Back-Reaction of Einstein's Gravitational-Waves as the Origin of Natal Pulsar Kicks

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At the early core-bounce of a supernova collapse rapid convective overturn along with gradients in density and temperature in the neutrino-decoupling zone drives anisotropic neutrino flux. If then active-to-sterile ($\nu_{\bar{\tau},\bar{\mu}} \leftrightarrow \nu_s$) neutrino oscillations in the dense core take place, gravitational radiation should be emitted all the way the oscillation length. Since the oscillation feeds mass-energy up into (or drains it from) the new species, the large neutrino mass-squared difference ($10^4\mathrm{eV}^2 \lesssim \Delta m^2 \lesssim 10^8\mathrm{eV}^2$) implies a huge amount of energy is released as gravity waves then in either neutrino convection and cooling or perturbed matter distributions. I identify the *b*ack-reaction force (mass and current multipoles) of the *g*ravitational wave burst generated over the oscillation timescale as the pulsar thruster.

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ASTROPHYSICAL MOTIVATION

Most stars in our galaxy drift with $\sim$ 30 kms^{-1} [1], while observed pulsars move with mean spatial (3-D) velocities of $\sim$ 450 kms^{-1}[2, 3]. Such large velocities should be imparted to the proto-neutron star (PNS) at its birth[4, 5]. The mechanism responsible for pulsar kicks has not been properly identified yet, despite of interesting proposals have been advanced[4, 5], and tight constraints on potential mechanisms put forward[6]. This *L*etter suggests that the *r*adiation-reaction force (RRF) applied by the *g*ravitational wave burst generated at the supernova (SN) early postbounce by neutrino oscillations: the spin-flavor conversion discussed here[7], or via either parametric resonance or MSW effect; generalized elsewhere[8], is the most likely mechanism triggering pulsars' kicks (the escaping sterile νs produce a gravitational wave (GW) with memory, as well).

A GW exerts a back reaction on a given source by virtue of carring away energy, angular and linear momentum from it. Whatever energy and momentum the wave takes away will be reflected in this back reaction. The RRF of the GWs from ν oscillations may apply several (pair-folded) thrusts to the nascent neutron(N^0) star, coupled to them via the *w*eak interaction, so as to make it move and rotate at birth. Since no other known mechanism can do as much during such a short timescale ($\sim$ 1ms, i.e., early bounce), the claimed effect could be the most fundamental mechanism driving pulsar spins and drift velocities as evidenced in pulsar surveys[2]. The electroweak coupling of N^0 and ν inside the PNS (the ultimate responsible for pulsar kick in this picture) is described by the Lagrangean

$$L^{int}_{N^0 \leftrightarrow \nu} = [\frac{G_F}{\sqrt{2}}](\bar{N}^0\gamma_\mu(1-\gamma_5)N^0)[\bar{\nu}\gamma^\mu(1-\gamma_5)\nu], \quad (1)$$

with the ν field satisfying the time-dependent Dirac equation

$$\left(i\gamma^0\partial_0 + i\gamma^\alpha\partial_\alpha + \rho(t)v_\beta\gamma^\beta[\frac{(1-\gamma_5)}{2}] - m_\nu\right)\Psi = 0. \quad (2)$$

Above G_F is the Fermi constant, and v_β the ν 4-velocity. In dense matter (at rest) the neutron 4-vector current density $J^\mu = <\bar{N}^0\gamma_\mu(1-\gamma_5)N^0> \equiv (\rho, 0, 0, 0)$ adquires a non-zero expectation value. Here $\rho = \frac{G_F}{\sqrt{2}}N^0_n$, with N^0_n the neutron number density.

ENLARGED ν AND GWS LUMINOSITY FROM ν-OSCILLATIONS

It is well-known that neutrino outflow from a PNS after the SN core bounce is a source of GWs[7, 9–13]. Numerical simulations by Müller and Janka[12] have shown that in general the fraction of the total binding energy emitted as GWs by pure neutrino convection is: $E^\nu_{GW} \sim [10^{-10} - 10^{-13}]\mathrm{M}_\odot\mathrm{c}^2$, for a ν luminosity: $L_\nu \sim 10^{53}$erg s^{-1} (see Ref.[12] for further details). This GWs energy has been shown not to be enough so as to kick the PNS.

Unlike GWs produced by convection of neutrinos [12], in the production of GWs by neutrino oscillations[7] from active-to-sterile there exists two main reasons for expecting a major enhancement in the GWs luminosity (which determines the strength of the GWs RRF) during the transient: a) the conversion itself, which makes it the overall luminosity of a given neutrino species to grow by a large factor. The enhancement stems from the mass-energy ($L_\nu \lesssim 10\%$, see below) being given to (or taken from) the new species into which oscillations take place.

