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New Concepts in Particle Physics from Solution of an Old Problem

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Abstract

Recent ideas on modular localization in local quantum physics are used to clarify the relation between on- and off-shell quantities in particle physics; in particular the relation between on-shell crossing symmetry and off-shell Einstein causality. Among the collateral results of this new non-perturbative approach are profound relations between crossing symmetry of particle physics and Hawking-Unruh like thermal aspects (KMS property, entropy attached to horizons) of quantum matter behind causal horizons which hitherto were more related with Killing horizons in curved spacetime than with localization aspects in Minkowski space particle physics. The scope of this framework is wide and ranges from providing a conceptual basis for the d=1+1 bootstrap-formfactor program for factorizable d=1+1 models to a decomposition theory of QFT's in terms of a finite collection of unitarily equivalent chiral conformal theories placed a specified relative position within a common Hilbert space (in d=1+1 a holographic relation and in higher dimensions more like a scanning). Although different from string theory, some of its concepts originated as string theory in the aftermath of the ill-fated S-matrix bootstrap approch of the 60^{ies}. Some remarks on the relation to string theory can be found at the end.

1 Introduction

Theoretical physicists, contrary to mathematicians, rarely return to their old unsolved problems; they rather prefer to replace them by new inventions. The content of the present article on some new concepts in particle physics is an exception. The old problems it addresses and partially solves are those of the relation between off-shell and on-shell quantities (or between fields-particles) and in particular of crossing symmetry in local quantum physics (LQP)[1]. These structures also led to the invention of the dual model and string theory.

The most prominent of on-shell quantities is the S-matrix of a local QFT, whereas fields and more general operators describing localized situations, and in the algebraic setting belonging to local subalgebras, are "off-shell". In this paper we will have to consider a new kind of operators which, as a result of their weak semiinfinite (wedge-like) localization and their close relation to the S-matrix, are to be considered as on-shell. This on-shell operators are essential for our new approach which avoids pointlike fields at the beginning and starts with generators of wedge-localized algebra. The off-shell compactly localized operators and local field generators are then obtained via intersections of wedge algebras. Here and in the sequel the word localization region always stand for the causal completion of a spacetime region; these are the regions which one obtains by intersecting wedges.

Besides these two extremes there are intermediate possibilities where on-shell and off-shell aspects appear together. The most prominent and useful mixed objects are bilinear forms on scattering vector states i.e. matrix elements of local operators A (either pointlike fields or bounded operators localized in smaller than wedge regions) taken between incoming and outgoing multiparticle scattering states (in terms of Feynman graphs, one leg is off-shell).

$$^{out}\left\langle q_1,...q_{n-1},q_n\left|A\right|p_n,p_{n-1},...p_1\right\rangle^{in} \tag{1}$$

which we will call (generalized) formfactors. These objects fulfil the important crossing symmetry which acts on the on-shell momenta.

The S-matrix whose matrix elements result from the previous formula for $A=1$, is *the* observable of particle physics par excellence: it is totally intrinsic and independent of field coordinatizations, although strictly speaking only (inclusive) cross sections are directly measured; a fact which is especially important if interactions between zero mass particles leads to infrared problems. However most of our physical intuition about causality and charge flows in spacetime is based on (off-shell) local fields or local observables, the new on-shell fields are somewhat more hidden and in particular not obtainable by Lagrangian or more generally by any kind of quantization approach.

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[1] We will often use the name "local quantum physics" (LQP) instead of QFT [1], if we have in mind the physical principles of QFT implemented by different concepts than those of the various quantization formalisms (canonical, quantization via path integrals etc.) which most of the readers are familiar with from the various textbooks. To the extend that the reader does not automatically identify QFT with those formalisms, he may without danger of misunderstandings continue to use the good old name QFT.

The old problems on which there has been significant recent progress which will be presented in the sequel can be compressed in terms of the following questions

- Does a physically admissable S-matrix fulfilling unitarity, crossing symmetry and certain analytic properties needed in its formulation, have an underlying unique local QFT? This one may call the *inverse problem of QFT associated with scattering.* It is a problem of principle interest to take notice not only of the well-known fact that fields and local observable lead to scattering, but that also local equivalence classes of fields[2] or nets of local observables are in turn determined by particle scattering data.

