ISSN 0029-3865

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS
Rio de Janeiro

Notas de Física

CBPF-NF-059/99
November 1999

Facts and Fictions about Anti de Sitter Spacetimes with Local Quantum Matter

Bert Schroer

CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico

Facts and Fictions about Anti de Sitter Spacetimes with Local Quantum Matter

Bert Schroer Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany
presently: CBPF, Rua Dr. Xavier Sigaud, 22290-180 Rio de Janeiro, Brazil
schroer@cbpf.br

November 1999

Abstract

Viewing the AdS_{d+1} spacetime in the context of the conformal- compactification and covering formalism, it is natural to view Rehren's algebraic holography between the bulk AdS quantum matter and that of CFT as a kind of "pull-back" of the global d-dimensional conformal (block) decomposition theory (which results from the resolution of the apparent Einstein causality paradox in conformal QFT) when one substitutes the lightcone in $\mathbb{R}^{(d,2)}$with an hyperboloid. Whereas in the case of lightcone this is possible in terms of fields, the use of algebras is more natural for the AdS hyperboloid. We also mention other ways of associating (a finite set of) chiral field theories with higher dimensional QFT's.

1 Historical and sociological background

There has been hardly any problem in particle physics which has attracted as much attention as the problem if and in what way the **Anti de Sitter** spacetime and the one dimension lower conformal field theory are related and if this could possibly contain clues about quantum gravity. In more specific quantum physical terms the question is about a conjectured [1][2] (and meanwhile in large parts rigorously understood) correspondence between two quantum field theories in different spacetime dimensions, the lower-dimensional conformal one being the "holographic image or projection" of the AdS.

Virtually the entire globalized community of string physicist (~3000) has placed this problem in the centre of their interest and there have been approximately around 100-150 papers per month during the last half year. Even if one takes into account the steep population growth in the number of particle physicist during the last decades and looks only at the percentage of involved particle physicist and compares it with the relative number of participants in previous similar phenomena (the S-matrix bootstrap of Chew, Regge theory, the Gursey-Radicatti SU(6) symmetric quark theory, to name some of them) which also led to press-conferences, interviews and articles in the media (e.g. time magazine), it remains still an impressive sociological phenomenon. Just imagine yourself working on this problem and getting up every morning turning nervously to the hep-th server in order to check that nobody has beaten you on similar results. What a life in an area which used to require a contemplative critical attitude! This is clearly a remarkable sociological situation in the exact sciences which warrants an explanation.

Leaving the explanation of these mass manifestations (on a subject which is more remote from tangible physics than anything before in this rich physics century) to historians or sociologists of the exact sciences, I will limit myself to analyze the situation from the point of view of a quantum field theorist with a 30 year professional experience who still nourishes a certain curiosity about string theory with its many successful formal mathematical consistency tests, but who has no active experience nor ambitions in that area. Whether you consider this is an advantage or disadvantage depends on your background, age and personal point of view.

The AdS model of a curved spacetime has a long history [3][4] as a theoretical laboratory of what can happen in a universe which is the extreme opposite of globally hyperbolic in that it processes a self-closing time, whereas the proper de Sitter spacetime was once considered among the more realistic models of the universe. The recent surge of interest about AdS came from string theory and is totally different in motivation and more related to the hope (or dream) to attribute a meaning to "Quantum Gravity" from a string theory viewpoint.

Fortunately for a curious outsider (otherwise I would have to quit right here), this motivation has no bearing on the conceptual and mathematical problems posed by the would be AdS-conformal QFT correspondence, which turned out to be an entirely quantum field theoretical problem in a particular curved spacetime. As a result of this peculiarity (related to the mentioned historical role of the AdS model), there are some fundamental and interesting aspects which this problem generates in QFT which are all related to causality, charge transport, superselection sectors and the plea for an intrinsic formulation of QFT [5] away from "coordinatizations" (of algebras of observables) in terms of pointlike covariant fields. All these issues are related to real-time physics and in most cases their meaning in terms of euclidean continuation (statistical mechanics) remains obscure, but this does not make them less physical.

This note is organized as follows. First I elaborate the kinematical aspects of the AdS_{d+1}-conformal QFT_d situation as a collateral result of the compactification formalism for the "conformalization" of the d-dimensional Minkowski space. For this reason the seemingly more demanding problem of QFT in CST can be bypassed. From this relation between the two spacetimes we see that apart from the boundary at infinity of AdS there is no pointlike relation between the spaces but rather a relation between bulk sets which includes in particular the "wedges" whose field theoretic distinction already appeared in many other contexts. Therefore the field theory in the third section follows closely the recent (conceptually as well as mathematically) rigorous solution of the AdS-conformal QFT correspondence presented by Rehren [5] and some closely related work which supports and extends those results [7] [8].

Since one of the most important messages is the "holographic" aspects, we then present in the fourth section some recent more general results obtained by the methods of algebraic QFT which relate the structure of an arbitrary QFT to a finite set of chiral quantum field theories and their relative position in a common Hilbert space. In massive d=1+1 theories this amounts to lightray holography, but in higher dimensions it is more appropriate to refer to this process as "scanning". Although such an extension of holographic ideas is not required for the understanding of the AdS-conformal correspondence, it gives a good glimpse of the future perspective of fresh attempts at a nonperturbative constructive approach to QFT [9] and hopefully also some clues on quantum gravity (commented on in the last section). In that last section we also return to the theme of "facts and fictions" theme by including the string theoretical point of view as mentally processed by a curious, critical and altogether sceptical quantum field theorist.