This augment gets reflected in the species mass-squared difference (energy conservation), and their relative numbers: one species is number-depleted while the other gets its number enlarged. But, even if the energy increase is smaller, b) the abrupt conversion over the transition time, which is set up by the oscillation length λ_{osc} and the neutrino velocity of diffusion $\bar{V}_\nu$ ($\sim 10^9$ cms^{-1} for convectives[12]), also magnifies transiently L_ν. Being the neutrino flavor transition the *k*ey piece to produce GWs by this mechanism, then we need to estimate how many of them can actually oscillate. This quantity is measured by the transition probability: $P_{a\to s}(|\vec{x}-\vec{x}_0|)$, we calculate next (for a thorough accounting see Ref.[7]).

To be efficient in producing GWs, neutrinos must be able to escape the core without thermalizing with the stellar material. For active neutrino species of energies ≈ 10MeV, this is not possible as long as the matter density is $\gtrsim 10^{10}$gcm^{-3}. Since the production rate of neutrinos is a steeply increasing function of matter density (production rate $\propto \rho^n$, where ρ is the matter density and $n > 1$), the overwhelming majority of the νs of all species produced are trapped. So the contribution to the GWs amplitude is negligible, irrespective of the ν conversions taking place within the active nu flavors.

Sterile neutrinos, on the other hand, would be able to escape the core. Though they are not directly produced inside the star, if any active ν species can be copiously converted into sterile νs through oscillations, it may be possible to dramatically increase the number of escaping νs, this way producing the GWs burst. This effect can be significant only if these active $\leftrightarrow$ sterile transitions take place inside the neutrinospheres of the active νs, i.e. at $\rho \gtrsim 10^{10}$gcm^{-3}.

Vacuum ν oscillations

Let us first consider the case of vacuum oscillations, assuming for simplicity that the oscillations take place between only two neutrino species (ν_a and ν_s, say). For a ν_a state produced at $\vec{x}_0$ which does not undergo any collisions before reaching $\vec{x}$, the probability that it will be observed at $\vec{x}$ as a ν_s is

$$P_{as}(|\vec{x}-\vec{x}_0|) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E_\nu}|\vec{x}-\vec{x}_0|\right), \qquad (3)$$

where θ is the $\nu_a \leftrightarrow \nu_s$ vacuum mixing angle, Δm^2 is the mass-squared difference between the two neutrino mass eigenstates in vacuum, and E_ν is the neutrino energy. In order for the estimates, let us consider only the neutrinos travelling radially outwards, so that the problem is reduced to a one dimensional problem, with x and x_0 now representing the distance from the centre of the star. The probability that this neutrino would interact within a distance dx is given by

$$dP(x, x_0) = (1 - P_{as}(x - x_0))\mathrm{d}x/\lambda(x), \qquad (4)$$

where $\lambda(x)$ is the mean free path of ν_a, given by $\lambda(x) \equiv [N(x)\sigma(x)]^{-1}$. Here $N(x)$ is the number density of the relevant scatterers at x while $\sigma(x)$ is the cross section for the neutrino scattering at x. From Eqs.(3,4), the probability of a ν_a reaching x from x_0 without interacting is (Mohapatra & Pal 1998)

$$P_{surv}(x, x_0) = \exp\left[-\int_{x_0}^{x}\frac{\mathrm{d}x}{\lambda(x)} + \sin^2 2\theta\int_{x_0}^{x}\sin^2\left(\frac{\Delta m^2}{4E_\nu}(x-x_0)\right)\frac{\mathrm{d}x}{\lambda(x)}\right]. \qquad (5)$$

The first integral in the exponential is the answer one gets in the absence of oscillations. It is this term that determines the size of the neutrinosphere for the active neutrino. The second term represents the enhancement of the survival probability due to oscillations.