- Is there a constructive procedure in which, similar to the d=1+1 bootstrap-formfactor program for factorizing d=1+1 models (which in fact reappears as a special case), the S-matrix and the generalized formfactors enter as important constructive elements in order to obtain off-shell objects as fields or local observables? In particular can one formulate such a constructive approach in a conceptually intrinsic manner i.e. without any quantization parallelism to classical field theory and without the use of field coordinatizations and short-distance divergence problems? This could be of tremendous practical importance.

Remark 1 *The most profound on-shell property which was discovered in the 60^{ies} is crossing symmetry. It is in a way deeper than TCP-symmetry, the symmetry derived from causality which among other things requires the existence of an antiparticle to each particle. In fact it is a kind of individual TCP-transformation which effects only one particle at a time within the multiparticle incoming ket configuration and carries it to the outgoing bra configuration as an antiparticle. In spite of its name it is not a quantum theoretical (Wigner) symmetry, since that crossing process involves an on-shell analytic continuation. For a formfactor we have*

$$\begin{aligned} & {}^{out}\left\langle q_1,...q_{n-1},q_n\left|A\right|p_n,p_{n-1},...p_1\right\rangle^{in} \\ = \quad & {}^{out}\left\langle -\bar{p}_1,q_1,...q_{n-1},q_n\left|A\right|p_n,p_{n-1},...p_2\right\rangle^{in} \end{aligned} \tag{2}$$

where the analytic continuation is carried out in the rapidity parametrization by an $i\pi$-shift: $\theta \rightarrow \theta + i\pi$, and the bar denotes the antiparticle. The difficulties in physical interpretation and conceptual placement of this relation (about which rigorous information outside of perturbation theory of sufficient generality are scarce) reflects the lack of its understanding. Although its meaning remains vague, most physicist think that it should be viewed as a kind of on-shell "shadow" of Einstein causality. One of the results of the new conceptual framework presented here is a an interpretation of the crossing property in terms of "wedge localization" and the ensuing thermal Hawking-Unruh properties which are usually associated exclusively with black hole quantum physics, but in fact turn out to be general properties of any local quantum description including of particle physics in Minkowski space.

[2] That an S-matrix cannot determine individual fields had been known since the late 50^{ies}.

After having sketched our physical motivation, we now briefly present our main mathematical tool which will be used for investigating physical problems of (quantum) localization: (Tomita's) modular theory of von Neumann algebras in standard position. These concepts, which for the first time clarified the on/off-shell relation and in particular the spacetime interpretation of on-shell crossing symmetry (its relation with off-shell Einstein causality [2]), were not available at the time of the S-matrix bootstrap of the 60^{ies} nor at the time of the invention of the dual model by Veneziano. The latter is mentioned here because it represents the first comprehensive attempt to incorporate the on-shell S-matrix ideas, including the property of crossing symmetry (in the stronger dual version), into a particle physics model. Although at the end the model did not increase the conceptual understanding of crossing symmetry, it was widely used in particle phenomenology and later led to the old string theory.

Chew's pure S-matrix approach based on unitarity and crossing symmetry, which tried to "cleanse" all off-shell notions from particle physics, slowly became the first failed attempt[3] at a "theory of everything" (except gravity) in the setting of quantum theory. The dual model on the other hand soon developed into (the old) string theory as a kind of off-shell extension of the S-matrix. Although here we are not dealing with string theory, we find it nevertheless interesting to emphasize the common roots through the implementation of crossing symmetry. Apart from the historical aspects, there is also some similarity in the results, even though the physical concepts and the mathematical formalisms are totally different. In the last section we will return to these comparisons.

The present line of research directly addresses the mentioned old unsolved problems with new physical concepts and mathematical tools. The main new mathematical tool is briefly described in the sequel; a more detailed description can be found in [3]. Since not only the mathematical formalism but also the concepts themselves may be somewhat unfamiliarand I would like to reach also particle physicist beyond a very small circle of specialists, I will try use the rest of this introduction for a panoramic and semi-rigorous presentation (including remarks on the history of the subject).

