CBPF-NF-059/95

On the Vacuum Stability in the Efimov-Fradkin Model at Finite Temperature.

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Abstract

We investigated the behavior of the non-truncated and truncated Efimov-Fradkin models ($\mathcal{L}_{\rm int} = -\sum_{n=3}^{N} \lambda_n \varphi^n$) at finite temperature in a generic D-dimensional flat spacetime. The thermal contribution to the renormalized mass and coupling constants are obtained in the one-loop aproximation by the use of a mix between dimensional and the Epstein zeta function analytic regularization and a modified minimal subtraction procedure. We proved that for $D_c(N-1) \leq D$ there is not a temperature for which at least one of the renormalized coupling constants becomes zero, where $D_c(N-1)$ is the critical spacetime dimension for the renormalized coupling constant λ_{N-1}. For $D_c(N) \leq D < D_c(N-1)$ only the renormalized coupling constant λ_{N-1} becomes zero at some temperature β_{N-1}^{-1}. For $D < D_c(N)$ the renormalized coupling constants $\lambda_{N-1}(\beta)$ and $\lambda_N(\beta)$ become zero at temperatures β_{N-1}^{-1} and β_N^{-1} respectivelly. In the latter situation, for temperatures $\beta_{N-1}^{-1} < \beta^{-1} < \beta_N^{-1}$ the effective potential has a global minimum. For temperatures above β_N^{-1} the system can develop a first order phase transition, where the origin corresponds to a metastable vacuum. In the nontruncated model, corresponding to a non-polynomial Lagrange density, for $D \geq 2$ all the coupling constants remain positive for any temperature.

Key-words: Effective potential; Metastability.

PACS Index: 11.10.Ef, 11.10.Gh.

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1 Introduction

In this paper an attempt is made to understand the vacuum stability mechanism in scalars models at finite temperature assuming polynomial and non-polynomial Lagrange densities. It is of common knowledge that the ultraviolet divergences that arises in models with non-polynomial Lagrange densities are not worse graph by graph than those encountered in polynomial renormalizable models [1]. This result was obtained using a summation method introduced by Efimov and Fradkin [2][3]. The idea of the method is to investigate the Borel summability of the divergent perturbative series [4]. The interaction Lagrange density of these models may be expanded in a power series of the type,

$$\mathcal{L}_{\text{int}} = -\lim_{N\to\infty}\sum_{n=3}^{N}\lambda_n\varphi^n(x), \tag{1}$$

where $\varphi(x)$ is a hermitian scalar field and λ_n are the coupling constants of the model.

Instead of regularizing the model using a ultaviolet cutoff Λ in the Euclidean momenta, or assuming the existence of a spacetime microscopic structure, characterized by a lattice spacing a, we prefered to regularize it by using a combination of two different methods: the dimensional [5] and a analytic regularization methods [6]. The advantage of this technique lies in the fact that the dependence of mass and coupling constant with the temperature appear in a very straighforward way. A recent discussion on the relation between the cutoff method and analytical regularization procedures to obtain the Casimir energy in an arbitrary ultrastatic spacetime with or without boundaries, may be found in Svaiter and Svaiter [7]. Upon the application of the analytic regularization method a mass parameter μ is introduced, in order to deal with dimensionless quantities in the analytic extensions. It is not dificult to show that the canonical dimension of the coupling constants of the model are given by

$$\lambda_n = \mu^{D-\frac{n}{2}(D-2)} \tag{2}$$

where D is the spacetime dimension. Each coupling constant in the expansion given by eq.(1) has a critical dimension $D_c(n)$. By critical dimension of each coupling constant we mean a spacetime dimension such that bellow it the coupling constant may be a large quantity due to its positive dimension $D-\frac{n}{2}(D-2)$ in terms of μ (or using the critical phenomena language, in terms of the original scale $\frac{1}{a}$ where a is the lattice spacing). We define the critical spacetime dimension $D_c(n)$ as the spacetime dimension where the renormalized coupling constant λ_n is dimensionless. Bellow $D_c(n)$ the model is super-renormalizable. We demonstrate that in the super-renormalizable case above some temperature the system may suffer a first order phase transition.

In two recent papers studying the $\lambda\varphi^4$ model, the possibility to change the sign of the renormalized coupling constant was raised [8][9]. In the first one, the thermal and topological contributions to the renormalized mass and renormalized coupling constant in the one-loop approximation were obtained [8]. In the second one we extended the study of the $\lambda\varphi^4$ model at finite temperature to a generic D-dimensional spacetime with trivial topology of the spacelike section and we also discussed the behavior with the temperature of the Gross-Neveu model, which is an ultraviolet asymptotically free model. In the Gross-Neveu model, we proved that for $D = 3$ the thermal contribution to the renormalized coupling constant is zero. In the other hand, for $D \neq 3$ our results are inconclusives [9]. Studying the $\lambda\varphi^4$ model we obtained more concrete results. Still using the effective potential and the one-loop aproximation, we presented the thermal contribution to the renormalized mass and coupling constant. The thermal renormalized coupling constant is given by

$$\lambda(\beta) = \lambda(\infty) + \Delta\lambda(\beta), \tag{3}$$

where $\lambda(\infty)$ is the temperature independent renormalized coupling constant and $\Delta\lambda(\beta)$ is its thermal correction. Using the fact that $\Delta\lambda(\beta)$ is negative, we proved that for $D < 4$, at sufficiently high temperatures, the system may suffer a first order phase transition with a metastable vacuum at the origin.

In the majority of the papers in the literature the temperature dependence of the renormalized coupling constant is neglected. This appoach is reasonable if we are interested in study a second order phase transition. In this case the variation of the squared mass with the temperature is the most important fact. Therefore it is sufficient to consider the renormalized coupling constant as independent of the temperature and the sign of the squared mass drives the second order phase transition. The situation which we are interested in discussing here is quite different, since the goal of our investigation is not the behavior of the system in the neighboorhood of a second order phase transition. Our intention is to study the model in high temperature regime (far from a critical temperature) where the possibility of vanishing some renormalized coupling constant witha first order phase transition at some temperature arises.

