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Dyadosphere bending of light

*V. A. De Lorenci**

Instituto de Ciências – Escola Federal de Engenharia de Itajubá
Av. BPS 1303 Pinheirinho, 37500-000 Itajubá, MG – Brazil

H. H. Fliche†

Universite Aix-Marseille III - Laboratoire de Modélisation en Mécanique et Thermodynamique
U.P.R.E.S. E.A. 2596 Case 322, Avenue Escadrille Normandie-Niemen
13397 Marseille Cedex 20 – France

M. Novello‡

Centro Brasileiro de Pesquisas Físicas
Rua Dr. Xavier Sigaud 150 Urca, 22290-180 Rio de Janeiro, RJ – Brazil

Abstract

In the context of the static and spherically symmetric solution of a charged collapsed star, we present the expression for the bending of light in the region just outside the event horizon where vacuum polarization effects are taken into account – the *dyadosphere.*

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*Electronic mail: `lorenci@cpd.efei.br`
†Electronic mail: `henri-hugues.fliche@meca.u-3mrs.fr`
‡Electronic mail: `novello@lafex.cbpf.br`

I. INTRODUCTION

Light velocity described by effective nonlinear electromagnetic theories has its magnitude depending on the field dynamics. Such dependence implies an effective modification of the flat background metric into a curved one, and it is accentuated when gravity process are taken into account. The most famous examples concerning these aspects are the well known implications of QED in curved spacetimes. The weak field limit of the complete one loop QED is known as the Euler-Heisenberg Lagrangian [1], and it yields several important results as, for instance, birefringence phenomenon [2], that means distinct velocity of light propagation for each polarization direction. There are many works dealing with the applications of nonlinear electrodynamics, specially in what concern its coupling with gravitational field. In this context we could mention the paper of I. T. Drummond and S. J. Hathrell [3] where they showed the possibility of superluminal velocities in certain spacetime configurations. Other interesting cases can be found in references [4–7].

Recently, R. Ruffini [8] and also G. Preparata, R. Ruffini and S. S. Xue [9] called our attention to a special region just outside black holes horizons where the electric field goes beyond its classical limit, implying a natural situation where effects of vacuum fluctuations should be considered. They denoted such region by *dyadosphere*. Assuming the existence of such region we could ask about the possible consequences for the trajectories of light rays that are crossing it. In this work we are interested in analyzing the consequences for bending of light phenomenon when vacuum fluctuation effects are taken into account.

In section II we present some aspects of the bending of light in the static and spherically symmetric solutions of general relativity theory. In section III we perform the coupling between nonlinear electrodynamics and gravity. We calculate the correction for Reissner-Nordstron metric from the first contribution of the weak field limit of one loop QED. In section IV we present the light cone condition for the case of the wave propagation in the Reissner-Nordstron spacetime modified by QED vacuum polarization effects - dyadosphere region. Finally, in section V, we derive the field equations for such situation and evaluate its contribution for the bending of light.

II. BENDING OF LIGHT IN SCHWARZSCHILD AND REISSNER-NORDSTRON SPACETIME

The purpose of this section is to present a short resume of the standard derivation of the bending of light in Schwarzschild and Reissner-Nordstron spacetimes. Einstein gravitational equation is given by

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

where κ is written in terms of the Newtonian constant G and light velocity c as $\kappa = 8\pi G/c^4$. The Schwarzschild solution represents a spherically symmetric and static solution of Einstein gravitational equation with exterior metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 \tag{2}$$

with $m = GM/c^2$. The motion of material test particles or light can be obtained from the geodesic equations

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \tag{3}$$

or else from the variational principle

$$\delta \int ds = 0. \tag{4}$$

For the above metric (2) we obtain the following equations of motion

$$\left(1 - \frac{2m}{r}\right) \dot{t} = const \doteq h_o \tag{5}$$

$$r^2 \dot{\varphi} = const \doteq l_o \tag{6}$$

$$\dot{r}^2 = h_o^2 - \frac{l_o^2}{r^2} \left(1 - \frac{2m}{r}\right) \tag{7}$$

where dots denote differentiation with respect to the parameter s, and we have adjusted initial conditions for the motion, that are:

$$\theta = \frac{\pi}{2}\,; \quad \dot{\theta} = 0. \tag{8}$$

If we consider the derivatives related to angular coordinate φ, and also performing the convenient change of variable $r = 1/v$, we obtain

$${v'}^2 - \frac{h_o^2}{l_o^2} + v^2 (1 - 2mv) = 0 \tag{9}$$

where primes denote derivatives with respect to φ. Taking the second derivative of the latter, it results[4]

$$v'' + v = 3mv^2. \tag{10}$$

This equation can be solved by means of perturbative techniques, resulting

$$v = \frac{1}{r_o} \sin\varphi + \frac{3m}{2r_o^2} \left(1 + \frac{1}{3} \cos 2\varphi\right) \tag{11}$$

where r_o is the minimal distance from the center of the star to the trajectory of light rays. Thus, going back to the original variable r and setting the asymptotic conditions $r \to \infty$, we obtain the amount of angular deflection on the trajectory of light rays, that is given by

$$\Delta\varphi = \frac{4m}{r_o}. \tag{12}$$

Indeed, the general formula for the bending of light can be directly obtained by integrating the equation (9), resulting