Definition 2 *A von Neumann algebra $\mathcal{A}$ (weakly closed operator sub-algebra of the full algebra $B(H)$ on a Hilbert space H) is in "standard" position" with respect to a vector $\Omega \in H$, denoted as $(\mathcal{A}, \Omega)$, if Ω is a cyclic and separating vector for $\mathcal{A}$. In this situation Tomita defines the following involutive antilinear but unbounded operator (the Tomita involution S)*

$$SA\Omega := A^*\Omega \tag{3}$$

where the star operation is the hermitian conjugate in operator algebras. Its closability property (as physicists we use the same notation for the closure) is the prerequisite for the polar decomposition

$$S = J\Delta^{\frac{1}{2}} \tag{4}$$

[3] In retrospect it appears certainly the most successful among all failed theories in this century. Many useful concepts as e.g. Weinberg's ideas on effective interactions resulted from reading on-shell S-matrix properties back into off-shell QFT. It left an impressive imprint on particle physics and its demise is more related to its ideological reservations against QFT than with false concepts.

where the angular part J (the modular involution) is antiunitary with $J^2 = 1$ and Δ is unbounded positive and therefore leads to a unitary group Δ^{it}.

Theorem 3 *(Tomita 1965, with significant improvements from Takesaki): The modular involution maps $\mathcal{A}$ on its von Neumann commutant $\mathcal{A}'$ in H:*

$$AdJ \cdot \mathcal{A} = \mathcal{A}' \tag{5}$$

The unitary Δ^{it} defines a "modular" automorphism group by

$$Ad\Delta^{it} \cdot \mathcal{A} = \mathcal{A} \tag{6}$$

(analogy to a dynamical law for the algebra).

The awe of an unprepared physicist in front of such a powerful and nontrivial mathematical theorem [4] is somewhat mitigated by the remark that three physicist (Haag, Hugenholtz and Winnink) were led to a closely related independent discovery in their pursuit of conceptual problems in quantum statistical mechanics which arise if one works directly in the thermodynamic limit [1]. As it is well known, the Gibbs representation formula

$$\begin{aligned} \langle A_V \rangle_\beta^{(V)} &= \frac{tre^{-\beta H_V} A_V}{tre^{-\beta H_V}} \\ A_V &\in algebra\ of\ box-quantization \end{aligned} \tag{7}$$

ceases to make sense[4] for infinite volume, although the weak convergence i.e. the convergence in the sense of states on algebras (instead of state vectors in Hilbert space) is secured under very mild assumptions [1]. The three named authors found out that the intrinsic GNS-construction, i.e. the canonical construction of a cyclic representation $\pi(\mathcal{A})$ in a Hilbert space H, and a reference vector $\Omega \in H$ with

$$\langle A \rangle_\beta = (\Omega_\beta, \pi(A)\Omega_\beta) \tag{8}$$

elevates the so-called KMS-condition[5] to a very fundamental attribute of a thermal state on an algebra $\mathcal{A}$. This KMS property then merged with Tomita's modular theory and in this form e.g. entered Connes classification of type III von Neumann algebras and characterization of certain invariance properties of foleii of states on a C*-algebra. In the 70^{ies} Haag and collaborators were able to derive the KMS condition directly from stability properties under local deformations and Pusz and Woronowicz found a direct link to the second law of thermodynamics [1]. These ideas were recently used for the derivation of thermal properties of quantum matter in an anti-de Sitter spacetime [5].

The relation with the Einstein causality of observables and locality of fields in QFT was made around 1975 in a series of papers by Bisognano and Wichmann [1]. Specializing to wedge algebras $\mathcal{A}(W)$ generated by Wightman fields, they proved the following theorem

[4] In a box the bounded below hamiltonian aquires a discrete spectrum and $e^{-\beta H}$ is of trace class ($\Omega_\beta = e^{-\frac{1}{2}\beta H}$ is H.S.), a property which is lost in the infinite volume limit.

[5] This analytic property was used by Kubo, Martin and Schwinger as a trick which substitutes the calculation of Gibbs traces with some easier analytic boundary problem.

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[5] This analytic property was used by Kubo, Martin and Schwinger as a trick which substitutes the calculation of Gibbs traces with some easier analytic boundary problem.