2 Conformal Compactification and AdS

One possibility to get to conformal QFT is to notice that the Wigner representation theory for the Poincaré group for zero mass particles allows an extension to the conformal symmetry: Poincaré group(d)$\rightarrow$ $SO(d,2)$. Besides scale transformations, this larger symmetry also incorporates the fractional transformations (proper conformal transformations)

$$x' = \frac{x - bx^2}{1 - 2bx + b^2x^2} \tag{1}$$

It is often convenient to view this formula as the translation group transformed with the hyperbolic inversion

$$x \longrightarrow \frac{-x}{x^2} \tag{2}$$

acting as an equivalence transformation within an extended group. For fixed x and small b the formula (1) is well defined, but globally it mixes finite spacetime points with infinity and hence requires a more precise definition in particular in view of the positivity energy-momentum spectral properties in its action on quantum fields. Hence as preparatory step for the quantum field theory concepts one has to achieve a geometric compactification. This starts most conveniently from a linear representation of the conformal group SO(d,2) in 6-dimensional auxiliary space $\mathbb{R}^{(d,2)}$ (i.e. without field theoretic significance) with two negative (time-like) signatures

$$G = \begin{pmatrix} g_{\mu\nu} & & \\ & -1 & \\ & & +1 \end{pmatrix} \tag{3}$$

and restricts this representation to the (d+1)-dimensional forward light cone

$$LC^{(d,2)} = \{\xi = (\boldsymbol{\xi}, \xi_4, \xi_5\} \,; \boldsymbol{\xi}^2 + \xi_d^2 - \xi_{d+1}^2 = 0\} \tag{4}$$

where $\boldsymbol{\xi}^2 = \xi_0^2 - \vec{\xi}^2$ denotes the d-dimensional Minkowski length square. The compactified Minkowski space is obtained by adopting a projective point of view (stereographic projection)

$$M_c^{(d-1,1)} = \left\{ x = \frac{\boldsymbol{\xi}}{\xi_d + \xi_{d+1}} ; \xi \in LC^{(d,2)} \right\} \tag{5}$$

It is then easy to verify that the linear transformation which keep the last two components invariant consist of the Lorentz group and those transformations which only transform the last two coordinates yield the scaling formula

$$\xi_d \pm \xi_{d+1} \to e^{\pm s}(\xi_d \pm \xi_{d+1}) \tag{6}$$

leading to $x \to \lambda x, \lambda = e^s$. The remaining transformations, namely the translations and the fractional proper conformal transformations, are obtained by composing rotations in the $\boldsymbol{\xi}_i$-ξ_d and boosts in the $\boldsymbol{\xi}_i$-ξ_{d+1} planes.

The so obtained spacetime is most suitably parametrized in terms of a "conformal time" τ

$$\begin{aligned} M_c^{(d-1,1)} &= (sin\tau, \mathbf{e}, cos\tau), \; e \in S^3 \\ t &= \frac{sin\tau}{e^d + cos\tau}, \; \vec{x} = \frac{\vec{e}}{e^d + cos\tau} \\ e^d + cos\tau &> 0, \; -\pi < \tau < +\pi \end{aligned} \tag{7}$$

so that the conformally compactified Minkowski space is a piece of a multi-dimensional cylinder which is carved out between two d-1 dimensional boundaries which lie symmetrically around $\tau = 0, \mathbf{e} = (\mathbf{0}, e^d = -1)$ where they touch each other [11]; but the projective aspect of Hilbert space vectors as representing physical states demands that we use the universal covering space which is the full cylinder (which has a tiling into infinitely many ordinary Minkowski spaces)

$$\widetilde{M_c^{(d-1,1)}} = S^{d-1} \times \mathbb{R} \tag{8}$$

Indeed in order not to be limited by the narrow confines of Huygen's principle (which tends to limit relativistic system to non-interacting ones [8]), the "nature" of local quantum physics demands the use of the covering space (or as the substitute a conformal decomposition theory of local fields into irreducible components with respect to the center of the conformal covering) as will become clear in the next section. The relevance of this covering space for the notion of relativistic causality was first pointed out by I. Segal [10] and nicely presented within the context of QFT by Luescher and Mack [11]. Formally it solves the "Einstein causality paradox of conformal quantum field theory" [12] which originated with would be conformal models of quantum field theory as the massless Thirring model which violated Huygens principle. The naive reason for this apparent violation was that there exist continuous curves of conformal transformations which lead from spacelike separations via the lightlike infinity to timelike separation which obviously generates a contradiction with the structure of the Thirring model whose timelike anti-commutator unlike the spacelike one does not vanish. The covering structure formally solves this causality problem by emphasizing that the path through lightlike infinity was in fact a path which led into another sheet and it is only the unjustified projection of one of the end-point back into the rhomboid Minkowski space (7) which has a timelike distance and not the point itself (which remains causally disjoint). If one depicts the covering space as a cylinder, then it contains infinitely many copies of the original Minkowski space (in d=1+1) of which the above one (7) is (for d=1+1) a compact region which allows a nice picture [11].