The effect of the second term is negligible if the vacuum mixing angle is small ($\sin^2 2\theta \approx 0$) or when the oscillation length is large compared to the radius of the neutrinosphere ($4E_\nu/\Delta m^2 \gg R_\nu$). The latter condition is satisfied for $E_\nu \approx 10$MeV only for $\Delta m^2 \lesssim 10^{-3}$eV^2. For Δm^2 larger than this value, the effect of the second term is an effective reduction in the radius of the neutrinosphere by a factor ζ, which can be estimated by taking $N(x) \propto x^{-\alpha} \longrightarrow \zeta \approx C^{\frac{1}{\alpha-1}}$, where

$$C \equiv 1 - \sin^2 2\theta\int_0^\infty \sin^2\left(\frac{\Delta m^2}{4E_\nu}x\right)\frac{\mathrm{d}x}{\lambda(x+R_\nu)} \quad . \qquad (6)$$

The enhancement is maximum when C is smallest, which happens with maximal vacuum mixing angle ($\sin^2 2\theta = 1$), and with the maximum value of the integral in Eq.(6).

Pure vacuum oscillations would take place inside the star only when $\Delta m^2/(2E) \gg \sqrt{2}G_F\rho/m_N$, where G_F is the Fermi constant and m_N is the nucleon mass. Since $\rho \gtrsim 10^{10}$ gcm^{-3} inside the neutrinosphere, this condition is satisfied for $E \sim 10$MeV neutrinos only for $\Delta m^2 \gg 10^4$eV^2. In this parameter range, the integrand is rapidly oscillating and the integral reduces to 0.5. Even in the most favorable scenario, thus, $C > 0.5$. Then, with $\alpha \approx 3$, we get $\zeta > \sqrt{2}$. Hence, the radius of the effective neutrinosphere cannot change by a large factor, which indicates that the oscillations into sterile neutrinos cannot increase the number of escaping neutrinos dramatically in this parameter range (where matter effects can be neglected).

ν oscillations in dense matter

Interaction with matter, as in Eq.(1), may help in allowing more νs to escape if resonant conversions into sterile νs occur inside the neutrinosphere of the active νs (see Ref.[7] for further discussions). In the case of $\nu_e \leftrightarrow \nu_s$ oscillations, the resonance occurs if

$$\sqrt{2}G_F\left[N_e(x) - \frac{1}{2}N_n(x)\right] \equiv V(x) = \frac{\Delta m^2}{2E_\nu}\cos 2\theta. \quad (7)$$

Here $N_e(x)$ is the electron number density (given by $N_{e^-} - N_{e^+}$), and $N_n(x)$ is the neutron number density. In the case of $\nu_{\mu,\tau} \leftrightarrow \nu_s$ the N_e term is absent, while in the case of antineutrinos, the potential changes by an overall sign. Numerically, for $\nu_e \leftrightarrow \nu_s$ oscillations,

$$V(x) = 7.5\times 10^2(\frac{\mathrm{eV}^2}{\mathrm{MeV}})(\frac{\rho_m(x)}{10^{10}\mathrm{g/cm}^3})\times(\frac{3Y_e}{2} - \frac{1}{2}), \quad (8)$$

where Y_e is the electron number fraction. For $\nu_{\mu,\tau} \leftrightarrow \nu_s$ oscillations, the last term in parenthesis is changed to $(\frac{Y_e}{2} - \frac{1}{2})$, assumed henceforth to be of order one.

Neutrino conversions in the resonance region can be strong if the adiabaticity condition is fulfilled: the oscillation probability is $P_{as} = \cos^2\theta$, which is close to 1 in the case of small mixing angles. Moreover, after the resonance region, the newly created sterile νs have very a small probability ($P_{sa}^{\mathrm{average}} = \frac{1}{2}\sin^2 2\theta$) of oscillating back to active νs, which could be potentially trapped. For the resonance condition to be satisfied, we require

$$10^4\mathrm{eV}^2 \lesssim \Delta m^2\cos 2\theta(\frac{10\mathrm{MeV}}{E_\nu}) \lesssim 10^8\mathrm{eV}^2, \quad (9)$$

while the adiabaticity condition is satisfied for

$$\frac{\Delta m^2\sin^2 2\theta}{2E_\nu\cos 2\theta}(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x})^{-1}_{x=x_{\mathrm{res}}} \gg 1, \quad (10)$$

where x_{res} is the position of the resonance layer. Inside the core,

$$(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x})^{-1}_{x=x_{\mathrm{res}}} \equiv \lambda_{osc} \sim 1\mathrm{km}, \quad (11)$$

and therefore the adiabaticity condition is satisfied if

$$\Delta m^2\frac{\sin^2 2\theta}{\cos 2\theta} \gg 10^{-3}\mathrm{eV}^2(\frac{E_\nu}{10\mathrm{MeV}}), \quad (12)$$