Theorem 4 *The Tomita modular theory for the wedge algebra and the vacuum state vector* $(A(W), \Omega)$ *yields the following physical identifications*

$$\begin{aligned} \Delta^{it} &= U(\Lambda_W(2\pi t)) \\ J &= TCP \cdot U(R_x(\pi)) \end{aligned} \tag{9}$$

Here Λ_W denotes the boost which leaves the wedge W invariant. If we choose the standard t-x wedge then the rotation which aligns the TCP with Tomita's J is a rotation around the x-axis by an angle π.

Now I come to my own contributions which are of a more recent vintage [9]. They result from the desire to invert the Bisognano-Wichmann theorem i.e. to use Tomita's modular theory for the actual construction (and classification) of (a net of) wedge algebras belonging to interacting theories with the final goal to intersect of wedge algebras in order to obtain a net of compactly localized double cone algebras. For the arguments which show that the particle physics properties, in particular the scattering matrix and formfactors of distinguished fields (conserved currents) can be abstracted from the net observables, I refer to [1] [22][19]. The nets can also be coordinatized by more traditional pointlike fields and a rigorous derivation for chiral nets can be found in [16]. For the derivation of LSZ scattering theory one makes the same assumption as in the old-fashioned Kramers-Kronig dispersion approach to particle physics, namely the existence of a mass gap. With this one immediately realizes that, whereas the connected part of the Poincaré group is the same as that of the free incoming theory, the disconnected part containing time reversals and in particular the modular involution J carry the full interaction

$$\begin{aligned} \Delta_W^{it} &= \Delta_{W,in}^{it} =: e^{-iKt} \\ J_W &= S_{sc} J_{W,in} \end{aligned} \tag{10}$$

Here $J_{W,in}$ refers to the Tomita involution (or TCP reflection) of the wedge algebra generated by the incoming field. If the theory is not asymptotically complete (i.e. the vacuum is not cyclic with respect to th incoming fields) these relations have to be modified, but here we discard such pathologies for which no physical illustration exists. Since we do not want to temper with historically grown notations, we have added a subscript to the S-matrix S_{sc} in order to distinguish it from Tomita's S. The modular "Hamiltonian" K defined in the first equation (the boost generator= Hamiltonian of a particular uniformely accelerated observer) has always symmetric instead of one-sided spectrum.

The last relation (10) is nothing but the TCP-transformation law of the S-matrix rewritten in terms of modular objects associated to the wedge algebra. The above role of the S-matrix as a kind of relative modular invariant of the wedge algebra (relative to the free one) is totally characteristic for *local* quantum physics and has no counterpart in quantum mechanics. It is this "semilocal" (referring to the semiinfinite extension of wedge regions) new aspect of the invariant S-matrix which together with the standard global scattering aspect (which is also present in nonrelativistic particle scattering) which, as we will see below, opens the gateway to a new realm of particle physics different from the various quantization approaches and the QFT-formalisms of the text books, such as interaction picture, time-ordered functions, euclidean functional integrals etc.

In order to achieve this, one needs one more concept which has no counterpart in the "old" quantum field theory, i.e. with is totally hidden from quantization. This is the existence of polarization-free generators ("PFG's") of the wedge algebra. This is deeply related to the characteristic vacuum structure of QFT, which was first observed in the old days by Heisenberg, Euler, Weisskopf and many others. Their observations rephrased in the modern LQP conceptual framework and adapted to one-particle state vectors suggests that any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles, unless the system is free i.e. the operator algebra is generated by a free field which is linear in the creation/annihilation operator. More specifically it leads to the impossibility of having a local generation of pure one-particle vectors unless the system is interaction-free. In this respect the situation can be viewed as a generalization of the Jost-Schroer theorem for pointlike fields (see Streater-Wightman [6]). In fact, as I learned from Detlev Buchholz, the smallest region for which the proof of this No Go theorem against interactions breaks down is the wedge region! The existenc of the PFG's shows that not only its proof, but also theorem itself ceises to be true. Indeed the wedge-localized algebras can be shown to support nontrivial PFG operators whose existence generates in a rich physical content [9].

Definition 5 *Let $T(W)$ be the space of Schwartz test functions with support in the wedge W. We call a set of operators F labelled by $\hat{f} \in T(W)$ which create one particle state vectors (in an interacting theory) without polarization contributions:*

$$F(\hat{f})\Omega = 1 - particle\ vector \tag{11}$$

polarization free (wedge) generators or PFG's.