We would like to emphasise that the study of the dependence of the coupling constant with the temperature is not new in the literature. Many authors have been studied such dependence in scalar models [10] and also in non-abelian gauge theories [11]. In the former case, since QCD is an asymptotically free theory, it can be shown that as the temperature increases, the temperature dependent renormalized coupling constant goes to zero. As we discussed, in the $\lambda\varphi^4$ model if $D < 4$, for temperatures β^{-1} above the temperature β_T^{-1} the renormalized coupling constant $\lambda(\beta)$ becomes negative and the origin is a metastable vacuum. Such kind of problems occours with non-asymptotically models. The growth of the coupling constant at large momenta corresponds to the temperature growth (in modulus) of the renormalized coupling constant.

Even in the absence of temperature, the instability of the vacuum of models using scalar fields has been discussed in the literature. An enlightening discussion has been done by Linde [12]. Studying the $O(N)$ model and performing an $\frac{1}{N}$ expansion of the effective potential, this author showed that the effective potential is a double-value function of the field φ (where the field $\Phi = (\varphi_1, .., \varphi_N)$ has a classical part $\Phi = \sqrt{N}(\varphi, 0, .., 0)$). In the upper branch appears a tachyonic pole which leads to disregard it as a non-physical one, remaning the effective potential described by a unique curve which for large values of the classical field is not bounded from bellow.

We would like to stress that the situation treated in this paper is very similar to the examples where renormalized quantities depend on the geometric parameters of the spacelike section. The simplest example is the renormalized vacuum energy of scalar fields confined in a parallelepipedal box, where the sign of the energy may depend on the relative lengths of the cavity. Indeed the sign of the Casimir energy may depend on the spacetime dimension, the type of boundary conditions, etc. [13], but we would like to emphasize only the dependence of the Casimir energy on the ratio of the sizes of the box (imposing Dirichlet b.c.) to give a rough idea of what kind of behavior we expect in situations where a regularization and renormalization procedures are obligatory. Note that the possibility of obtaining a negative renormalized coupling constant in the $\lambda\varphi^4$ model was conjectured by Nash a long time ago [14].

In this paper, we will investigate the one-loop renormalization of the truncated and non-truncated Efimov-Fradkin model assuming thermal equilibrium with a reservoir at temperature β^{-1}. Using the one-loop effective potential discussed briefly in ref.[8] and ref. [9], we will show that if $D \geq D_c(N-1)$, all the renormalized coupling constants of the truncated model are positive for any temperature (note that for reasons of stability in the tree level N must be even). For $D_c(N-1) > D \geq D_c(N)$ only the renormalized coupling constant $\lambda_{N-1}(\beta)$ becomes zero at some temperature β^{-1}_{N-1}. For $D_c(N) > D$ the renormalized coupling constants $\lambda_{N-1}(\beta)$ and $\lambda_N(\beta)$ become zero at the temperatures β^{-1}_{N-1} and β^{-1}_N respectivelly.

The outline of the paper is the following: in section II the effective potential is presented. In section III the thermal contribution to the renormalized mass and coupling constant are presented in the truncated model ($N = 4$). In section IV we repeat the calculations of the truncated model for ($N > 4$) and in the non-truncated model. Finally, we discuss some aplications of our results in curved spacetime and high order behavior of perturbation theory. Conclusions are given in section V. In this paper we use $\hbar = c = 1$.

2 The one-loop effective potential of the Efimov-Fradkin model at zero and finite temperature.

In this section we will generalize some results obtained in Ref.[8] and Ref.[9]. Supose a D-dimensional flat spacetime with trivial topology of the spacelike section and Bose fields in thermal equilibrium with a reservoir at temperature β^{-1}. Let us assume the following Lagrange density associated with a massive neutral scalar field.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \sum_{n!}^{\infty}\frac{\lambda_n}{n!}\varphi^n + counterterms. \tag{4}$$

Since the model is non-renormalizable, the counterterms have meaning only in the context of a finite number of loops. Note that we are not assuming inversion simmetry in the model, i.e. $V(\varphi_0) = V(-\varphi_0)$. If we assume it, the only surviving terms will be the even powers of the field. The time ordered products of the fields can be continued analytically to imaginary times and we define an Euclidean action integrating the analytic continuation to imaginary times in the Lagrange density. After a Wick rotation, defining the normalized expectation value of the field by $\varphi_0 = \frac{<0|\varphi|0>}{<0|0>}$, the zero temperature effective potential is given in the one-loop aproximation by

$$V(\varphi_0) = V_I(\varphi_0) + V_{II}(\varphi_0) \tag{5}$$

where:

$$V_I(\varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \sum_{n=3}^{\infty}\frac{\lambda_n}{n!}\varphi_0^n + counterterms, \tag{6}$$

and

$$V_{II}(\varphi_0) = \sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}\left(\sum_{n=3}^{\infty}\frac{1}{(n-2)!}\lambda_n\varphi_0^{n-2}\right)^s \int \frac{d^Dq}{(2\pi)^D}\frac{1}{(\omega^2+\vec{q}^{\,2}+m^2)^s}. \tag{7}$$

There is no difficulty to extend the above results assuming that the system is in thermal equilibrium with a reservoir at temperature β^{-1}. In the study of quantum fields at finite temperature two different approaches are currently used. The first one is the real time formalism in the canonical [15] or path integral approach [16]. The second one, is the Euclidian time formalism and will be used from now on in this paper. After a Wick rotation, the functional integral runs over the fields that satisfy periodic boundary conditions in Euclidian time. The effective action may be defined, as in the zero temperature case, by a functional Legendre transformation. Regularization and renormalization procedures follow the same steps taken in the zero temperature case, since temperature effects do not change the ultraviolet behavior of the model. Summing up, to study temperature effects in Bose fields we must perform the following replacements in the Euclidian region:

$$\int\frac{d\omega}{2\pi} \to \frac{1}{\beta}\sum_{n'} \tag{8}$$

and

$$\omega \to \omega_{n'} = \frac{2\pi n'}{\beta} \tag{9}$$

where $\omega_{n'} = \frac{2\pi n'}{\beta}$ are the Matsubara frequencies. Introducing a mass parameter μ and defining the dimensionless quantities,

$$c^2 = \frac{m^2}{4\pi^2\mu^2}, \tag{10}$$

$$(\beta\mu)^2 = a^{-1}, \tag{11}$$

and

$$k^i = \frac{q^i}{2\pi\mu}, \tag{12}$$

the Born terms plus the one-loop contributions to the effective potential are given by,

$$V(\beta,\varphi_0) = V_I(\varphi_0) + V_{II}(\beta,\varphi_0)$$

where

$$V_I(\beta,\varphi_0) = \frac{1}{2}m^2\varphi_0^2 + \sum_{n=3}^{\infty}\frac{\lambda_n}{n!}\varphi_0^n + counterterms, \tag{13}$$

and

$$V_{II}(\beta,\varphi_0) = \sqrt{a}\mu^D\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}\left(\sum_{n=3}^{\infty}\frac{\lambda_n\varphi_0^{n-2}}{4\pi^2\mu^2(n-2)!}\right)^s\sum_{n'=-\infty}^{\infty}\int d^dk\frac{1}{(an'^2+\vec{k}^2+c^2)^s}. \tag{14}$$