Using the above parametrization in terms of $\mathbf{e}$ and the "conformal time" τ, one can immediately globalize the notion of time like distance and one finds the following causality structure

$$\begin{aligned}&(\xi(\mathbf{e},\tau)-\xi(\mathbf{e}',\tau'))^2>0,\ hence \\ &\tau-\tau'>2Arcsin\left(\frac{\mathbf{e}-\mathbf{e}'}{4}\right)^{\frac{1}{2}}=Arccos\left(\mathbf{e}\cdot\mathbf{e}'\right)\end{aligned} \tag{9}$$

In order to get the AdS manifold into the game, we may instead of using the directions on the light cone also use the asymptotic directions on the forward hyperboloid $\xi^2=R^2$. This is admittedly a round-about way to obtain the conformal compactified Minkowski space, but a very interesting one; as it turns out this (d+1)-dimensional hyperboloid defines (after a $\mathbb{Z}_2$ identification $\xi\leftrightarrow-\xi$) the anti de Sitter spacetime. If one only wants to use this AdS spacetime in order to describe the compactified geometry including the covering aspects, one may simply parametrize the AdS as (for R=1)

$$\begin{aligned}\eta^0&=\sqrt{1+r^2}sin\tau\\ \eta^i&=r\mathbf{e}^i,\ i=1,..d\\ \eta^{d+1}&=\sqrt{1+r^2}cos\tau\end{aligned}$$

and verify that the ensuing covering parametrization of AdS for r$\rightarrow\infty$ (properly rescaled) approaches the covering parametrization of the conformal boundary.

If it would be only for this asymptotic statement, the relation between AdS and the conformal space would not be very exiting. But as the d-dimensional conformal spacetime and the local quantum physics on it can be pulled back onto the d+1 dimensional forward light cone by defining correlation functions fulfilling certain homogeneity properties [11], one could ask whether such a "pull-back" is also possible into the AdS hyperboloid instead of the light cone. If one relaxes the requirements and goes beyond points and lines (direction) to more general sets, such a correspondence is indeed possible within the conceptual framework of AQFT [5]. The idea of how to formulate such a correspondence comes from the fact that $AdS^{(d,1)}$ and $M_c^{(d-1,1)}$ and their coverings share the same spacetime symmetry namely $SO(d,2)$ resp. $\widetilde{SO(d,2)}$. Therefore one may start from a wedge region (reaching into the boundary) in the d+2 dimensional auxiliary space which is defined in terms of two light rays and left invariant under an appropriate boost transformation. If we call the projection of an ambient wedge in $\mathbb{R}^{(d,2)}$ onto AdS "(AdS-) wedge" and notice that its projection on the conformalized Minkowski spacetime is generically a double cone (a "conformal" wedge is a special case since it is conformally equivalent to a double cone), then we have a starting point from which we can build up a correspondence. It would be natural (and without alternative) to relate the correspondence by letting the symmetry group act in both worlds and extend this correspondence to transformed regions and their intersections. We refer to a paper by Rehren [5] where this has been carried out and on which this and the following section rely heavily. Now the old problem which historically led to AdS, namely the existence of timelike closed lines and the apparently weird causality situation of local quantum physics in such a world, can receive a helping hand from the conformal side[1] where it has a more hidden counterpart in the form of an apparent "Einstein causality paradox" whose solution in terms of conformal blocks was already found a quarter century ago [13] and which we will present in the present context in the next section.

3 Conformal QFT pulled back into AdS

The formulation in terms of conformal covering space would be useful if the world, including laboratories of experimentalists, would also be conformal, which certainly is not the case. Therefore it is helpful to know that there is a way of re-phrasing the physical content of local fields (which violate the Huygens principle and instead show the phenomenon of "reverberation" [12] inside the forward light cone) without running into the trap of the causality paradox of the previous section. Such fields, although behaving irreducibly under infinitesimal conformal transformations, transform reducibly under the action of the global center of the covering $Z(\widetilde{SO(d,2)})$. As a unitary abelian group it is generated by the 2π-translation in the conformal time τ. A local covariant field $A(x)$ (local in the sense of the causal structure of $\widetilde{SO(d,2)}$) corresponding unitary operator $Z \in Z(\widetilde{SO(d,2)})$ can be decomposed as [13]

$$A_d(x) = \int_0^! A_d^\xi(x) d\xi \tag{10}$$

[1] Meanwhile K.-H. Rehren informed me that one of his collaborators is studying this problem in detail.

with A_d^ξ formally given by

$$A_d^\xi(x) = \sum_{n=-\infty}^{\infty} Z^n A_d(x) Z^{-n} exp\left[in\pi(d-2\xi)\right] \tag{11}$$

from which one gets

$$AdZ A_d^\xi(x) = exp\left[-i\pi(d-2\xi)\right] A_d^\xi(x) \tag{12}$$

The notation is the following: d is the scaling dimension of the local (causal in the covering sense) field $A_d(x)$ and the ξ-integration is the decomposition into its centrally irreducible components. These component fields, unlike the original globally causal fields, do not fulfill the Reeh-Schlieder theorem (sometimes referred to as the field—field-state-vector correspondence), rather they have a source and a range and their application to a non-matching source subspace vanishes. Their physical interpretation is easily obtained from the conformal analysis of 3-point functions

$$\begin{aligned} \langle C_{dc}(x) A_d(y) B_{d_b}(z) \rangle &= \left\langle C_{dc}(x) A_d^\xi(y) B_{d_b}(z) \right\rangle \\ \xi &= \frac{1}{2}(d + d_b - d_c) \ \mathrm{mod}(1) \end{aligned} \tag{13}$$