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[4] From now on, we will not consider the unstable circular solution $v' = 0$.

$$\Delta\varphi = -\pi + 2\int_{r_o}^{\infty} dr \frac{1}{\sqrt{\frac{h_o}{l_o} r^4 - r^2 + 2mr}}. \tag{13}$$

The method described above can be used for any kind of metric structure. In the Reissner-Nordstron geometry the only modification on the structure of light propagation will arise by the introduction of the charge Q. The geometry takes the form

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{\kappa Q^2}{2r^2}\right) dt^2 - \left(1 - \frac{2m}{r} + \frac{\kappa Q^2}{2r^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2. \tag{14}$$

Following the same steps as we did before we obtain

$$v'^2 = \frac{h_o^2}{l_o^2} - v^2\left(1 - 2mv + \frac{\kappa Q^2 v^2}{2}\right) \tag{15}$$

which yields

$$v'' + v = 3mv^2 - \kappa Q^2 v^3. \tag{16}$$

The charge term appears at the third order of approximation in the radial variable. One could expect that the contribution for the bending of light from this term should appear like Q^2/r_o^2 which is considerable smaller than m/r_o in usual situation. Therefore, in the case of a star with small radius, one should expect some appreciable contribution. In this vein we will look for the correction in this order from the coupling with nonlinear electromagnetism, which is valid, for instance, in the region just outside the horizon of black holes, where polarization effects must be taken into account.

III. GRAVITY TO NONLINEAR SPIN ONE THEORY COUPLING

Let us consider a class of spin-1 theories defined by the general Lagrangian $L = L(F)$, where $F = F^{\mu\nu}F_{\mu\nu}$. The corresponding energy momentum tensor has the form

$$T_{\mu\nu} = -Lg_{\mu\nu} + 4L_F F_{\mu\alpha}F_\nu{}^\alpha \tag{17}$$

where L_F denotes the derivative of L with respect to F. By considering minimal coupling of gravity to nonlinear spin-1 theory, we obtain the following equations of motion, besides Einstein equations (1),

$$(L_F F^{\mu\nu})_{||\nu} = 0 \tag{18}$$

$$\overset{*}{F}{}^{\mu\nu}{}_{||\nu} = 0 \tag{19}$$

where double bar ($||$) represents the covariant derivative with respect to the curved background $g_{\mu\nu}$, and $\overset{*}{F}_{\mu\nu}$ is the dual tensor. Let us take the static and spherically symmetric solution. The geometry presents the form

$$ds^2 = A(r)dt^2 - A(r)^{-1}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\varphi^2 \tag{20}$$

where $A(r)$ is determined by the field equations. We set the only non zero component of electromagnetic tensor to be $F^{01} = f(r)$. Thus, the combined system of electromagnetism and gravity, equations (1), (18) and (19), reduces to the set

$$r\frac{\partial A(r)}{\partial r} + 2A(r) = \kappa\left[r^2 L + 4r^2 L_F f(r)^2\right] \tag{21}$$

$$L_F f(r) = -\frac{Q}{4r^2}. \tag{22}$$

We are interested in the analysis of the weak field limit of the complete one-loop approximation of QED, given by the effective Lagrangian [1]

$$\mathcal{L} = -\frac{F}{4} + \frac{1}{8\pi^2}\int_0^\infty ds \frac{\mathrm{e}^{-m_e^2 s}}{s^3}\left[\frac{e^2 s^2 G}{4}\frac{\mathrm{Re}\cosh\sqrt{2e^2s^2(F+iG)}}{\mathrm{Im}\cosh\sqrt{2e^2s^2(F+iG)}} - \frac{2e^2s^2F}{12} - 1\right] \tag{23}$$

where $G \doteq \overset{*}{F}_{\mu\nu}F^{\mu\nu}$. In the limit of low frequency $\nu << m_e c^2/h$ one obtains the Euler-Heisenberg Lagrangian [1]

$$L = -\frac{1}{4}F + \frac{\mu}{4}\left(F^2 + \frac{7}{4}G^2\right) \tag{24}$$

with

$$\mu \doteq \frac{2\alpha^2}{45m_e^4}. \tag{25}$$

Since we are only considering the electric component of $F_{\mu\nu}$ there is not any contribution due to the invariant G. Using Lagrangian (24), the integration of equation (21) results