Owing to the discreteness of the Matsubara frequencies, an analytic regularization procedure will be used. Defining the inhomogeneous Epstein zeta function as

$$A_N^{c^2}(s,a_1,a_2,..,a_N)=\sum_{n_1,n_2..n_N=-\infty}^{\infty}(a_1n_1^2+a_2n_2^2+...+a_Nn_N^2+c^2)^{-s}, \tag{15}$$

we will see that its analytic continuation will be used to regularize the model. Before showing how this analytic continuation works and in order to simplify eq.(14), it is convenient to define g_n and ϕ as the new coupling constants and an adimensional (for $D=4$) vacuum expectation value of the field

$$g_n=\frac{\lambda_n}{4\pi^2\mu^{4-n}(n-2)!} \tag{16}$$

and

$$\frac{\varphi_0}{\mu}=\phi. \tag{17}$$

Substituting eq.(16) and eq.(17) in eq.(14) we obtain

$$V_{II}(\beta,\phi)=\mu^D\sqrt{a}\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{2s}\Big(\sum_{n=3}^{\infty}g_n\phi^{n-2}\Big)^s\sum_{n'=-\infty}^{\infty}\int d^dk\frac{1}{(an'^2+c^2+\vec{k}^2)^s}. \tag{18}$$

Since the spatial section of the spacetime is non compact, in order to deal with the divergences in the integral of eq.(18), we will first use dimensional regularization. From the well known formula,

$$\int\frac{d^dk}{(k^2+a^2)^s}=\frac{\pi^{\frac{d}{2}}}{\Gamma(s)}\Gamma(s-\frac{d}{2})\frac{1}{a^{2s-d}}, \tag{19}$$

and defining $f(D,s)$ as

$$f(D,s)=f(d+1,s)=\frac{(-1)^{s+1}}{2s}\pi^{\frac{d}{2}}\Gamma(s-\frac{d}{2})\frac{1}{\Gamma(s)} \tag{20}$$

it is possible to write $V_{II}(\beta,\phi)$ in terms of the inhomogeneous Epstein zeta function as:

$$V_{II}(\beta,\phi)=\mu^D\sqrt{a}\sum_{s=1}^{\infty}f(D,s)\Big(\sum_{n=3}^{\infty}g_n\phi^{n-2}\Big)^sA_1^{c^2}(s-\frac{d}{2},a). \tag{21}$$

The terms $s\le\frac{D}{2}$ are divergent which implies that the effective potential is not yet regularized. To complete the regularization, let us assume that each term in the series of the one-loop effective potential $V(\beta,\phi)$ is replaced by its analytic extension, defined at the begining in a open connected set of points of the complex plane s. Since we discussed carefully the process of the analytic continuation in the previous works, here we will only sketch this derivation. First, it is necessary to write eq.(21) in terms of the modified inhomogeneous Epstein zeta function as we did in the above citated works. For $Re(s)>\frac{N}{2}$, the modified inhomogeneous Epstein zeta function, $E_N^{c^2}(s,a_1,a_2,..a_N)$ converges and represent an analytic function of s, so $Re(s)>\frac{N}{2}$ is the largest possible convergence domain of the series. Then, using a Mellin transform it is possible to find its analytic continuation. After some calculations using the results of ref.[17] we rewrite eq.(21) as:

$$V_{II}(\beta,\phi)=\mu^D\sum_{s=1}^{\infty}\Big(\sum_{n=3}^{\infty}g_n\phi^{n-2}\Big)^s h(D,s)\Big(\frac{1}{2^{\frac{D}{2}-s+2}}\Gamma(s-\frac{D}{2})(\frac{m}{\mu})^{D-2s}+\sum_{n'=1}^{\infty}\Big(\frac{m}{\mu^2\beta n'}\Big)^{\frac{D}{2}-s}K_{\frac{D}{2}-s}(mn'\beta)\Big) \tag{22}$$

where:

$$h(D,s)=\frac{1}{2^{\frac{D}{2}-s-1}}\frac{1}{\pi^{\frac{D}{2}-2s}}\frac{(-1)^{s+1}}{s}\frac{1}{\Gamma(s)}. \tag{23}$$

Although eq.(22) is ill defined, in the one-loop aproximation it is possible to find the exact form of the counterterms in such a way that, mass and coupling constants (and consequently) the effective potential are finite quantities. To extract the singularities from the analytic extensions, let us define the mass squared as the value of the inverse propagator at zero momentum and the coupling constant λ_n as the proper n-point function at zero external momentum. In the next section, we will develop such idea in a very simple case: the truncated $(N=4)$ Efimov-Fradkin model and subsequently present the temperature dependent renormalized squared mass and coupling constants of the model.