Hence the quantum number of the irreducible components is related to the dimensional spectrum (critical indices) of the theory and the ξ-dependent phase factors enter the transformation law which comes close to the naive classical transformation

$$U(b) A_d^\xi(x) U^{-1}(b) = \frac{1}{\left[\sigma_+(b,x)\right]^{d-\xi} \left[\sigma_-(b,x)\right]^{\xi}} A_d^\xi(x) \tag{14}$$

whereas the more complicated law for the local field follows from the decomposition formula (10). In the case of d=1+1 for which the group (as well as its center) factorizes $\widetilde{S(2,2)} = \widetilde{SU(1,1)} \times \widetilde{SU}(1,1)$ and one obtains the well-known BPZ [14] conformal block decomposition theory with the additional remark that it was already discovered 10 years before[2]. In order to facilitate the reading of our 74/75 papers [13] on the subject, I have used exactly the same normalizations and notation. There is a special aspect of this chiral decomposition theory. It is the only case for which one has a classification theory of the possible spectra of dimensions/critical indices. It is given by the spectrum of phases of the so-called statistical parameter which occurs within the superselection theory of AQFT and in the case at hand is inexorably related to the braid group exchange algebra structure of the nonlocal irreducible components A_d^ξ.

Now the path to a nontrivial i.e. interacting QFT on AdS which can withstand the causality challenge appears in a clearer light: pull-back the nontrivial centrally irreducible conformal block (superselected charge sector) algebras instead of fields, using the wedge techniques of Rehren. By construction the pulled-back charge sector algebras (and their intersections) will satisfy the correct phase relations with the generator of the center Z which are needed for the avoidance of causality paradoxes. For AdS_2

[2] The new result in the BPZ work were the nonabelian illustrations (minimal models) which went far beyond the abelian exponential Bose field and current algebra of one decade before.

the operators from these block algebras are even expected to fulfill an exchange algebra with R-matrix structure constants. The pull-back method, unlike for the light cone pull-back cannot be done in terms of field-coordinatizations unless one admits also multiply localized field with a charge transport tube between them [5].

4 Generalized Holography in Local Quantum Physics

The message we can learn from the AdS-conformal correspondence is two-fold. On the one hand there is the recognition that there are situations where it is necessary to avoid the use of "field coordinates" in favor of directly working with local algebras. Although this idea is a rather old enrichment of advanced QFT and forms the backbone of AQFT, in most concrete situations there were always convenient field coordinatizations available in terms of which (in analogy to preferred coordinates in differential geometry) the calculations simplified. For the AdS-conformal correspondence the best way is however to stay intrinsic, i.e. to use the net of algebras.

The second message is that there may exist a holographic relation between QFT's and lower dimensional conformal QFT. Here we are entering a much more recent issue of QFT which is still in its infancy. Let us consider the simplest case: the chiral conformal holographic image of a two-dimensional massive QFT or, using an older terminology (which will hereby attain a rigorous and at least partially new meaning), the light ray restriction (quantization). Let us start from the right wedge algebra of a massive 2-d QFT (either generated from a Wightman field or from the algebraic net approach) $\mathcal{A}(W)$. We want to introduce the restriction of this theory on its upper horizon $\mathbb{R}_+$ which is half of the total line of the light ray $\mathbb{R}$. We first notice that compact intervals on $\mathbb{R}_+$ does not cast a two-dimensional causal shadow, in contradistinction to a spacelike interval. The physical reason is of course that each point in an ε-neighborhood below that interval is in the backward influence cone of the points on $\mathbb{R}_+$ which are outside that interval. The situation changes if we take *all of* $\mathbb{R}_+$. In that case the causal shadow is the wedge region; this holds only for the massive case, since for d=1+1 massless theories there are independent characteristic data on the lower light ray horizon of the wedge (doubling of the degrees of freedom). In the general approach to QFT the von Neumann algebra of a compact spacetime region is, according to the causal shadow property of AQFT (which is a local version of the time-slice property [15]), identical to the algebra of its causally completed region. Each Lagrangian field theory, to the extend that it exists beyond perturbation, fulfills this requirement[3]. If one tilts the spacelike interval into the lightlike direction, the causal shadow region becomes gradually smaller. The only way to counteract this shrinking is to extend the spacelike interval say gradually to the right in such a way that the larger lower causal shadow part becomes the full wedge in the limit. The correctness of this intuitive idea which

[3]Postulates as this in fact have been abstracted from the Lagrangian quantization setting with the idea to separate the good physical content from the Damocles sword of existence of Lagrangian QFT's.