$$A(r) = 1 - \frac{2m}{r} - \frac{\kappa}{r}\int^r dr\left[\frac{r^2 f(r)^2}{2} + 3\mu r^2 f(r)^4\right] \tag{26}$$

where, in order to set the value of the first constant of integration, we have assumed the Schwarzschild solution in the limit of vanishing charge. Calculating the function $f(r)$ from equation (22) in the appropriate order of approximation $\mathcal{O}(\mu)$, one gets

$$f(r) = \frac{Q}{r^2} - 4\mu\frac{Q^3}{r^6}. \tag{27}$$

Introducing this result in equation (26) we finally obtain the expression for $A(r)$:

$$A(r) = 1 - \frac{2m}{r} + \frac{\kappa Q^2}{2r^2} - \frac{\kappa\mu Q^4}{5r^6}. \tag{28}$$

The Reissner-Nordstron case arises from this solution for the limit case $\mu = 0$. The equations (20) and (28) yields the correct form of the spacetime geometry taking into account the one-loop approximation.

IV. LIGHT CONE CONDITION

In the case of nonlinear electrodynamics, e.g., Euler-Heisenberg effective theory, the wave propagation will suffer a correction due to vacuum polarization effects. Such correction is usually presented in terms of a light cone condition [10], which in our case, is given by

$$k^\alpha k^\beta g_{\alpha\beta} = -4\frac{L_{FF}}{L_F} F^{\mu\alpha} F^\nu{}_\alpha k_\mu k_\nu. \tag{29}$$

It is worth mentioning that there will be two different modes of propagation, one for each polarization direction. Therefore the only one that contributes to the deviation of the classical bending of light is the mode present in equation (29). The other one propagates through the background light cone defined as

$$k^\mu k^\nu g_{\mu\nu} = 0. \tag{30}$$

The condition (29) can be presented in a more appealing form as a slight modification of the background geometry

$$\left(g^{\mu\nu} + 4\frac{L_{FF}}{L_F} F^{\mu\alpha} F^\nu{}_\alpha\right) k_\mu k_\nu = 0. \tag{31}$$

This property allows us to introduce the concept of an effective geometry such that

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} + 4\frac{L_{FF}}{L_F} F^{\mu\alpha} F^\nu{}_\alpha \tag{32}$$

for which k_μ is a null vector. Hence, we can use the previous method to derive the bending of light in the presence of vacuum polarization effects due to nonlinear electrodynamics. Using such definition, k_μ is nothing but a null vector in the effective geometry.

The geometrical relevance of equation (32) goes beyond its immediate definition. Indeed, the integral curves of the vector k_ν are geodesics. In order to accomplish it, an underlying Riemannian structure for the manifold associated with the effective geometry will be required. In other words, this implies a set of Levi-Civita connection coefficients $\tilde{\Gamma}^\alpha{}_{\mu\nu} = \tilde{\Gamma}^\alpha{}_{\nu\mu}$, by means of which there exists a covariant differential operator (the *covariant derivative*), which we denote by semi-comma, such that

$$\tilde{g}^{\mu\nu}{}_{;\,\lambda} \equiv \tilde{g}^{\mu\nu}{}_{,\,\lambda} + \tilde{\Gamma}^\mu{}_{\sigma\lambda}\tilde{g}^{\sigma\nu} + \tilde{\Gamma}^\nu{}_{\sigma\lambda}\tilde{g}^{\sigma\mu} = 0. \tag{33}$$

From (33) it follows that the effective connection coefficients are completely determined from the effective geometry by the usual Christoffel formula.

Contraction of equation (33) with $k_\mu k_\nu$ results

$$k_\mu k_\nu \tilde{g}^{\mu\nu}{}_{,\,\lambda} = -2k_\mu k_\nu \tilde{\Gamma}^\mu{}_{\sigma\lambda}\tilde{g}^{\sigma\nu}. \tag{34}$$

Differentiating $\tilde{g}^{\mu\nu} k_\mu k_\nu = 0$, we have

$$2k_{\mu,\,\lambda} k_\nu \tilde{g}^{\mu\nu} + k_\mu k_\nu \tilde{g}^{\mu\nu}{}_{,\,\lambda} = 0. \tag{35}$$

From these expressions we obtain

$$\tilde{g}^{\mu\nu}\left(k_{\mu,\,\lambda} - \tilde{\Gamma}^\sigma{}_{\mu\lambda} k_\sigma\right) k_\nu = 0 \tag{36}$$

or yet

$$\tilde{g}^{\mu\nu} k_{\mu;\,\lambda} k_\nu = 0. \tag{37}$$

As the propagation vector $k_\mu = \Sigma_{,\,\mu}$ is a gradient, one can write $k_{\mu;\,\lambda} = k_{\lambda;\,\mu}$. Thus, equation (37) reads

$$\tilde{g}^{\mu\nu} k_{\lambda;\,\mu} k_\nu = 0 \tag{38}$$

which states that k_μ is a geodesic vector. By remembering it is also a null vector (with respect to the effective geometry $\tilde{g}^{\mu\nu}$), it follows that its integral curves are therefore null geodesics.

