, we take the low momentum limit by $p^2 \ll p$ and write $\mathcal{L}_2$ up to terms proportional to p (or better ∂). In words, we consider the kinetical terms *small* as compared with others.

[6]Let us remind the London's *ansatz* for superconductivity: $j_\mu = \kappa A_\mu$. Despite of the nature of the fields (A_μ is a gauge field and $\tilde{G}_\mu$ a gauge invariant quantity), both forms are quite similar.

back reaction on the latter does not influence the conditions that allow *'monopole formation'*. However, if the density of charges becomes very high and the energy of the system of electric and magnetic fields is comparable to the energy density of the background, then our assumptions would be jeopardised. In short, we understand that we are relying on the approximation that the sources do not affect the background.
This background current seems to be very formal, introduced only to accomodate our monopole-like solution. The interesting question that now we raise is how to systematically propose a potential in the Dirac's equation in such a way that its solution, ψ, leads to a current $\vec{J}$ such that (27) is fulfilled. From our analysis, we have obtained that an arbitrary potential, V, yielding a current given by (27) does not lead to a separable form of the Dirac's equation. Imposing that $\vec{J}$ is known, V is not uniquely fixed, i.e., different families of non-separable V lead to the same expression for $\vec{J}$, and we are attempting at an explicit solution for ψ as a result from the Dirac's equation with a particular potential.

Now, writing the non-relativistic Lagrangean for the system: $L_p = \frac{1}{2}m\dot{r}^2 + q\vec{A}\cdot\dot{\vec{r}}$, with $\vec{A} = \vec{A}^D$, and search for a conserved angular momentum vector, we find[7]

$$\vec{\mathcal{J}} = \vec{r} \wedge \vec{p} - \frac{gq}{4\pi c}\hat{r},$$

and quantising its radial component (here, treated as a quantum operator) according to Quantum Mechanics [21], we get:

$$\hat{\mathbf{r}} \cdot \mathcal{J} = \frac{n}{2} \quad \Longrightarrow \quad \frac{qg}{4\pi\hbar c} = \frac{n}{2} \qquad n = 1, 2, \ldots . \tag{28}$$

Therefore, we obtain a *quantisation condition* for the problem (analogous to eq. (2)). [However, the difference put between the two types of such Abelian monopoles must be remembered and taken into account]. The using of others procedures (e.g. single-valuedness of the wave-function or Wu-Yang's approach) shall lead us to the same result, eq. (28).

To close this section, we draw the attention to the fact that a similar treatment to Proca's theory would lead us to a quite analogous conclusion: this theory is compatible with the monopoles that were here introduced. On the other hand, we justify our choice by CSKR model because it presents another very interesting feature: the mass parameter appears in a more general quantisation condition. This will be the goal of the following section.

3 The 'non-minimal' coupling and mass quantisation

In this section we shall introduce a new kind of 'coupling' into the model. This will be done by the following gauge covariant derivative: $\nabla_\mu\psi(x) \equiv (\partial_\mu + \imath e A_\mu - \imath\sigma\tilde{G}_\mu)\psi(x)$,

[7]The first term is the angular momentum of a point-like object with momentum $\vec{p}$ and the second comes from the interaction between the electromagnetic fields of both particles. In addition, we know that in the quantum mechanical context its counterpartner operator must commute with the Hamiltonian operator and satisfy the $SU(2)$ algebra.

where σ is the parameter that measures the strength of the coupling between the fermions and the tensorial sector. Hence, the model is:

$$\mathcal{L}_3 = \mathcal{L}_1 + \overline{\psi}\,(i\nabla_\mu\gamma^\mu - m_f)\psi\,, \tag{29}$$

(here, we choose $e, \sigma > 0$, as we have already taken for μ_0) .[8]
The influence of non-minimal coupling on the 3-dimensional Maxwell-Chern-Simons model has been discussed in a series of works (some of them are listed in Ref. [22]).

From (29), there follow the dynamical eqs. for the femions:

$$\left[(\imath\partial_\mu - eA_\mu + \sigma\tilde{G}_\mu)\gamma^\mu - m_f\right]\psi(x) = 0$$

$$\overline{\psi}(x)\left[(\imath\overleftarrow{\partial}_\mu + eA - \mu - \sigma\tilde{G}_\mu)\gamma^\mu + m_f\right] = 0\,,$$

and those for the gauge fields:

$$\partial_\mu F^{\mu\nu} = -\mu_0\epsilon^{\nu\kappa\alpha\beta}\partial_\kappa H_{\alpha\beta} + eJ^\nu = -2\mu_0\tilde{G}^\nu + eJ^\nu\,, \tag{30}$$

$$\partial_\mu G^{\mu\nu\kappa} = +\epsilon^{\nu\kappa\alpha\beta}\partial_\alpha\left(\mu_0 A_\beta + \frac{\sigma}{2}J_\beta\right) = +\mu_0\tilde{F}^{\nu\kappa} + \frac{\sigma}{2}\epsilon^{\nu\kappa\alpha\beta}\partial_\alpha J_\beta\,, \tag{31}$$

also, the already known Bianchi's identities (8).