3 The renormalized mass and coupling constants in the truncated $(N=4)$ Efimov-Fradkin model.

The goal of this section is to study how temperature effects leds to instabilities in scalar massive models. For the sake of simplicity and in order to obtain some insight about the thermal contribution to the renormalized mass and coupling constants in the non-truncated Efimov-Fradkin model, let us suppose the truncated model, i.e. $\lambda_n=0$ for $n>4$. We remark that the theory defined only with the term λ_3 is not consistent in any spacetime dimension since the energy is not bounded below, and so no ground state can exist in the interacting theory. The inclusion of λ_4 introduce a global minimum at the model. Defining

$$f(D,s)=\frac{1}{2^{D/2-s+2}}\Gamma(s-\frac{D}{2}),$$

it is possible to write $V_{II}(\beta,\phi)$ as

$$V_{II}(\beta,\phi)=\mu^D\sum_{s=1}^{\infty}\sum_{k=0}^{s}h(D,s)C_s^k g_3^{s-k}g_4^k\phi^{s+k}\Big(f(D,s)(\frac{m}{\mu})^{D-2s}+\sum_{n=1}^{\infty}\Big(\frac{m}{\mu^2\beta n}\Big)^{\frac{D}{2}-s}K_{\frac{D}{2}-s}(mn\beta)\Big). \tag{24}$$

In order to find the exact form of the counterterms that will render the model finite, let us consider the renormalization conditions for the non-truncated model

$$\frac{\partial^2}{\partial\phi^2}V(\beta,\phi)|_{\phi=0}=\mu^2m^2 \tag{25}$$

and

$$\frac{\partial^n}{\partial\phi^n}V(\beta,\phi)|_{\phi=0} = \mu^n\lambda_n, \qquad n = 3, 4, ... \quad . \tag{26}$$

We should point out that, strictly speaking, there is no need for wave function renormalization because the vacuum expectation value of the field has been chosen to be a constant. Using eq.(6), eq.(24), eq.(25) and eq.(26) it is possible to find the exact form of the countertems in such a way that they cancel the polar parts of the analytic extensions. In the neighbourhood of the poles, the regular part of the analytic extension of the inhomogeneous Epstein zeta function has two contributions: one which is independent of the temperature (this contribution can be absorbed in the counterterms), and another that depends on it. The thermal contribution to the renormalized coupling constant is proportional to the regular part of the analytic extension of the inhomogeneous Epstein zeta function in the neighborhood of some poles for the ultraviolet divergent graphs. Of course non-ultraviolet divergent graphs do not need to be regularized giving a finite thermal contribution to the renormalized quantities.

Note that we are choosing the renormalization conditions at $\varphi_0 = 0$. This may be done in the truncated model even if the minimum of the effective potential is not at $\varphi_0 = 0$, since the renormalization point is totally arbitrary. The values of the renormalized quantities obtained using $\varphi_0 = 0$ as the renormalization point are related to the corresponding quantities obtained in the true vacuum $\varphi_0 = a$ by the equations,

$$m^2|_{\varphi_0=a} = \frac{1}{2}m^2 + \frac{1}{2}\lambda_3 a + \frac{1}{4}\lambda_4 a^2, \tag{27}$$

$$\lambda_3|_{\varphi_0=a} = \frac{1}{3!}(\lambda_3 + \lambda_4 a), \tag{28}$$

and finally

$$\lambda_4|_{\varphi_0=a} = \lambda_4. \tag{29}$$

Analysing the sign of the thermal corrections to the renormalized physical parameters evaluated at $\phi = 0$, the sign of eq.(27) and eq.(29) does not change. This is expected since the metastable behavior and the existence of a global minimum cannot depend upon the choice of the renormalization point. For the case of the coupling constant $\lambda_3(\beta)$ with the restrictive condition $|a\lambda_4| < |\lambda_3|$, all the forthcoming conclusions also apply.

Let us call $\Delta m^2(D, \beta, m, \lambda_3, \lambda_4, \mu)$ and $\Delta\lambda_n(D, \beta, m, \lambda_3, \lambda_4, \mu)$, $n = 3, 4$ the thermal squared mass and thermal coupling constants respectivelly. In the following, in order to simplify the notation we keep explicitly only the β dependence of the renormalized quantities. Thus

$$m^2(\beta) = m^2 + \Delta m^2(\beta), \tag{30}$$

$$\lambda_n(\beta) = \lambda_n + \Delta\lambda_n(\beta) \quad n = 3, 4, \tag{31}$$

and for the sake of simplicity in the notation in the rest of this section, we call $\lambda_3(\beta) = \sigma(\beta)$ and $\lambda_4(\beta) = \lambda(\beta)$. From now on we will disregard the combinatorics factors in front of Feynman diagrams, since they are always real positive numbers, and cannot change qualitativelly the forthcomming results concerning the sign of the renormalized physical parameters. We are interested only in the connected 1 particle irreducible diagrams (1PI) which means that in the approximation we are making here, we have two graphs that contribute to the temperature dependent renormalized squared mass (see fig.(1)), the terms $s = 1, k = 1$ (of order λ) and $s = 2, k = 0$ (of order σ^2). It is not difficult to show that the case $s = 1, k = 1$ gives a positive contribution

$$\Delta m^2_{g_4}(\beta) - \Delta m^2_{g_4}(\infty) = \mu^{D-2} h(D,1) g_4 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta). \tag{32}$$

In the same way, the contribution from the term $s = 2, k = 0$ is negative and it is given by:

$$\Delta m^2_{g_3}(\beta) - \Delta m^2_{g_3}(\infty) = \mu^{D-2} h(D,2) g_3^2 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \tag{33}$$

Using general properties of the Bessel function $K_n(z)$ we obtain that the leading contribution comes from the graph given by fig (1.a), i.e. $\Delta m^2_{g_4}(\beta) - \Delta m^2_{g_4}(\infty) + \Delta m^2_{g_3}(\beta) - \Delta m^2_{g_3}(\infty) > 0$ and the thermal correction to the renormalized squared mass is always positive. Using the proper time method, Braden obtained the same expression for the thermal mass in the $\lambda\varphi^4$ model (see eq. 32). This author also discussed the two-loop correction to the mass and proved that the counterterms are temperature independent [18]. Note that the non-leading contribution coming from the graph of fig.(1.b) is negative, going in the direction of the vanishing of the mass. In other words, in the truncated model with only non-zero λ_3 coupling constant (disregarding the problem of the unboudedness of the effective potential even in the tree level aproximation), the thermal squared mass will becomes zero and negative at high temperatures. Various investigations have been made in theories with a cubic coupling. Gross, Perry and Yaffe [19] calculated the thermal mass of a graviton coupled with massless fermions in the one-loop aproximation. These authors founded that the thermal mass squared is negative the graviton develop an imaginary mass at some temperature. This lead the authors to conclude that the hot flat spacetime is unstable. The thermal graviton one-loop correction was also analysed by Kikuchi, Moriya and Tsukahara and and Holstein [20]. It was also shown that the thermal effects desestabilize the hot curved spacetime.