which is easily satisfied by $\Delta m^2 \gtrsim 10^4\mathrm{eV}^2$ as long as $\sin^2\theta \gg 10^{-7}$. Thus, we find that a substantial fraction of νs may get converted to sterile νs, and escape the core of the star, if the sterile νs mass is such that $10^4\mathrm{eV}^2 \lesssim \delta m^2_{as} \lesssim 10^8\mathrm{eV}^2$. Such a mass difference cannot solve the observed solar and atmospheric neutrino problem, but the possibility of three active neutrinos explaining these anomalies and a heavy sterile neutrino of mass $m^s_\nu \sim$ keV still stays open. Moreover, the number of νs escaping, and their angular distribution, is sensitive to the instantaneous distribution of production sites. It was argued that these inhomogeneities can give rise to quadrupole moments that generates GWs[7, 12]. Ref.[7] also showed that the $f$raction of sterile νs that can actually escape in the first few milliseconds is: $P_{a\to s}(|\vec{x} - \vec{x}_0|) \lesssim 10\%$ of the total ν-flux. Next we focus on the way the PNS couples (and responds) to the back-action of the GWs produced by these ν oscillations.

ν COUPLING TO NEUTRON STAR MATTER

Neutron-ν coupling

In proviso, we propose a phenomenological approach to get some insight into the fashion this back-reaction effect proceeds (a self-consistent description may demand a kinetic theory formalism for νs[8, 14] in full general relativity), a task that we address elsewhere[8]. However, the PNS configuration before the oscillation to take place can be figured out as a general relativistic gas sphere of degenerate neutrons (plus some protons and electrons) and neutrinos in hydrostatic and thermal β-equilibrium: $N^0 + \bar{\nu} \leftrightarrow p + e^-$, to which Eq.(1) applies. The PNS energy-momentum tensor should contain contribution from both N^0s and massive active νs. What the collisionless massive-ν gas adds to the total pressure and density in the PNS can be computed if νs are described in the momentum space as having a distribution function[15]

$$dn(r) = \frac{g}{h^3}\left[\frac{1-e^{[\epsilon-\epsilon_c(r)]/kT}}{e^{[\epsilon-\mu(r)]/kT}+1}\right] d^3p(\epsilon), \quad \text{for} \quad \epsilon \leq \epsilon_c(r), \quad \text{and} \quad dn(r) = 0, \quad \text{for} \quad \epsilon > \epsilon_c(r). \tag{13}$$

Here $\epsilon_c(r) \sim 25$MeV is the ν cut-off energy in the SN-core, $g = 2s_\nu + 1$ the spin multiplicity of quantum states, $\mu(r)$ the chemical potential and $kT = [50-100]$MeV, the fermionic thermodynamic temperature. Since when trapped in the core neutrinos are relativistic particles, namely $\bar{V}_\nu \sim c$, then Eq.(13), the Einstein total energy relation for νs: $P^\beta P_\beta = m^2_{(0)\nu} c^4 u_\alpha u^\alpha$, and the thermodynamics definition of pressure (in terms of Juttner-like transformations: $\sinh\theta \equiv \frac{p}{mc}$[16]) leads to write (the same relations holds for the N^0 gas in the PNS, see details elsewhere[8])

$$\rho(r) = K(\sinh(t) - t), \qquad P(r) = \frac{K}{3}\left(\sinh(t) - 8\sinh(\frac{t}{2}) - 3t\right), \tag{14}$$

where $K = m_\nu^4 c^5/(32\pi^2 h^3)$, and the parameter t is related to the maximum momentum value, $p_0 = p_0(r)$, in the Fermi distribution at a distance r from the star's center as: $t = ln\left[\frac{p_0}{m_\nu c} + \sqrt{1+(\frac{p_0}{m_\nu c})^2}\right]$, and the ν number density reads: $n_\nu = \frac{m_\nu^3 c^3}{3\pi^2 h^3}\sinh^3(t/4)$.

By using Einstein's equations in a spherically symmetric space-time, with a total energy-momentum tensor for relativistic degenerate collisionless N^0+ν gas with almost the same 4-velocity ($u^\nu_\mu \sim u^{N^0}_\mu$): $T_{\mu\nu} = T^\nu_{\mu\nu} + T^{N^0}_{\mu\nu}$, we get an Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = -\left(\frac{\rho^{all} + P^{all}}{c^2}\right)\frac{\left[\frac{GM(r)}{r^2} + \frac{4\pi G}{c^2} r P^{all}\right]}{(1-\frac{2GM(r)}{c^2 r})}, \tag{15}$$

where $\rho^{all} = \rho^\nu + \rho^{N^0}$, $P^{all} = P^\nu + P^{N^0}$, and $M(r)$ is defined as: $\frac{dM(r)}{dr} = 4\pi r^2([\rho^\nu(r) + \rho^{N^0}(r)])$.