suggests

$$\mathcal{A}(\mathbb{R}_+) = \mathcal{A}(W) \tag{15}$$

can be checked against other rigorous results. One rigorous result from Wigner representation theory (which therefore is limited to free field theories) together with the application of the Weyl or CAR (for halfinteger spin) functor is the statement that the cyclicity spaces for an interval I on $\mathbb{R}_+$ agree with the total space [16]

$$\overline{\mathcal{A}(I)\Omega} = \overline{\mathcal{A}(\mathbb{R}_+)\Omega} = \overline{\mathcal{A}(W)\Omega} = H \tag{16}$$

i.e. the validity of the Reeh-Schlieder theorem on the light ray subalgebra. In fact this holds for all positive energy representations including zero mass, except zero mass in d=1+1 in which case the decomposition in two chiral factors prevents its validity. The step from spaces to (15) is done with the help of Takesaki's theorem (mentioned later) The second argument is that any field theory for which the wedge algebra has polarization free generators (PFG) generators F automatically fulfills (15). Here PFG means that

$$\begin{aligned} F\Omega &= \int f(p)\,|p\rangle\,\frac{d^dp}{2\omega} \\ locF &= W \end{aligned} \tag{17}$$

i.e. the one time application of the wedge localized generator onto the vacuum is a one particle state without any admixture of higher particle-antiparticle pair states (vacuum polarization clouds). It is highly surprising and nontrivial that this is possible in the presence of interactions. In some way the PFG's behave like free fields but in the presence of interactions and to resolve this apparent contradiction they must be sufficiently nonlocal (i.e. wedge-localized). In fact it can be shown that the wedge region is the smallest for which this is possible; any smaller spacetime region, in particular any compact region leads to the admixture of incoming scattering states with arbitrarily high particle number as a characteristic inexorable attribute of localization in the presence of interactions. The argument that (15) is a consequence of the fact that $\mathcal{A}(W)$ can be generated by such F's can be found in [9] and will not be repeated here. Again one shows first cyclicity (16) from the properties of PFG's and then the result follows by invoking a theorem of Takesaki on conditional expectations for subalgebras in relation to their modular groups. Since this theorem and its setting in AQFT is very deeply related to the noncommutative structure of local quantum physics and gives an excellent look into the "natural" workshop (i.e. the noncommutative aspect is intrinsic and not imposed by decorating (euclidean) commutative structures with noncommutative geometry) let me briefly explain it, although it is already somewhat outside the title of my note.

All structural investigations in AQFT start with a net of operator algebras as subalgebras of the algebras of the algebra of all operators $B(H)$ in a Hilbert space H. These subalgebras are indexed by spacetime regions $\mathcal{O}$ in Minkowski space

$$\mathcal{O} \to \mathcal{A}(\mathcal{O}) \tag{18}$$

and fulfill a few entirely physically motivated properties which constitute the principles of local quantum physics. This field-coordinatization independent approach brought a wealth of new physical results (including the old perturbative results which depend on the use of pointlike fields) but it always run into problems if one tried to extract from it an algorithm for construction of specific models which remained faithful to its intrinsic spirit. All this has changed recently, due to the recognition of the revolutionary role[4] of the modular Tomita-Takesaki theory in local quantum physics. Whereas the discovery that the wedge algebra $\mathcal{A}(W)$ together with the vacuum state leads to modular objects which are related with two important objects in QFT (the modular conjugation J with the TCP-operator and the modular group Δ^{it} with the wedge affiliated Lorentz boost) was already made in the middle of the 70^{ies}, its inverse namely the construction of a wedge algebra and its extension into a net by covariance and intersections is a more recent discovery. It turns out that the wedge algebra has on-shell generators, namely the above mentioned PFG's which are determined in terms of the physical S-matrix. Therefore in cases where the old S-matrix bootstrap program of Chew works i.e. for the factorizing d=1+1 models, the modular machinery can be directly applied. The modular program works especially well for chiral conformal theories for which the wedge is replaced by the halfline and instead of the physical S-matrix one deals with an unitary operator with another interpretation. In more general cases one has two options, either find a new kind of perturbation theory for these PFG's or one must find some way to decompose an actual theory into chiral conformal ones. The way of associating a chiral conformal theory with e.g. a d=1+1 massive theory is the following [17]. Start from the right wedge algebra $\mathcal{A}(W)$ with apex at the origin and let an upper lightlike translation a_+ (which fulfills the energy positivity!) act on $\mathcal{A}(W)$ and produce an inclusion

$$\mathcal{A}(W_{a_+}) \subset \mathcal{A}(W) \tag{19}$$

This inclusion is halfsided "modular", i.e. the modular group Δ^{it} of $(\mathcal{A}(W),\Omega)$ (which is the Lorentz boost, as previously stated) acts on $\mathcal{A}(W_{a_+})$ for t<0 as a compression

$$Ad\Delta^{it}\mathcal{A}(W_{a_+}) \subset \mathcal{A}(W_{a_+}),\ t<0 \tag{20}$$

The assumed nontriviality of the net i.e. the intersections[5] of wedge algebras entails that the relative commutant (primes on algebras denote their commutant in $B(H)$)

$$\mathcal{A}(W_{a_+}) \cap \mathcal{A}(W)' \tag{21}$$

is also nontrivial and insures that the inclusion is what has been termed "standard". But it is known that standard modular inclusions correspond to chiral conformal theories, i.e. the classification problem

[4] In mathematics it played a crucial role in the work of Alain Connes on the classification of von Neumann algebras and it is also present (but as a result of the assumed finiteness of algebras not as prominent) in Vaughan Jones work on subfactors; both contributions led to a Fields medal.