The situations where we are interested in discussing the ones when some renormalized coupling constant $\lambda_n(\beta)$ vanishes by temperature effects. As we discussed above, if $D < D_c(n)$ this situation can be realized. It is important to note that in this region the model is super-renormalizable, and when the fields are massless, perturbative expansion suffers from severe infrared divergences. Since the thermal squared mass is always positive and we are interested in high temperature regime, this problem does not afflict us i.e. infrared divergences never appear in our calculations at least in the one-loop aproximation. It should be noted that this fact does not occur in higher order-loop calculations. If we consider N self-energy insertions of $O(\lambda)$ (a ring correction) into a single loop, its contribution is infrared divergent in the case of the zero mass of the field. In other words, on the perturbative level the thermal mass generation does not prevent the appearance of infrared divergences in higher order-loop diagrams.

Let us now study the thermal contribution to the renormalized coupling constants. Initially for the thermal renormalized coupling constant $\sigma(\beta)$ we obtain,

$$\sigma(\beta) = \sigma(\infty) + \Delta\sigma_{g_3}(\beta) + \Delta\sigma_{g_3 g_4}(\beta). \tag{34}$$

As in the previous case it is necessary to study the cases $s = 2, k = 1$ and $s = 3, k = 0$ (see fig.(2)). In the case $s = 2, k = 1$ (of order $\sigma\lambda$ (see fig.(2.a)), it is not difficult to show that it gives a negative contribution

$$\Delta\sigma_{g_3 g_4}(\beta) - \Delta\sigma_{g_3 g_4}(\infty) = 2\mu^{D-3} h(D,2) g_3 g_4 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \quad (35)$$

For the second case, $s = 3, k = 0$ (of order σ^3, see fig.(2.b)) the contribution is positive,

$$\Delta\sigma_{g_3}(\beta) - \Delta\sigma_{g_3}(\infty) = \mu^{D-3} h(D,3) g_3^3 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-3} K_{\frac{D}{2}-3}(mn\beta). \quad (36)$$

The thermal correction to the renormalized coupling constant $\sigma(\beta)$ is given by,

$$[\Delta\sigma_{g_3 g_4}(\beta) - \Delta\sigma_{g_3 g_4}(\infty)] + [\Delta\sigma_{g_3}(\beta) - \Delta\sigma_{g_3}(\infty)]. \quad (37)$$

The term between the first brackets of eq.(37) dominates over the second one and the thermal correction to the renormalized coupling constant $\sigma(\beta)$ is negative. We have the interesting situation where the renormalized coupling constant $\sigma(\beta)$ attains its maximum at zero temperature ($\beta^{-1} = \infty$) and decreases monotonically as the temperature increases. In other words, the thermal contribution to the renormalized coupling constant $\Delta\sigma(\beta)$ is negative, and increases in modulus with the temperature. As we discussed in the previous sections, for $D < D_c(n)$ the coupling constant may be a large quantity. From eq.(2), it is not difficult to show that $D_c(n) = \frac{2n}{n-2}$. See fig.(3). Since the thermal contribuition to the renormalized coupling constant is negative there is a temperature β_3^{-1} where $\sigma(\beta)$ vanishes if $D < 6$. Above this temperature $\sigma(\beta)$ becomes negative. As we will see, even if $D < 4$, there is a finite range of temperatures where $\lambda(\beta)$ is still positive. The thermal contribution to the renormalized coupling constant $\lambda(\beta)$ also can be calculated. The complete expression for $\lambda(\beta)$ is

$$\lambda(\beta) = \lambda(\infty) + \Delta\lambda_{g_3}(\beta) + \Delta\lambda_{g_3 g_4}(\beta) + \Delta\lambda_{g_4}(\beta). \quad (38)$$

As in the previous case we need to study the graphs $s = 2, k = 2$, $s = 3, k = 1$ and $s = 4, k = 0$. See fig.(4). For the first case $s = 2, k = 2$ (of order λ^2), we get a negative contribution (see fig.(4.a)),

$$\Delta\lambda_{g_4}(\beta) - \Delta\lambda_{g_4}(\infty) = \mu^{D-4} h(D,2) g_4^2 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \quad (39)$$

For the case $s = 3, k = 1$ (of order $\sigma^2\lambda$, fig.(4.b)) we obtain a positive contribution

$$\Delta\lambda_{g_3 g_4}(\beta) - \Delta\lambda_{g_3 g_4}(\infty) = 3\mu^{D-4} h(D,3) g_3^2 g_4 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-3} K_{\frac{D}{2}-3}(mn\beta). \quad (40)$$

Finally, in the last case, $s = 4, k = 0$ (of order σ^4, fig.(4.c)) we obtain a negative contribution given by

$$\Delta\lambda_{g_3}(\beta) - \Delta\lambda_{g_3}(\infty) = \mu^{D-4} h(D,4) g_3^4 \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta})^{\frac{D}{2}-4} K_{\frac{D}{2}-4}(mn\beta). \quad (41)$$

The thermal correction to the renormalized coupling constant $\lambda(\beta)$ is the sum of the contributions from the three graphs of fig.(4) which gives

$$[\Delta\lambda_{g_3}(\beta) - \Delta\lambda_{g_3}(\infty)] + [\Delta\lambda_{g_3 g_4}(\beta) - \Delta\lambda_{g_3 g_4}(\infty)] + [\Delta\lambda_{g_4}(\beta) - \Delta\lambda_{g_4}(\infty)]. \tag{42}$$

The term between the last brackets in eq.(42) dominates over the others and since its contribution is negative the thermal correction to the renormalized coupling constant $\lambda(\beta)$ is negative. The important conclusion from the above discussion is the following: the critical dimension for λ is $D = 4$, which implies that if we take $D < 4$ there is a temperature such that $\lambda(\beta)$ becomes zero. Let us call β_4^{-1} this temperature. If the system is heated above this temperature β_4^{-1} the renormalized coupling constant $\lambda(\beta)$ becomes negative. Note that we have two different temperatures (for $D < 4$) where $\sigma(\beta)$ and $\lambda(\beta)$ vanish. First $\sigma(\beta)$ becomes zero at β_3^{-1} (where $\lambda(\beta)$ is still positive) and after at β_4^{-1} the renormalized coupling constant $\lambda(\beta)$ becomes zero (where $\sigma(\beta)$ is negative). For temperatures $\beta^{-1} > \beta_4^{-1}$ the system can develop a first order phase transition with decay of a false vacuum [21].