It is easy to see that as a result of this definition, which forces the "one-F-state-vector" (11) to lie on the mass shell, and the requirement that the F's act cyclically and separating, i.e. without nontrivial annihilators of the vacuum in $alg\left\{F(\hat{f}); supp\hat{f} \subset W\right\}$, the F-operators themselves must lie on the mass shell and hence:

$$\begin{aligned} F(\hat{f}) &= \int \hat{f}(x)F(x)d^{d+1}x,\ supp\hat{f} \subset W \\ F(x) &= \int \left(Z(p)e^{-ipx} + \bar{Z}^*(p)e^{ipx}\right)\frac{d^dp}{2\omega},\ x \in W \\ Z(p)\Omega &= 0 = \bar{Z}(p)\Omega \\ Z^*(p)\Omega &= \Psi(p),\ \ \bar{Z}^*(p)\Omega = \bar{\Psi}(p) \end{aligned} \tag{12}$$

This mass-shell property is the only property which the $F(x)$ share with free fields. Here the bar on the operators and the one-particle state vectors Ψ corresponds to antiparticles. The field theoretic notation $F(x)$ should be handled with great care because unlike for pointlike fields, the x is not the position of a spacetime localization but only a label on which Poincaré transformations act in such a way that the generating family for W is taken into one for the Poincaré transformed wedge. *It is the constructive use of such nonlocal*[6] *objects which is responsible for the disappearance of the ultraviolet divergency problem (and together with it the short-distance aspects of the renormalization problem).* This problem shows up in the standard perturbative approach if one introduces interactions via multiplying local fields at the same point (the meaning of which can only a posteriory be defined and changes with the model).

Such ideas of an approach to QFT which avoids the off-shell ultraviolett problems are not entirely new. Ignoring the afore-mentioned failed pure S-matrix attempt, there is the very successful but annoyingly special bootstrap-formfactor program which in its present form is limited to d=1+1 factorizing (a terminology which refers to the S-matrix) models. It achieves its success precisely by avoiding the various quantization approaches (canonical, functional integral) which are inexorably tight up with the standard ultraviolett short distance problems[7]. But these d=1+1 bootstrap-formfactor recipes had a very weak conceptual basis (related to the speciality of the situation which is described by these recipes). The new approach presented here is a vast generalization of the concepts including those behind these recipes. In a way it is a bit surprizing that the provocative challenging message of this more than 20 year old low-dimensional successful construction program has not been noticed and utilized before; it is hard to imagine that this could have happened in an earlier time when the relation between formalism and conceptual understanding was more balanced. Perhaps this is due to the fact that most particle physicist consider QFT as a basically settled issue with only nasty technical problems remaining. We will demonstrate in this paper that such a view is quite unwarranted.

Since according to the previous remarks PFG's do not exist (nontrivially) for smaller localization regions $\mathcal{O} \subset W$, and since they are physically uninteresting for regions larger than W, we omit the abbreviation for the region W from the PFG terminology.

PFG's generalize the free field structure in the presence of interactions into a controllable nonlocal direction which however always remains at the service of local theories, i.e. they are auxiliary quantities in the Hilbert space of the "would be"[8] local theory and coexist together with the operators of increasing locality obtained by "quantum localization" via intersections of wedge algebras. With other words, although they are nonlocal (semilocal in the sense of the noncompact wedge localization) and therfore in some sense contain a cut-off aspect, these properties are not introduced ad hoc, but of deep physical origin,

[6]Note that "nonlocal" does not refer to the theory (as in the case of momentum space cutt-offs) but rather to extended operators in a would be local theory whose locality becomes exposed only after computing intersections of wedge algebras.

[7]In fact outside the context of QM and certain superrenormalizable QFT's for which constructivists were able to show a good mathematical meaning, these quantization procedures are "artistic" since by the time one arrived at the physical results (the renormalized fields), the initial quantization requirements fail: no renormalized correlation function fulfils a euclidean functional representation. The algebraic approach was introduces in order to balance this remarkable but somewhat unfortunate situation.