[5] The nontriviality of the intersections are the algebraic counterpart of the "good" short distance behaviour in a quantization approach (which is a prerequisite for renormalizability); both are necessary (and in the algebraic case also sufficient) for the existence of the theory.

for the latter is identical to the classification of all standard modular inclusions. In the case at hand the emergence of the chiral theory is intuitively clear since the only "living space" in agreement with Einstein causality (within the closure of W and spacelike with respect to the open W_{a_+}) which one can attribute to the relative commutant is the lightray interval of length a_+ starting at the origin. From the abstract modular inclusion setting the Hilbert space which the relative commutant generates from the vacuum could be a subspace $H_+ \subset H$, $H_+ = PH$ of the original one. With the help of the L-boost (=modular group Δ^{it} of $(\mathcal{A}(W),\Omega)$ one then defines a net on the halfline $\mathbb{R}_+$ and a global algebra

$$\begin{aligned} \mathcal{A}(\mathbb{R}_+) &= alg\left\{\cup_{t<o} Ad\Delta^{it}(\mathcal{A}(W_{a_+})\cap\mathcal{A}(W)')\right\} \subset \mathcal{A}(W) \\ \overline{\mathcal{A}(\mathbb{R}_+)\Omega} &= H_+ \end{aligned} \tag{22}$$

The modular group Δ^{it} of the original algebra leaves this "lightray algebra" invariant and hence we are in the situation of the Takesaki theorem which states that a subalgebra together with the vacuum which is left invariant by the modular group of the larger invariant has modular objects which are restrictions of those of $(\mathcal{A}(W),\Omega)$

$$\begin{aligned} (\mathcal{A}^{res}(\mathbb{R}_+),\Omega) &\Rightarrow ((\Delta^{it})^{res}, J^{res}) \\ notation &: \quad B^{res} = PBP \end{aligned} \tag{23}$$

In addition the condition on the invariance of the subalgebra is the precise condition under which a conditional expectation E with $\mathcal{A}^{res}(\mathbb{R}_+) = E(\mathcal{A}(W)) = P\mathcal{A}(W)P$ (the noncommutative counterpart of the physicists decimation process with the help of "integrating out" degrees of freedom in the sense of Wilson/Kadanoff). The existence of PFG generators of $\mathcal{A}(W)$ insures that $H_+ = H$ and in that case the Takesaki theorem leads to the desired equality

$$\mathcal{A}^{res}(\mathbb{R}_+) = \mathcal{A}(\mathbb{R}_+) = \mathcal{A}(W) \tag{24}$$

The identity shows that identification of chiral conformal theories with zero mass is a prejudice caused by the naive confusion of covariances with localization favored by the quantization point of view. Whether chiral theories are describing massless or massive situations depends on the interplay of covariance and locality. In the present case locality and covariance coalesce in the case of a_+ lightray translations and L-boosts which restrict to dilations. These are the standard covariances which are used to define a net structure on $\mathcal{A}(\mathbb{R}_+)$. However the third covariance on $\mathcal{A}(W)$ namely the opposite lightray a_- translation, which acts locally on the $\mathcal{A}(W)$ net, becomes enormously nonlocal if one tries to interprete in terms of the $\mathcal{A}(\mathbb{R}_+)$ net structure. In fact the action is not geometric at all but rather totally "fuzzy" [18] : an a_- translation acting on a a_+-indexed interval spreads all over $\mathbb{R}_+$ bur does not loose its solid algebraic meaning in the sense of transforming the original interval-indexed subalgebra into another one which does not permit any spacetime indexing in the "lightray world". This second "hidden" symmetry[6] is important

[6] The original $\mathcal{A}(W)$-net also has hidden symmetries, e.g. the two conformal rotations belonging to chiral theories on $\mathbb{R}_\pm$.

for the finite mass spectrum: although the $P_\pm$ spectra (lightray momentum spectrum) separately goes to zero and mimics a zero mass situation, the product (not a tensor product as in the massless d=1+1 case!) P_+P_- has a mass gap.

The present modular inclusion approach to "lightray physics" and degrees of freedom reduction [19][20] clarifies some opaque features of "light cone quantization", notably those which have to do with locality and covariance, i.e. the reconstruction of the local massive d=1+1 net from its "lightray holography". On the one hand it brings into the open an old suspicion that lightray physics is nonlocal (and therefore threatened by lack of physical interpretability), but on the other hand shows that the situation, if treated with the proper concepts and methods, is not without redemption. It gives additional weight to a point which was made by Rehren in the above context of the AdS-conformal correspondence namely: field coordinatizations are not so useful for describing holographic relations, the more intrinsic net structure is better adapted. The lightray holography remains of course conceptually different from the holography through a boundary at infinity (for which the AdS-conformal correspondence is the only known example). The former two-dimensional illustration has a structurally very rich and only partially explored generalization to higher dimension. In that case the modular method applied to one wedge only transfers a small fraction of the structure of the original theory into a chiral conformal theory which localization-wise should really be associated with the upper light front horizon of the wedge. But since its transversal localization remains completely unresolved, the so obtained light front theory only contains the longitudinal localization data of a chiral conformal net. Let me explain the transversal resolution in the simplest case of a d=1+2 theory. In that case one tilts the wedge by a L-boost which leaves the upper defining light ray for the initial wedge invariant [9]. One then convinces oneself that this newly positioned second wedge has a modular associated chiral conformal theory which, though being unitarily equivalent to the first one, carries the missing information (which is needed for the reconstruction of the original d=1+2 theory) in form of its relative position in the common Hilbert space H. This method should be more appropriately called scanning by (a family of) chiral conformal theories. The idea is to approach the existence and construction problem of higher dimensional QFT's by the apparently simpler looking problem of studying a finite family of chiral conformal theories in a prescribed relative position. Almost all complex problems owe their solution to chopping up one difficult problem into several less difficult ones and the progress (outside perturbation) on this issue (of construction of QFT) left much to be desired just because the problem proved too viscous against such chopping up attempts.