Finally, the effective potential as a function of the temperature and the vacuum expectation value of the field for $D < 4$, $m^2 = \mu = \lambda = 1$ may be ploted in a "toy" model. The temperature is the parameter that allows us to interpolate between the two configurations: a stable vacuum at low temperatures and a metastable state at temperatures $\beta^{-1} > \beta_4^{-1}$, (see.fig.(5)). In the next section we will repeat the calculations that we have done in this section to the truncated $(N > 4)$ and also in the non-truncated model.

4 The renormalized mass and coupling constants in the truncated $(N > 4)$ and the non-truncated models.

In this section we will suppose a general truncated model i.e. $\lambda_n = 0$ for $n > N > 4$. Since we intend to disregard at the tree level the problem of the unboundedness of the energy density, we assume that N is an even integer. The calculations are now formally identical to the previous ones. The only difference is the richness coming from the distints graphs contributing to the thermal renormalized coupling constants. For reasons which will become clear latter we will study two different situations

(i) $D < D_c(N-1)$

(ii)$D \geq D_c(N-1)$.

For $D < D_c(N-1)$ let us investigate the thermal renormalized coupling constants $\lambda_{N-2}(\beta), \lambda_{N-1}(\beta)$ and $\lambda_N(\beta)$ separatelly. We must analise the leading diagrams giving contributions to the renormalized coupling constants $\lambda_{N-2}(\beta)$, $\lambda_{N-1}(\beta)$ and $\lambda_N(\beta)$. In this case it is not difficult to show that there is a positive contribution to the renormalized coupling constant $\lambda_{N-2}(\beta)$ given by the graph $s = 1$ in fig.(6). This is because the leading contribution comes from the graph with the smaller value of s. An example of nonleading contributions are those given by the graphs in fig.(7). The leading thermal contribution give us

$$\lambda_{N-2}(g_N,\beta) - \lambda_{N-2}(g_N,\infty) \cong \mu^{D-N+2} h(D,1) g_N \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-1} K_{\frac{D}{2}-1}(mn\beta). \qquad (43)$$

The result above can be generalized to the others coupling constants $\lambda_3, ...\lambda_{N-3}$. Thus, the renormalized coupling constants $\lambda_3, ..\lambda_{N-2}$ are always positive, for any D spacetime dimension. The situation changes in the case of the coupling constant $\lambda_{N-1}(\beta)$. In this case the leading graphs are given in fig.(8). The thermal contribution from these graphs to the renormalized coupling constant λ_{N-1} is given by

$$\begin{aligned} \lambda_{N-1}(g_N, g_{N-1}.., g_3, \beta) &- \lambda_{N-1}(g_N, g_{N-1}.., g_3, \infty) \cong \qquad (44) \\ &\cong \mu^{D-N+1} h(D,2)(g_N g_3 + g_{N-1} g_4 + .) \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta) \end{aligned}$$

which is a negative expression, implying that for $D < D_c(N-1)$ it must have a temperature where $\lambda_{N-1}(\beta)$ vanishes. Finally, for the coupling constant $\lambda_N(\beta)$ the leading graphs are given by fig.(9). A straighforward calculation gives for the thermal contribution to the renormalized coupling constant λ_N the value

$$\begin{aligned} \lambda_N(g_N, g_{N-1}.., g_3, \beta) &- \lambda_N(g_N, g_{N-1}.., g_3, \infty) = \qquad (45) \\ &= \mu^{D-N} h(D,2)(g_N g_4 + g_{N-1} g_5 + ..) \sum_{n=1}^{\infty} (\frac{m}{\mu^2 \beta n})^{\frac{D}{2}-2} K_{\frac{D}{2}-2}(mn\beta). \end{aligned}$$

As in the previous case the coupling constant λ_N also becomes zero at the temperature β_N^{-1} if $D < D_c(N)$. ¿From the same arguments related to the critical dimension of each coupling constant, for $D \geq D_c(N-1)$ all the renormalized coupling constants are positive for any temperature. A very interesting situation is the case where $D_c(N) \leq D < D_c(N-1)$. Although the coupling constant λ_{N-1} becomes negative above the temperature β_{N-1}^{-1}, the effective potential has a global minimum, since the renormalized coupling constant $\lambda_N(\beta)$ is positive for any temperature. In this case the ground state of the model is stable. Note that we are using the renormalization conditions at $\phi = 0$. Imposing only even powers of the field in eq.(1) all the above conclusions apply. Including odd powers of the field, the global minimum of the effective potential is not at $\phi = 0$. Let us suppose that the minimum occurs at some value $\phi = a$. It is possible to show that the results concerning the sign of the renormalized coupling constant $\lambda_N(\beta)$ and squared mass do not change. From a physical point of view this could not be otherwise, since the critical behavior of the system and the existence or not of vacuum decay should not be affected by a change of the renormalization point. Summing up, in the truncated model we have tunelling between different vacua if $D < D_c(N)$ where $D_c(N)$ is the critical dimension of λ_N.

The above discussions can be sumarized as follows. In a massive scalar super-renormalizable model at finite temperature, there is a temperature β_N^{-1} such that the renormalizable coupling constant $\lambda_N(\beta)$ becomes zero. Above such temperature there is tunelling between different vacua.

Particularly important is the connection between our investigations and instantons solutions as we have discused in the end of section (3), for the case $N = 4$. It is well known that in $D = 4$ the massive $\lambda\varphi^4$ model does not admit real instantons solutions. In the massless case also there are no real instantons solutions (with positive action), nevertheless a complex instanton solution (with negative action) is known [22]. The instanton solution is related to the fact that the renormalized coupling constant λ is negative. It is not difficult to see the connection between the mechanism studied by us and the possible existence of instantons, since in our case the renormalized coupling constant may become negative as the temperature changes. In the simplest case of massive $\lambda\varphi^4$ and $D < 4$ instantons could exist in the model for $\beta^{-1} > \beta_4^{-1}$.