[8]The construction of the wedge algebras does not yet solve the existence problem of models of local quantum physics, but one still needs to show the nontriviality ($\neq C1$) of the double cone algebras obtained by intersecting sufficiently many wedge algebras..

and as a consequence no limiting process for cutoff removal is required. Their existence is crucial for the linkage of the particle physics crossing symmetry with the thermal and entropical aspects of modular localization QFT. Although they were first noticed in form of the Bekenstein-Hawking-Unruh thermal properties of Killing horizons in black hole physics, they are not belonging exclusively to black hole physics but are rather part of general particle physics with or without curved spacetime. In other words these well-known "classical" thermal properties in CST have a quantum counterpart in which bifurcated Killing horizons are substituted by surfaces of *causally completed* Minkowski space *localization regions* e.g. the light cone surface of a double cone. The geometric Killing symmetry in the quantum setting passes to the (geometrically) hidden quantum symmetry defined by the modular group corresponding to the concrete situation. In Unruh's case of a wedge region or in the analogous case of conformal matter enclosed in a double cone, the quantum symmetry is equal to the one described by a Killing vector associated with the Lorentz- or conformal- group.

We have organized this paper as follows. The next section reviews and illustrates the field-coordinate-free approach for interaction-free theories and in d=1+1 factorizing model. In the latter case the Zamolodchikov-Faddeev algebra emerges in a natural way (without having been put in) and the Z-F operators for the first time acquire a spacetime interpretation in connection with the new PFG generators of wedge algebras. The presentation of polarization-free wedge generators is extended to systems which are not factorizing (i.e. theories with on-shell (real) particle creation) in section 3.

After a brief introduction of the AQFT framework in section 4, the fifth section treats the light ray/front restriction and holography in terms of associated chiral conformal field theories. There we also discuss the problem of undoing such maps (the "blow up" property) in terms of scanning a higher dimensional QFT by a finite family of chiral conformal theories. The mathematical technology used in this section is one of the most powerful which AQFT presently is able to offer (the theory of modular inclusions and intersections).

In the sixth section we take up the problem of associating entropy for localized matter. The previous association of chiral conformal theories to two- and higher dimensional models offers the identification the relevant degrees of freedom (e.g. those which contibute to the entropy) with those of the associated simpler chiral theories. In the same section we also review Rehren's presentation [7] of the AdS-conformal field theory isomorphism which is a more special and simpler kind of holography (no modular inclusions and intersections are needed) which happens through a conformal theory attached at the boundary at infinity rather than at the light cone. In contradistinction to the holography/scanning approach connected to a horizon (light ray/front holography) in section 6, this Maldacena-Witten AdS_{d+1} holography is correspondence between two equally unknown and difficult (apart from d=1) theories. Although this makes it less useful for a constructive approach in (CST) QFT, this isomorphism, as pointed out by Rehren, serves as an illustration par excellence for the necessity and the power of the field coordinate-free concept of AQFT.

The last section finally tries to confront our approach with string theory and explain (unfortunately without much success) why the two lead to similararities. Actually the differences, especially those on the issue of the still elusive quantum gravity, are probably more important than the similarities. The enigmatic power of these differences may in the long run turn out to determine the future path of particle physics.

This presentation is a survey of published [9][11][34] [26][2][15] as well as of new results. I plan to defer most rigorous proofs of new statements (i.e. to the extend that they have not appeared in the mentioned papers) to a separate publication with a more mathematical content. We appologize to the experts for some repetitions, but we think that to shed some light on a difficult subject from slightly different angles may actually be helpful for nonexperts.

2 Systems without Interactions and Factorizing Models

In trying to bring readers with a good knowledge of standard QFT in contact with some new (and old) concepts in algebraic QFT (AQFT) without sending him back with a load of homework, I face a difficult problem. Let us for the time-being put aside the intrinsic logic, which would ask for a systematic presentation of the general framework, and let us instead try to maneuver in a more less ad hoc (occasionally even muddled) way.