Doing the same considerations as before to introduce magnetic monopoles (static and point-like classical configuration onto a matter background), we get the following equations:

$$\nabla\wedge\vec{B}(\vec{r}) = +e\vec{J}(\vec{r}) - 2\mu_0\vec{\mathcal{E}}(\vec{r})\,,\quad \nabla\cdot\vec{E}(\vec{r}) = +e\rho(\vec{r}) - 2\mu_0\mathcal{B}(\vec{r}), \tag{32}$$

$$\nabla\wedge\vec{\mathcal{E}}(\vec{r}) = +\mu_0\vec{B}(\vec{r}) + \frac{\sigma}{2}\nabla\wedge\vec{J}(\vec{r}) \quad , \quad \nabla\cdot\vec{\mathcal{E}}(\vec{r}) = 0 \tag{33}$$

$$\nabla\cdot\vec{B}(\vec{r}) = \chi^0(\vec{r}) = g\delta^3(\vec{r}) \quad , \quad \nabla\wedge\vec{E}(\vec{r}) = 0, \tag{34}$$

$$\nabla\mathcal{B}(\vec{r}) = -\mu_0\vec{E}(\vec{r}) + \frac{\sigma}{2}\nabla\rho(\vec{r}). \tag{35}$$

Now, we see that both sets of equations (one mixing $\mathcal{B}$ and $\vec{E}$ and another relating $\vec{B}$ to $\vec{\mathcal{E}}$) present inconsistencies. Fortunately, what happens here is that the 4-dimensional *ansatz*, eq. (26), can solve all these problems So, implementing it in the above equations, we get (after ordering the equations):

$$\nabla\wedge\vec{B}(\vec{r}) = +e\vec{J}(\vec{r}) - 2\mu_0\vec{\mathcal{E}}(\vec{r}) = 0\,,\quad \nabla\cdot\vec{E}(\vec{r}) = +e\rho(\vec{r}) - 2\mu_0\mathcal{B}(\vec{r}) = 0 \tag{36}$$

$$\nabla\wedge\vec{\mathcal{E}}(\vec{r}) = \left(\frac{e\mu_0}{e - \sigma\mu_0}\right)\vec{B}(\vec{r}) \quad , \quad \nabla\cdot\vec{\mathcal{E}}(\vec{r}) = 0, \tag{37}$$

$$\nabla\cdot\vec{B}(\vec{r}) = +g\delta^3(\vec{r}) \quad , \quad \nabla\wedge\vec{E}(\vec{r}) = 0, \tag{38}$$

$$\nabla\mathcal{B}(\vec{r}) = -\left(\frac{e\mu_0}{e - \sigma\mu_0}\right)\vec{E}(\vec{r}). \tag{39}$$

[8]A question must be asked: why the fermions are coupled to $\tilde{G}_\mu$ and not to the gauge field $H_{\mu\nu}$ (as was done for A_μ)? We answer this question by saying that this is the simplest form to write such "coupling" in a Lorentz covariant way and, at the same time, preserving the gauge invariance of the model. Nevertheless, it is clear that this vertex is non-renormalisable. Here, such aspect brings no major problems, since we are dealing with a non-relativistic Quantum Mechanical treatment. Actually, another "coupling" allowed in this way is: $\overline{\psi}\tilde{G}_\mu\gamma^\mu\gamma_5\psi$, what is clearly non-parity invariant; but here, we are not dealing with aspects of parity breaking, so we return to our former choosing.

It is clear that, hereafter we shall be considering regimes of the model for which $e \neq \sigma\mu_0$ is satisfied.

Now, placing a particle (with electrical $q = eq'$ and "tensorial" $Q = \sigma$ charges[9]; mass m) into the external magnetic field (those assumptions done before, in Section 2, must be taken also here), we get its non-relativistic Lagrangian:

$$L_2 = \frac{1}{2}m\dot{r}^2 + q\vec{A}\cdot\dot{\vec{r}} - \sigma\vec{\mathcal{E}}\cdot\dot{\vec{r}}.$$

And, the conserved angular momentum vector reads:

$$\vec{\mathcal{J}} = \vec{r}\wedge\vec{p} - \frac{g}{4\pi c}\left(q + \frac{e\sigma\mu_0}{e-\sigma\mu_0}\right)\hat{r}.$$

Now, the second term, that is related with the *'electromagnetic'* angular momentum, brings us information about the tensorial gauge sector, by defining an *'effective charge'* as: $\left(q + \frac{e\sigma\mu_0}{e-\sigma\mu_0}\right)$.