As noted a long time ago by Dyson, in QED, for negative coupling constant e^2, the Hamiltonian is unbounded bellow and the vacuum is a metastable state. In this situation, particles and antiparticles would repel each other increasing the distance between them and pairs of particle and antiparticle would be continually created. In the vacuum energy (the sum of all connected diagrams having n vertices and no external legs) appears an imaginary part. As it was noted originally by Bender and Wu studying the quantum anharmonic oscillator, there is a relation between the nth Rayleigh-Schrodinger coefficient and the lifetime of the unstable states of a negativelly coupled anharmonic oscillator [24]. The idea was used also in Field Theory by Parisi and others [25][26][27]. Asymptotic estimates in perturbation theory can be obtained by computing the imaginary part of the Green's functions for small negative coupling constant. More recently Fainberg and Iofa also calculed the high order corrections to the instantons contribution to the Green's functions in the regime $\lambda < 0$ [28].

The effects we have described in this paper may also be applied to cosmological phase transition problems. The study of phase transition in cosmological models has been widely discussed in the literature. For a complete review see [29] and other references therein. It is shown that in the evolution of the universe, metastable vacuum states may appear. The decay of such metastable states is materialized in the Lorentzian spacetime as nucleation of a bubble of true vacuum in the false vacuum phase. Frequently, in the study of the false vacuum decay, it is assumed that the system is "prepared" in a metastable state. Such metastable states appear naturally in our formalism by temperature effects that change the sign of renormalized coupling constants. For instance, as we have seen before, in the truncated model ($N = 4$) for $4 \leq D < 6$, the coefficient of the φ^3 term becomes negative above the temperature β_3^{-1}. This is a natural realization of the potential studied by Gleiser et al [30] and Vilenkin and Ford [31]. If we assume that the universe expands and supercools, the possibility of the creation of bubbles of true vacuum arises nevertheless there are subtleties in this process. Back to Lorentzian time, let us define $\Delta\tau$ as the time necessary to the temperature of the enviroment to drop down to β_4^{-1}, where the vacuum state becomes stable. On the other side, if the mean life of the metastable state Δt is larger than $\Delta\tau$ there is no nucleation of the bubbles at all. Only if $\Delta t < \Delta\tau$, there would be a finite probability of nucleation of bubbles.

In the case of "real" cosmological evolution it is necessary to include gravity, non-trivial problems may appear, as for example the possibility of presence of horizont. For a carefull analysis of these situations, see [32]. We cannot disregard the possibility that particle creation associated with the tunneling process will destroy the above scenario.

Particle creation which occurs in the process of nucleation of bubbles was analized by Rubakov [33]. We still do not know how to introduce these effects in our model. The discussion of tunnelling effects, instantons and how they contribute to high order estimates in perturbation theory will be presented in a forthcomming paper [34]

5 Conclusion

The purpose of this paper has been to discuss the effect of keeping local terms with higher powers of the field in the Lagrange density of a neutral scalar field. We also assume that the system is in thermal equilibrium with a reservoir at temperature β^{-1}. We proved that in the truncated Efimov-Fradkin model:

(i) for $D \geq D_c(N-1)$ there is not a temperature where at least one of the coupling constants becomes zero,

(ii) for $D_c(N-1) > D \geq D_c(N)$ There exist a temperature β_{N-1}^{-1} where only the renormalized coupling constant $\lambda_{N-1}(\beta)$ becomes zero and all the others renormalized coupling constants remain positive,

(iii) for $D < D_c(N)$ the coupling constants $\lambda_{N-1}(\beta)$ and $\lambda_N(\beta)$ become zero at some temperatures β_{N-1}^{-1} and β_N^{-1} respectivelly.

It is clear that in the non-truncated case, all the renormalized coupling constants remain positive for $D \geq 2$. We would like to point out that some care must be taken in order to not extrapolate the results of this paper to regions outside the domain of validity of the aproximation we have done, i.e. beyond one-loop level. As we discussed in the previous section, a natural extension of the ideas of the paper is to include gravitation, although there are some subtleness related with this approach. The techniques of the paper with the Euclidian path approach can only be implemented in some special cases (for example Schwarzschild or de Sitter spacetime), i.e. to continue analytically to Euclidian space the metric must have a section in the complexified spacetime on which the metric is real and positive definite. In spacetime metrics where this properties works all the calculations can be repeated of course with the subtleness of the curved metric.

6 Acknowlegement

We would like to acknowledge Prof.S.W.MacDowell, Prof.A.Grib, Prof.C.A.Aragao de Carvalho and Dr.B.F.Svaiter for many helpful discussions. Also we are grateful to H.Levcovitz and S.Joras by the help in the graphs and in the draws. This paper was suported by Conselho Nacional de Desenvolvimento Cientifico e Tecnologico do Brazil (CNPq).

Fig.(1) - The two graphs that contributes to the temperature dependent renormalized mass m^2. Note that they are ultraviolet divergent for $D = 4$.

Fig.(2) - The two graphs that contributes to the renormalized coupling constant $\sigma(\beta)$. Note that only the first one is ultraviolet divergent for $D = 4$.

Fig.(3) - The critical dimension as a function of n for each coupling constant λ_n.

Fig.(4) - The three graphs that contributes to the renormalized coupling constant $\lambda(\beta)$. Again, only the first is ultraviolet divergent in $D = 4$.

Fig.(5) - The effective potential as a function of the vacuum expectation value of the field and the inverse of the temperature. For low temperatures it has a global minimum and for temperature $\beta^{-1} > \beta_{g_4}^{-1}$, the potential has a metastable vacuum.

Fig.(6) - The graph that gives the leading contribution to the renormalized coupling constant $\lambda_{N-2}(\beta)$.

Fig.(7) - The graphs that gives non-leading contributions to the renormalized coupling constant $\lambda_{N-2}(\beta)$.

Fig.(8) - The graphs that gives the leading contribution to the renormalized coupling constant $\lambda_{N-1}(\beta)$.

Fig.(9) - The graphs that give leading contributions to the renormalized coupling constant $\lambda_N(\beta)$.