V. THE INFLUENCE OF QED ON THE TRAJECTORY OF LIGHT

Taking the Lagrangian (24), and setting the only non-zero component of electromagnetic tensor to be $F^{01} = f(r)$, it follows

$$F = -2f(r)^2 \tag{39}$$

$$L_F = -\frac{1}{4} - \mu f(r)^2 \tag{40}$$

$$L_{FF} = \frac{\mu}{2}. \tag{41}$$

Since we are analyzing the spherically symmetric and static solution of a charged star, we set the components of the background metric to be the same as the ones presented in equation (20). Thus, the non-vanishing components of the effective metric $\tilde{g}^{\mu\nu}$, up to terms quadratic on the constant μ, are

$$\tilde{g}^{00} = A(r)^{-1}[1 + 8\mu f(r)^2] \tag{42}$$

$$\tilde{g}^{11} = -A(r)[1 + 8\mu f(r)^2] \tag{43}$$

$$\tilde{g}^{22} = g^{22} \tag{44}$$

$$\tilde{g}^{33} = g^{33} \tag{45}$$

where $A(r)$ is given by (28). In what follows we will consider only the stronger term arising from quantum corrections. All other terms will be neglected. The function $f(r)$ is calculated from the equations of the electromagnetic field, and its value is set by equation (27). Using these results we obtain the line element:

$$d\tilde{s}^2 = \left[1 - 8\mu f(r)^2\right]\left[A(r)dt^2 - A(r)^{-1}dr^2\right] - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2. \tag{46}$$

From the variational principle

$$\delta \int d\tilde{s} = 0 \tag{47}$$

the following equations of motion are obtained:

$$\left[1 - 8\mu f(r)^2\right] A(r)\dot{t} = const \doteq h_o \tag{48}$$

$$r^2\dot{\varphi} = const \doteq l_o \tag{49}$$

$$\dot{r}^2 = \frac{h_o^2}{\left[1 - 8\mu f(r)^2\right]^2} - \frac{l_o^2 A(r)}{r^2\left[1 - 8\mu f(r)^2\right]}. \tag{50}$$

As before, dots mean derivatives with respect to parameter s, and we have adjusted the initial conditions $\theta = \pi/2$ and $\dot{\theta} = 0$. Performing the change of variable $r = 1/v$ and expressing the derivatives with respect do the angular variable φ we obtain from (48) to (50):

$$v'^2 = \frac{h_o^2}{l_o^2}\frac{1}{[1-8\mu f(r)^2]^2} - \frac{A(r)v^2}{1-8\mu f(r)^2} \tag{51}$$

where $v' = dv/d\varphi$. In the required order of approximation, functions $A(v)$ and $f(v)$ are given by

$$f(v) = Qv^2 \tag{52}$$

$$A(v) = 1 - 2mv + \frac{\kappa Q^2 v^2}{2} - \frac{\kappa\mu Q^4 v^6}{5}. \tag{53}$$

Thus, taking the derivative of equation (51) and using the above results, follows

$$v'' + v = 3mv^2 - \left(1 - 32\frac{\mu h_o^2}{\kappa l_o^2}Q\right)\kappa Q^2 v^3 + \mathcal{O}(\mu^2, v^4). \tag{54}$$

This shows that the contribution coming from QED appears in the same order of magnitude, in the radial variable, as the classical Reissner-Nordstron charge term.

VI. CONCLUSION

In this work we presented the contribution from weak field limit of the complete one-loop QED for the bending of light in the region just outside an event horizon of a black hole. We showed that this contribution appears in a significative order in terms of radial variable. Indeed, it is the same order of the charged term that arises from Reissner-Nordstron geometry. Equation (54) shows that the correction term depends on the ratio h_0/l_0, besides the charge Q. Thus, for photons with the same frequency, the effect will be greater for those whose trajectory is closer to the center of attraction.

An interesting feature of this result consists in the fact that such deviation of the classical bending of light is applicable only to one mode of light wave propagation, associated with a specific polarization direction. Thus, there will be a polarization of the light waves crossing the dyadosphere region.

A possible continuation of the work consists of the numerical calculation of the correction found here and its application to the observational astrophysics.

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