¿From the precedent discussion it becomes clear that once the oscillation occurs almost all the sterile νs being created escape the star taking a huge amount of energy (and momentum) density ($\lesssim 10\%\rho^\nu$) away with them. The coupling energy to matter of the former active νs that convert into steriles is lost in the transition. This breaks the PNS equilibrium, makes it pulsate, rotate and move due to a rocket-like thrust, whose force ($\bar{F}^{back}_\nu$) stems from $L_\nu \equiv \bar{F}^{back}_\nu \times \bar{V}_\nu$, and applies back to the remaining active-ν fluid to finally push the star away through the $\nu - N^0$ matter coupling as expressed by Eq.(1).

GWs energetics from ν luminosity

The ν oscillation timescale is defined as: $\Delta t_{osc} \equiv \lambda_{osc}/\bar{V}_\nu$. It leads to the neutrino luminosity:

$$L_\nu \equiv \frac{\Delta E_{\nu_a \longrightarrow \nu_s}}{\Delta t_{osc}} \sim \frac{3\times 10^{52}\text{erg}}{1\times 10^{-4}\text{s}} = 3\times 10^{56}\text{ergs}^{-1}\ . \tag{16}$$

Note that this L_ν agrees quite well with the one numerically computed by Pons et al. (2001)[17]. Hence, the GWs luminosity, L_{GW}, as a function of the ν luminosity can be obtained by relating the GWs flux: $\frac{c^3}{16\pi G}|\dot{h}|^2 = \frac{1}{4\pi R^2} L_{GW}$, to the GWs amplitude given as[10, 12]:

$$h^{TT}_{ij} = \frac{2G}{c^4 R}\int_\infty^{t-R/c} dt' L_\nu(t')\alpha(t') e_i \otimes e_j, \longleftrightarrow h = \frac{2G}{c^4 R}\left[\Delta t L_\nu \alpha\right], \tag{17}$$

where $e_i \otimes e_j$ is the GW polarization tensor. Be aware that because the ν-flux in not a slowly-moving GWs source this formula is not subjected to any multipole decomposition of the GW field, and as such it was used in Ref.[7] to estimate the GWs amplitude h. However, to describe the motion of the PNS fluid, below, the post-Newtonian expansion is called for. This yields:

$$L_{GW} = 3.0 \times 10^{52} \left[\frac{L_\nu}{3 \times 10^{56} \frac{\rm erg}{s}}\right]^2 \left(\frac{\alpha}{4 \times 10^{-1}}\right)^2 \frac{\rm erg}{\rm s}, \tag{18}$$

where $0.2 < \alpha < |-0.8|$ is the anisotropy parameter[26] as defined by Burrows and Hayes[10]. It turns out that the GWs energy radiated in the process: $E_{GW} \equiv L_{GW} \times \Delta t_{osc}$, is about two orders of magnitude greater than the one sterile neutrinos carry away:

$$E_\nu = 10^{57}|_{\nu_s} \times 10^4 {\rm eV}|_{\nu_s} [\frac{10^{-33} {\rm gc}^2}{\rm eV}] \sim 2 \times 10^{-6} {\rm M}_\odot {\rm c}^2. \tag{19}$$

The quoted GWs energy is $\sim 10^5$ larger than both the current estimates from the fluid motion of the PNS constituents[12] and the one inferred from the proper motion of pulsars: $E^{Pulsar}_{surveys}(GW) \sim 4 \times 10^{-6} {\rm M}_\odot c^2$[19]. The latter estimate follows from inquiring what would the recoils be if the kick's energy source were merely GWs. With so much an energy it is possible to kick the PNS up to the observed velocities[2]. Moreover, surveys suggest that runaway pulsars are canonical NS: M$\sim 1.4{\rm M}_\odot$, B$\sim 10^{12}$G, then it becomes clear in this picture that the linear momentum of the escaping sterile ν-flux is *n*ot to compensate for the pulsar kick, but rather the back-reaction of GWs from their oscillations. This is so because for νs to do the job an extremely high B$\sim 10^{15}$G should be endowed by the PNS, as in the Kusenko and Segrè ν-driven pulsar kick mechanism[20], which is not the case if one stands on the pulsar surveys.

GWS BACK-REACTION ON NASCENT PULSARS

Because the PNS is a slowly moving source compared to the relativistic νs, next we use a post-Newtonian (PN) expansion to compute the GWs back-reaction effect on it due to the coupling Eq.(1).