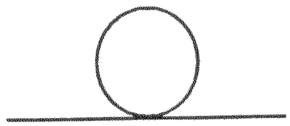


Fig.(1.a)

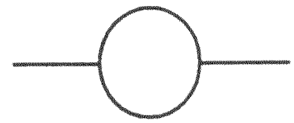
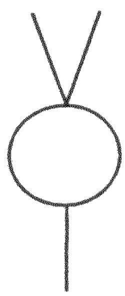


Fig.(1.b)



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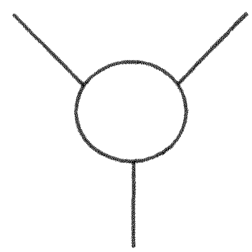

Fig.(2.b)

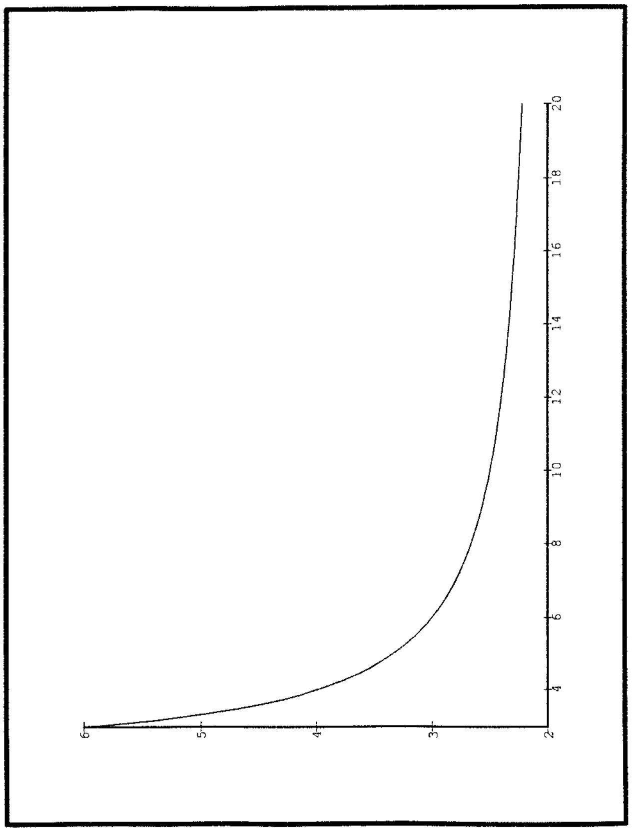

Fig. 3

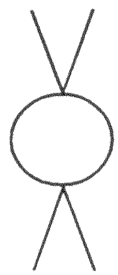

Fig.(4.a)

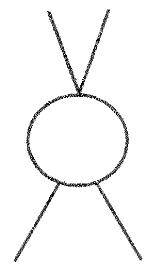

Fig.(4.b)

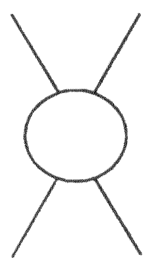

Fig.(4.c)

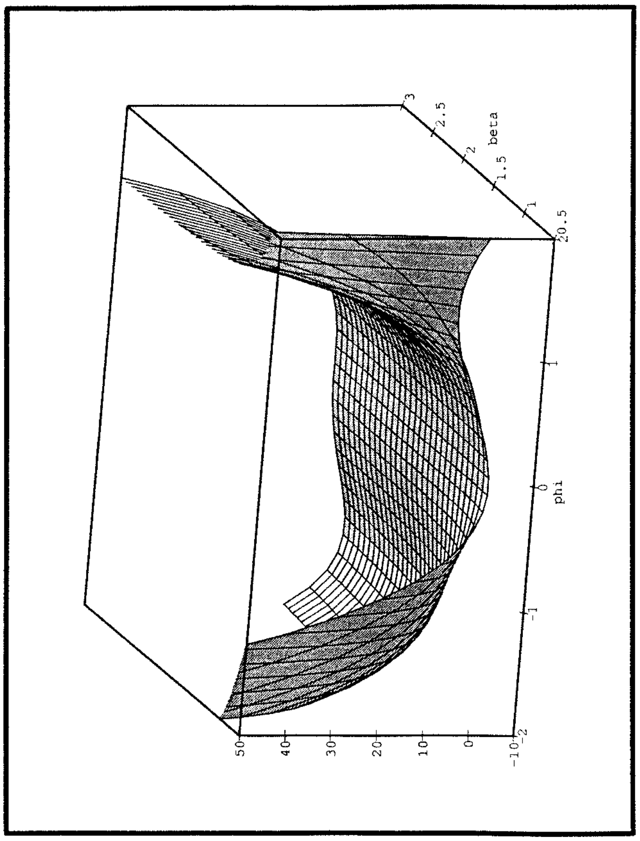

Fig. 5

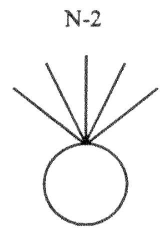

Fig.(6)

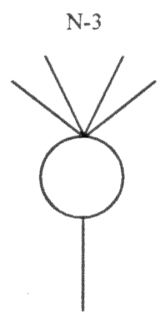

Fig.(7 .a)

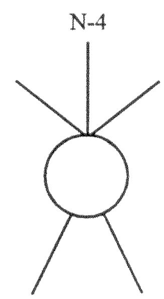

Fig.(7 b)

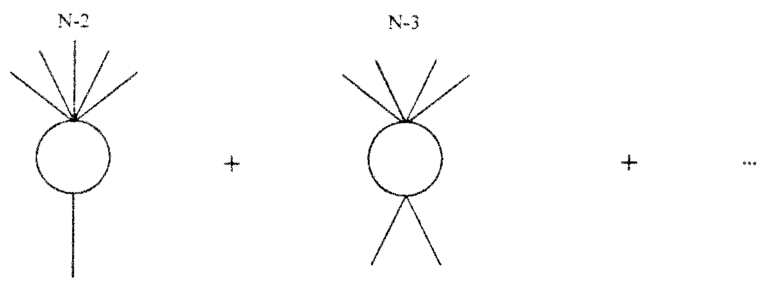

Fig.(8)

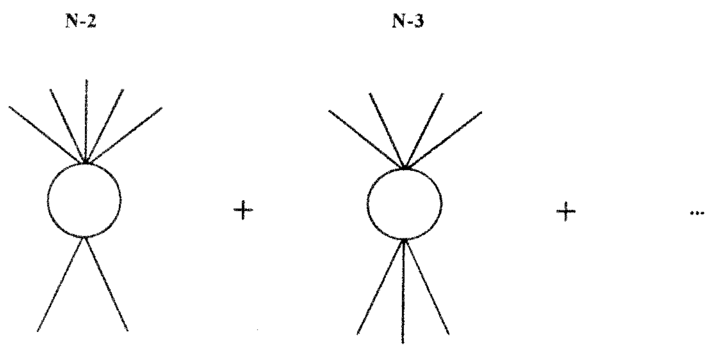

Fig.(9)

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