For a more detailed analysis with emphasis on thermal aspects (localization temperature and entropy) we refer to [9]

5 Lessons on Quantum Gravity: Fact or Fiction?

Despite the interesting messages about the nature of the AdS-conformal correpondence, there is yet no hint at possible properties which could point into the direction of quantum gravity (which after all was the prime reason why string theorist revived the interest in the AdS model).

The strange socio-scientific situation described in the introduction requires an explanation. Even if one takes into account the new marketing aspect which accentuates fashions on a globalized scale (and from which also other areas of exact sciences were not spared), one still finds it difficult to explain the mere size if over a thousand papers in a relative short time on a special model for only one special purpose (to get a glimpse of quantum gravity) in view of the fact that there are many interesting and theoretically challenging problems around (even more than at any other time in this century). One's surprise and bewilderment (I am talking about physicists who had a professional life already before the new string time) even increases if one notices, as we did in this note, *that there is a nice solution [5] of the problem which is not only clear from a mathematical viewpoint but hardly leaves anything to be desired from a conceptual point of view.* The reason why this has been totally ignored in my view is not just that it argues in a framework outside of the differential-geometric setting of string theory. Rather one gets the suspicion that it is related to the fact that, similar to the present work, it does not confirm those hopes of string theoreticians about quantum gravity. This is why (even though some of their AdS conjectures were made precise and rigorously proven!) they seem to remain unhappy about that work. Since as a theoretical physicist I am not qualified to comment on sociological aspects, the only thing I can do is to go back and try to understand from where this stubborn believe in the inexorable connection if string theory with quantum gravity is coming from. This is not possible without a little excursion into history.

The split of string theory away from local quantum field theory can be traced back to the time of the invention of the dual model. Before there was an attempt at a pure on-shell S-matrix theory which got bogged down on the insufficient physical (and consequently also mathematical) conceptual understanding of crossing symmetry and on the fundamentalistic cleansing rage against off-shell (field theoretic) concepts. Phenomenologist introduced a very special form of crossing called "duality". It consisted in demanding that crossing symmetry for two-particle scattering should already be enforceable with infinitely many one particle states which via crossing (between the Mandelstam s t and u channels) should form a bootstrapping situation ("nuclear democracy") already among one-particle contributions. It had no basis in QFT (nobody ever found a quantum field theoretic realization) and it is almost impossible to convey to a contemporary physicist the strange physical ideas which led to it (and which are presumably totally unknown among younger string theorist). But it did serve as the prime motivation for the introduction of Veneziano's dual model[7] which in turn allowed an interpretation in terms of a

[7] Even in this new setting duality in its original one particle form could not be maintained but had to receive unitarity corrections.

quantum string (the birth of the old string theory). But it should be emphasized that it was not a string in the sense of objects with semiinfinite spacelike localization as envisaged on a formal intuitive level by Mandelstam in local gauge theory or more rigorously as AQFT describes the spacetime carriers of d=1+2 braid group statistics (or more generally of topological charges in the sense of AQFT [6]). Rather the string of string theory is a "spectral string" in the sense of an oscillator like spectrum for the eigenvalues of the mass operators (i.e. an infinite particle tower). The question of spacetime localization, so the experts explained at that time (and still keep telling) is ill-posed in the present string formalism; one has to first have a second quantized form ("string field theory") and then one can address this question[8]. To make this point a bit more palatable, string theorist invariably point to an alleged analogue situation between relativistic particle Lagrangian (square root of the Minkowski line element) and quantum field theory. This is done on a basis of some prescriptions of how to handle such extremely formal functional integral expressions which are then formulated in such a way that the result is the well-known Schwinger alpha-parameter representation of the free field Feynman propagator. What is usually forgotten is to say that a theory of interacting particles on that line of thinking simply does not exist and for precisely this reason one does QFT! Hence this analogy seems to contain a somewhat detrimental and undermining message. In the end it does of course not interest in what way one arrives at an idea as long as it produces some testable experimental results or relates to existing theoretical principles in an interesting way which also could mean transcending (but never ignoring) existing principles.

The next step, namely the gravity interpretation of string theory (the old string was a kind of nonperturbative proposal for strong interactions) consisted in identifying the string tension constant with the Planck length and meant initially just sliding up the energy scale without but keeping the same mathematical formalism. It was by far the most adventurous step in particle physics, if not in the entire history of physics (May be this explains the conspicuous absence of the string theorist of the first generation as e.g. Neveu and most of his collaborators). Many years later, with the ascend of differential geometric knowledge among particle physicist, arguments in the spirit of the differential-geometric sigma model were proposed in favor of a gravity interpretation for the spin=2 component (in the spectrum of all spins which a spectral string theory always contains and which made it useful for Regge phenomenology for a number of years). But it cannot be overstressed that it was the sigma-model reading and not any physical principle or concept which created the quantum gravity aspect of string theory. The power of rational arguments in particle physics is of course limited as in any other human activity. They are generally not sufficient to undo ingrained prejudices or fundamentalistic believes (especially if they are related to TOE´s).