Beamed GWs from ν oscillations

If the back-reaction of GWs released is to kick the nascent pulsar, then GWs must be beamed up to some degree. It is well-known that the PNS magnetic field distorts the neutrinosphere and aligns the escaping neutrinos along its dipole axis: the stronger the dipole field the stronger the neutrino alignment, and thus the residual beaming effect on the GWs emitted, and consequently on its back-reaction force. With the occurrence of some beaming in the GWs emission we can also expect that not only the matter mass-quadrupole tensor but higher multipoles, in particular the matter current-quadrupole moment, to play an important role in kicking the star because of its association with axial perturbations.

7/2-PN expansion of GWs back-reaction field

The GWs *m*ultipolar radiation field needed to describe the back-action onto the PNS neutron fluid due to the coupling Eq.(1) reads [21–23]

$$h^{TT}_{ij}(\mathbf{X}, t) = \frac{2G}{c^4 R} P_{ijkl}(\mathbf{N}) \Bigg[I^{(2)}_{kl} + \frac{1}{3c} N_a I^{(3)}_{akl} + \frac{1}{12c^2} N_a N_b \times I^{(4)}_{abkl} + \frac{4}{3c}(\epsilon_{ab(k} J_{l)}{}^{(2)}_{a} N_b - \epsilon_{ab(k} N_{l)} N_c J^{(2)}_{ca} N_b) + \frac{1}{2c^2} \epsilon_{ab(k} J_{l)}{}^{(3)}_{ac} N_b N_c + O\left(\frac{1}{c^3}\right)\Bigg] (t - \frac{R}{c}), \tag{20}$$

plus terms of $O\left(\frac{1}{R^2}\right)$, here dropped off. The $I^{(n)}$-terms correspond to the mass-quadrupole, mass octupole and mass-2^4-pole moments, respectively, while the $J^{(n)}$-terms represent the current-quadrupole and octupole moments, respectively. Note that $I^{(n)}(t) \equiv d^n I/dt^n$, $R = |\mathbf{X}|$ is the source distance, and $X_{(kl)} = \frac{1}{2}(X_{(kl)} + X_{(lk)})$. $P_{ijkl} \equiv (\delta_{ik} - N_i N_k)(\delta_{jl} - N_j N_l) - \frac{1}{2}(\delta_{ij} - N_i N_j)(\delta_{kl} N_k N_l)$ describes the traceless-transverse (TT) projection operator onto the plane orthogonal to the outgoing wave direction $\mathbf{N}$. Note that we have incorporated the result given in Eq.(61) of Ref.[23] into the original Eq.(6.8) of Ref.[22]. Thus the pure quadrupole mass and current terms in Eq.(20) reduce to:

$$h^{TT}_{ij}(\mathbf{X}, t) = \frac{2G}{c^4 R} P_{ijkl}(\mathbf{N}) \left[\frac{\partial^2}{\partial t^2}[Q_{kl}](t - R/c) + \frac{4}{3c}(\epsilon_{ab(k} J^{(2)}_{l)a} N_b - \epsilon_{ab(k} N_{l)} N_c\, J^{(2)}_{ca} N_b)\right], \tag{21}$$

being $Q_{kl}(t) = \int d^3x \rho(\mathbf{x}, t) \left[x_i x_j - 1/3 \delta_{ij} \mathbf{x}^2\right]$ the trace-free part of the *m*ass-quadrupole tensor, and

$J_{ij}(t) \equiv \int d^3\rho(\mathbf{x},t)\epsilon_{kl(i}J_{j)l}x_kV_l$ the current-quadrupole tensor of the PNS matter distribution, and V_l the ν-fluid 3-velocity vector.

NS matter mass-current contribution to RRF.- The wave solution, Eq.(20), effectively include radiation-reaction effects in the hydrodynamical description of a source driven by pure GWs emission. The spacetime metric $g_{\mu\nu}$ was expanded to post-Newtonian orders higher than $n=5$ in the parameter $(v/c)^{1/2}$, where v is the fluid internal velocity. Blanchet achieved order $(v/c)^{7/2}$, which exhibits the role of mass-current multipoles, and used it to evolve inspiraling compact binaries[22]. Very recently Rezzolla et al.[23] revised Blanchet's formalism shown inadequate for numerical treatments of GWs sources. Their new results were applied to evolve rotating neutron stars driven by axial perturbations: the r-mode instability[23]. To our goal what is relevant here is to obtain useful expressions for the RRF that can explicitly account at least for the matter current-quadrupole RRF "densities", namely; an expansion up to order $(v/c)^{7/2}$. Following Ref.[23], the pure time-dependent matter current-quadrupole moments of the GWs RRF reads