Now, in the context of Quantum Mechanics, we quantise the radial component of the conserved angular momentum operator:

$$\hat{\mathbf{r}}\cdot\mathcal{J} = \frac{n}{2} \quad\Longrightarrow\quad \frac{g}{4\pi\hbar c}\left(q + \frac{e\sigma\mu_0}{e-\sigma\mu_0}\right) = \frac{n}{2}\,, \tag{40}$$

(with n integer). Hence, we obtain what we have announced: that the mass parameter might be present in a quantisation condition. Two limits of this relation take importance:

$$\lim_{\sigma\to 0} \quad\longmapsto \frac{qg}{4\pi\hbar c} = \frac{n}{2} \qquad \text{and} \qquad \lim_{\mu_0\to 0} \quad\longmapsto \frac{qg}{4\pi\hbar c} = \frac{n}{2} \tag{41}$$

From (41), we see that, in the limit $\sigma \to 0$ we recover the result obtained in Section 2, which is expected. But, if we take $\mu_0 \to 0$ we recover the same result. This seems to state that the interaction between the fermions and the tensorial sector is performed by means of the topological term, that linkes both gauge symmetries.

It is noteworth to mention that the topological mass parameter does not get shifted by 1_loop corrections induced by loop of matter fields (scalars and/or spinors) minimally coupled to A_μ, but non-minimally coupled to $H_{\mu\nu}$. Indeed, by computing the self-energy diagram that exhibits A_μ to $H_{\nu\rho}$ on the external legs, it has been shown that the (finite) fermionic 1_loop contribution does not shift the mass parameter μ_0, so that the quantisation condition displayed in (40) does not suffer from (finite) renormalization effects on

[9] The "current equation" for the tensorial sector may be writen as:

$$\partial_\mu G^{\mu\nu\kappa} = j^{\nu\kappa} \quad \text{with} \qquad j^{\mu\nu} = \begin{cases} j^{0i} \equiv (\vec{j}_1)^i \\ j^{ij} \equiv \epsilon^{ijk}(\vec{j}_2)^k \end{cases}.$$

(it's clear that the conservation equation for $j_{\mu\nu}$ is $\partial_\mu j^{\mu\nu} = 0$). From this, we see that this sector carries no charge attribute. What happens is that all fermions carry the same *charge* with respect to the tensorial gauge group, $Q = \sigma$.

μ_0. Such a Feynman graph and its answer (for the case of scalar matter fields) are quoted in the Appendix.

Concluding Remarks

The main motivation of the present paper was the investigation of the possibility for the existence of Dirac's monopole solutions associated to the massive spin_1 model described by the mixing of a $U(1)$ gauge field to a rank-2 tensor gauge field according to CSKR. We have concluded that no such monopole emerges if matter is absent. Indeed, we have been able to work out possible conditions on the matter background so as to trigger monopole formation. We would however like to understand better the rôle of the matter background on the physics of the monopole. For example, quantisation of the latter in the presence of the background; or still, possible bounds on the monopole mass as dictaded by the background.

Our quantisation relation involving the topological mass parameter does not mean that the latter is quantised as it is the case for the topological mass parameter in Abelian [23] or non-Abelian [24] Chern-Simons theory in (2+1) dimensions . All we get here is a quantisation condition where all the parameters are mixed. If we assume electric (as well as magnetic) charge quantisation, all we get is the quantisation of the product $\sigma\mu_0$. However, this quantisation condition should be more deeply exploited.
Moreover, in the attempt of take some light to our motivating question, we have noticed that the non-coexistence of massive vector boson and Dirac's monopole might lie in the way Electrodynamics-type models are built up, i.e., in terms of 2-form field-strength (containing the classical physical fields); and not in the way of mass generation, as we had initially suspected.

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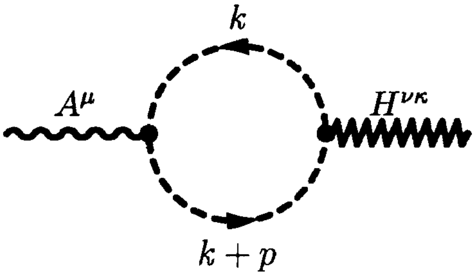
Appendix

The Feynman graph that exhibits A^μ and $H^{\nu\kappa}$ on the external legs with a loop of scalars[10] is depicted below:

[10] A similar graph with spinor loop (instead of scalar one) may lead us to a slightly different result, but no shift of the mass parameter will occur by finite 1_loop contributions.



The result of the above graph, after dimensional regularisation has been adopted, reads as follows:

$$
\begin{aligned}
I_\mu{}^{\beta\lambda}(p) &= -i\pi^2\epsilon^{\nu\alpha\beta\lambda}\,p_\alpha\left\{p_\mu p_\nu\int_0^1 dz\,(1-2z)^2\ln[p^2z(1-z)-m^2]+\right.\\
&\quad -\frac{1}{3}\left(\frac{2}{\delta}-k\right)p_\mu p_\nu-2\eta_{\mu\nu}\left[\left(\frac{2}{\delta}-k+1\right)\left(m^2-\frac{p^2}{6}\right)\right.\\
&\quad \left.\left.+\int_0^1 dz\,(p^2z(1-z)-m^2)\ln[p^2z(1-z)-m^2]\right]\right\},
\end{aligned}
$$

here, δ $(=4-D)$ is the dimensional regularisation parameter and $k\equiv\gamma+\ln\pi$ (γ is the Euler's constant); $m^2=2\mu_0^2$ is the mass parameter.
The finiteness of these integrals (written in terms of Feynman parametrisation) is evident.

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