As a mathematical physicist the present situation permits me to draw the conclusion (from the facts presented in this note) that the solution of AdS-conformal correspondence does not shed any more light on this problem than any other curved spacetime model with a local quantum matter content. Besides

[8] There were some courageous attempts in the light cone gauge [22], but it seems that they have been ignored.

string theory there are other ideas [21][9]. To the extend that they use methods of algebraic QFT, they rely heavily on the modular localization properties of local quantum matter enclosed behind Killing or pure "quantum horizons" and use many of the concepts touched upon in the previous section (extended into the direction of thermal properties of such localized matter, including "localization entropy"). In such a framework it is suggestive to introduce an equivalence class of theories which coalesce at the horizon in the hope to find properties of that class which one could associate with quantum gravity. But a lot of work has to be carried out in order to get more physical breath into (or to bury) such ideas.

Note added: The well-known relation of complex spacetime geometry related to the analytically continued (tube, extended tube, permuted extended tube) correlation functions of covariant fields or more general localized operators (the BWH-theory [23]) may be adapted to the AdS spacetime in order to find a complex counterpart of the real spacetime formalism of this paper. One expects that the only significant change (for QFT which are genuinely interacting and therefore violate the Huygens principle in an even number of spacetime dimensions) would consist in the loss of the uni-valuedness of the correlation functions in the AdS-adjusted BWH-domain. In d=1+1 this phenomenon is well-known and results from the change from Fermi/Bose spacelike commutation relations to plektonic ones which reflect the R-matrix structure of the exchange algebras. However in higher dimensions the analytic ramification properties of the vacuum expectation values of the block components of centrally reducible local fields are still a terra incognita. I believe one has two options, either use operators and real geometry/analysis as done here and also in [5], or study state vectors obtained by multiple application of fields to the vacuum or correlation functions with the help of complex geomety/analysis. It is however not possible to use complx methods for operators and therefore the terminology "holomorphic fields" is an extremely unfortunate one.

In a recent paper by G. W. Gibbons [24] such complex methods were used, but since the relation to QFT correlation functions has not been made, it is not possible to say anything about the precise relation to [5] or to the present work.

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, (1998) 231

[2] E. Witten, Adv. Theor. Math. Phys. **2**, (1998) 253

[3] C. Fronsdal, Phys. Rev. **D10**, (1974) 589

[4] S. J. Avis, C. J. Isham and D. Storey, Phys. Rev. **D18**, (1978) 3565

[5] K.-H. Rehren, *Algebraic Holography*, hep-th/9905179

[6] D. Buchholz, *Current trends in axiomatic quantum field theory*, hep-th/9911233

[7] M. Bertola, J. Bros, U. Moschella and R. Schaeffer, *AdS/CFT correspondence for n-point functions, hep-th/9908140*

[8] D. Buchholz, M. Florig and S. J. Summers, *Hawking-Unruh temperature and Einstein Causality in anti-de Sitter space-time,* hep-th/9905178

[9] B. Schroer, *New Concepts in Particle Physics from Solution of an Old Problem*, hep-th/9908021 and references therein

[10] I. E. Segal, *Causality and Symmetry in Cosmology and the Conformal Group*, Montreal 1976, Proceedings, Group Theoretical Methods In Physics, New York 1977, 433 and references therein to ealier work of the same author.

[11] M. Luescher and G. Mack, Commun. Math. Phys. **41**, (1975) 203

[12] M. Hortacsu, B. Schroer and R. Seiler, Phys. Rev. **D5**, (1972) 2519

[13] B. Schroer and J. A. Swieca, Phys. Rev. **D10**, (1974) 480, B. Schroer, J. A. Swieca and A. H. Voelkel, Phys. Rev. **D11**, (1975) 11

[14] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. **B247**, (1984), 83

[15] R. Haag and B. Schroer, J. Math. Phys. **3**, (1962) 248

[16] D. Guido, R. Longo, J.E. Roberts and R. Verch, *Charged Sectors, Spin and Statistics in Quantum Field Theory on Curved Spacetimes*, math-ph/9906019

[17] B. Schroer and H.-W. Wiesbrock, *Modular Constructions of Quantum Field Theories with Interactions,* hep-th/9812251, Rev. in Math. Phys., in print

[18] B. Schroer and H.-W. Wiesbrock, Modular Theory and Geometry, math-ph/9809003, Rev. in Math. Phys. in print, B. Schroer and H.-W. Wiesbrock, *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models*, hep-th/9901031, Rev. in Math. Phys. in print

[19] L. Susskind, J. Math. Phys. **36**, (1995) 6377

[20] G. ´t Hooft, Dimensional reduction in quantum gravity, in Salam-Festschrift, A. Ali et al. eds., World Scientific 1993, page 284

[21] S. Carlip, *Black Hole Entropy from Conformal Field Theory in Any Dimension*, hep-th/9812013, and references therein.

[22] J. Dimock, *Locality in Free String Theory*, math-arc 99-311 and references therein

[23] R. F. Streater and A. S. Wightman, PCT, Spin&Statistics, and All That, W. Benjamin, New York 1964

[24] G. W. Gibbons, *Holography and the Future Tube*, hep-th/9911027