$$F_i^{7/2\mathrm{PN}} = \rho[-\partial_i({}_9\alpha) - \partial_t({}_8\beta_i) - \partial_j({}_8\beta_i)v_j + \partial_i({}_8\beta_j)v_j - \partial_t({}_7h_{ij}v_j) - v_k\partial_k({}_7h_{ij}v_j) + \frac{1}{2}v_jv_k\partial_i({}_7h_{jk})], \tag{22}$$

where ${}_7h_{ij} = \frac{32G}{45}x_k\epsilon_{kl(i}J^{(4)}_{j)l}$ is the coefficient of the term c^{-7} in the expansion, and the even ${}_n\beta_j$ functions in Eq.(22) are shown to make it null contributions to the source dynamics[23]. The dependence on time-varying mass-quadrupole moments (order $(v/c)^{5/2}$) is encoded in the term:

$$\delta F_i^{7/2\mathrm{PN}}({}_7\alpha, 6\beta_i, {}_5h_{ij}) \equiv \rho[-\partial_i({}_7\alpha) - \partial_t({}_6\beta_i) + v_j\partial_i({}_6\beta_j) - v_j\partial_j({}_6\beta_i) - \partial_t({}_5h_{ij}v_j) - {}_5h_{ij}v_k\partial_kV_j], \tag{23}$$

with ${}_5h_{ij} = -\frac{4G}{5}\bar{I}^{(3)}_{ij}$ the coefficient of term c^{-5} (see[23] for details of the derivation and definitions).

Natal pulsar recoil velocities

For the pure current-quadrupole RRF (axial effects), $F^{(react)}_{(c4-pole)}$, we can derive the pulsar recoil velocity $\vec{V}_{rec}$ from the dynamical relation:

$$L^{GW}_{(c4-pole)} = \frac{16G}{45c^7}\left|\left\langle J^{(3)}_{ij}J^{(3)}_{ij}\right\rangle\right|_{1\mathrm{ms}} \equiv F^{(react)}_{(c4-pole)}V_{rec} \times \cos(\vec{F}^{(react)}_{(c4-pole)}, \vec{V}_{rec}), \tag{24}$$

where $F^{(react)}_{(c4-pole)} = F^{7/2(PN)}\bar{\rho}R^3_{PNS}$. To obtain order of magnitude estimates in all the computations henceforth we use $|J_{ij}| = MR^2|V_\nu|$, with M the PNS mass, R the star radius, $|x_i| = \bar{d}$ the position at which the potential is being considered, and $\frac{d}{dt} = T^{-1}_{dyn} \equiv \left(R^3/GM\right)^{1/2} = f^{-1}_{GW} \sim 1$ms for the dynamical timescale, as demonstrated by numerical simulations[11, 12] (see also Figure 6 in Ref.[9]). For a typical proto-neutron star: $M = 1.44\mathrm{M}_\odot$, $R_{PNS} = 30$km[5], and $\bar{\rho} = 1\times10^{14}\mathrm{gcm}^{-3}$, we can get the largest recoil velocities ($V_{rec} \lesssim 1500$ kms^{-1}) observed in pulsar surveys[1, 2] as:

$$V_{\mathrm{rec}} \sim 1.5\times10^8[\frac{\cos^{-1}(\vec{F}^{(react)}_{(c4-pole)}, \vec{V}_{rec})}{48^0}]\ \mathrm{cms}^{-1}, \tag{25}$$

for values of the angle between the GWs reaction force and the effective pulsar velocity: $\cos^{-1}(\vec{F}^{(react)}_{(c4-pole)}, \vec{V}_{rec}) \sim \cos^{-1}(0.672) \geq 48$ degrees. This relative angle is not fixed for all the cases, but rather vary from one pulsar to another. This fact, together with the way RRF applies, implies *no* correlation at all of the final direction of motion and a particular axis on the PNS. These features fit constraints put by Ref.[6].

Initial pulsar spin periods

Finally, GWs spin down the pulsar rotation by carrying off angular momentum, J_i, at the total rate (mass- and current-quadrupoles):

$$\frac{dJ_i}{dt} = \sum_A \varepsilon_{ijk} x^A_j F^{A(react)}_k = -\varepsilon_{ijk} \left| \frac{2G}{5c^5} \left\langle \bar{I}^{(2)}_{ij} \bar{I}^{(3)}_{ij} \right\rangle + \frac{32G}{45c^7} \left\langle J^{(2)}_{jl} J^{(3)}_{kl} \right\rangle \right| . \tag{26}$$

The RRF acting at an appropriate lever-arm may limit the initial rotation frequency of the star. If the RRF acts at the centroid of the *f*orce per unit length distribution (the GW field on each quadrant in the plane wave approximation), then it must produce a torque on the star which is proportional to that lever-arm: $\vec{d} \times \vec{F}^{(react)} \equiv \bar{I}\vec{\alpha}$. Here the induced angular acceleration is: $|\vec{\alpha}| \equiv \Delta\omega/\Delta t \sim \omega/T_{dyn}$. The centroid is located at the lever-arm $x^A_j = \bar{d}$ respect to the star center. This position corresponds to a given phase of the emitted GWs, say $\varphi = \pi/8 \longrightarrow \bar{d} \sim 25\text{km} < R_{PNS}$[5]). Thus the *n*atal pulsar rotation frequency

$$\bar{\omega} \sim 0.6 \times 10^4 \text{rads}^{-1} \longrightarrow P_{init} \sim 10^{-3}\text{s}. \tag{27}$$

It is easy to see that the lowest initial spins observed could be obtained for a lever-arm $\bar{d}$ half of the one inferred to above. Furthermore, this RRF should act upon the PNS during the GWs damping timescale[24]:

$$\Delta T^{(react)}_{GWs} \sim 0.40\text{s} \left(\frac{1.4M_\odot}{\text{M}}\right) \left(\frac{10^6 cm}{\text{R}_{\text{NS}}}\right)^2 \left(\frac{\text{P}}{1\text{ms}}\right)^4 . \tag{28}$$

Such a time interval fits quite well the duration $\Delta t \sim 0.32$s for the four thrusts supposed to be applied to the PNS in the Spruit and Phinney kick model[5]. Thus the GWs RRF kick mechanism provides a consistent picture that agrees quite well with the observational initial spin periods and runaway velocities of pulsars, and as such it constitutes a realization of the Nazin and Postnov conjecture[19].

CONCLUSIONS

In summary, there is strong theoretical evidence for acceleration of black holes that have been impinged by a powerful GW[25]. If such a strong GW is able to kick a black hole to the substantial velocities discussed in Ref.[25], then it is expected a similar acceleration impulse to occur onto the PNS during the SN postbounce due to GWs kicks at birth. If the proposed mechanism actually operates when the pulsar is born, this would be a valuable second example of the influence of GWs on real astrophysical processes. And as such, the overwhelming population of high velocity pulsars would point also towards Einstein's theory of gravitation as the most likely realized in nature.

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[26] Two warning notes are due at this stage. Firstly, in computing the GWs luminosity the numerical value of this anisotropy parameter was "absolute averaged" between the two extreme limits numerically computed by Burrows and Hayes [Phys. Rev. Lett. 76, 352 (1996)][10]. That value was used merely as to an order of magnitude estimate. Such a procedure would suggest the ν-fluid anisotropy is "frozen" during the timescale over which the oscillation process is expected to take place, what is clearly not the case. Certainly, the ν-fluid anisotropy is a dynamical parameter, and as such a realistic and correct description of the process here envisioned should envolve a hydrodynamical follow-up of the ν outflow with appropriate time resolution, altogether with a self-consistent checking whether the conditions for conversions to occur are met. Secondly, we do stand on the assumption that this anisotropy manifest itself, not only at the ν diffusion phase but rather, from the very beginning of the SN core bounce, the time interval over which the process must develop. To support this premise we recall the simulations of coalescing neutron star binaries by Ruffert and Janka [18], in which an elaborated treatment of the neutrino leackage is introduced. Although this is a very different astrophysical context, the relevant physics therein is highly suggestive of the one to be expected in the process under focus here. Finally, although we used the anisotropy parameter from Ref.[10], we stress that its difference with respect to the one derived by Muller and Janka [Astron. Astrophys. 317, 140 (1997)], which differs in about two (2) orders of magnitude, it is not enough to invalidate the result here obtained. According to the calculations just presented the resulting overall GWs luminosity would read: $L_{\rm GW} \sim 10^{48}$erg s^{-1}, which is still enough to make its contribution to the final pulsar kick more crucial than anyone stemming from the ν momentum asymmetry, as in Kusenko and Segre [Phys. Rev. Lett. 77, 4872 (1996)].