In a pedestrian approach the problem of constructing nets of interaction free systems from Wigner's one particle theory may serve as a nice pedagogical exercise. Since Wigner's representation theory (we only need irreducible *positive energy representations*) was the first totally intrinsic quantum theory without any quantization parallelism to classical particle theory, it is reasonable to expect in general that, if we find the right concepts, we should be able to avoid covariant pointlike fields alltogether in favor of a more intrinsic way to implement the causality/locality principle. Rather the local fields should be similar to coordinatizations of local observables in analogy to differential geometry. This viewpoint is indeed consistent and helpful [9][10]. By using a spatial variant of Tomita's theory for the wedge situation (i.e. by defining a kind of Tomita S on the Wigner representation space without a von Neumann algebra), one obtains a real closed subspace $H_R(W)$ of the Wigner space H of complex multi-component momentum space wave functions as a say $+1$ eigenspace of a Tomita-like quantum mechanical operator s in H, where s is defined to be the $i\pi$ continued boost (obtained by the functional calculus associated with the spectral theory) multiplied by the one-particle version of the j-reflection. For this one only needs to extend the Wigner representation to include the disconnected Poincaré transformations (reflections with determinant one) which together with the analytically continued boost yields an unbounded antilinear involution on the quantum mechanical Wigner space [9][10]. The substitute for the von Neumann commutant in this spatial case is the symplectic (or real orthogonal) complement of $H_R(W)$ in H. It turns out that this situation is "standard" in a spatial sense

$$H_R(W) + iH_R(W) \text{ is dense in } H \tag{13}$$
$$H_R(W) \cap iH_R(W) = \{0\}$$

As in the algebraic case, the modular formalism characterizes the localization of subspaces, but is not able to distinguish individual elements in that subspace (particular covariant x-space wave functions or testing functions with particular support properties inside an x-space wedge). There is a good physical reason for that because as soon as one tries to do that, one is forced to leave the unique Wigner (m,s) representation framework and pick a particular representation by selecting one specific intertwiner among the infinite set of u and v intertwiners which link the unique Wigner (m,s) representation to the countably infinite many covariant possibilities [9]. With other words, one is in the framework explained and presented in the first volume of Weinberg's book [8]. Any selection of a specific covariant description, vis. by invoking Euler-Lagrange equations and the existence of a Lagrangian, may be convenient for doing computations or as a mnemotechnical device for classifying polynomial interaction densities[9], but is not demanded as an intrinsic attribute of physics. In the above spatial modular manner, the uniqueness of the (m, s) Wigner theory can be transported (via application of the Weyl resp. CAR functor) directly to the QFT and results in uniqueness of the local net which is obtained by intersections from the more coarse-grained wedge net. If we would have taken the conventional route via interwiners and local fields as in [8], then we would have been forced to use Borchers construction of equivalence classes in order to see that the different free fields associated with the (m,s) representation with the same momentum space creation and annihilation operators in Fock space are just different generators of the same coherent families of local algebras i.e. yield the same net. This would be analogous to working with particular coordinates in differential geometry and then proving at the end that the objects of interests are invariant and therefore independent of coordinates.

On the mathematical side we meet for the first time the "modular machine" which is capable to encode informations about spacetime geometry into the more technical looking domain properties of operators. This is achieved by those strange antilinear Tomita involutions S which are unbounded and which create via their domain properties a host of antiunutary mirror transformations and modular automorphisms with (sometimes only partial) manifest geometric meaning.

It is amusing that this spatial modular formalism in Wigner space also preempts the particle statistics by producing a mismatch in the case of half-integer spin between the real symplectic (orthogonal) complement and the result of a geometric π-rotation of the wedge into its opposite. The functorial way of associating modular localized subalgebras with real subspaces of Wigner space only uses (exponentiated Weyl-like in case of integer spin) momentum space creation and annihilation operators related to the

[9]The causal approach permits the transformation of a polynomial interaction from one coordinatization to any other whereas a formalism using classical actions involving free field Lagrangians $\mathcal{L}_0$ is restricted to the use of Euler-Lagrange field coordinatizations.

Fockspace extension by forming tensor products of Wigner spaces; nowhere one is forced to use individual pointlike fields.

Of course we cannot use these nets in order to replace those interaction densities to be used in a Stückelberg-Bogoliubov causal perturbation theory; this standard perturbative approach only works with pointlike fields. Proofs that the same physics could have been obtained in terms of different free field coordinatizations (rewriting the interaction polynomials) tend to be quite involved.

The implementation of interactions in the framework of nets requires a radical rethinking of the formalism, even if we are only interested in perturbative aspects of the nets. In order to get a clue, let us first ask a less general question. It is well-known that there exists a special class of theories in d=1+1 which are factorizing in the sense of the multiparticle S-matrix (which commutes with the incoming number operator $\mathbb{N}_{in}$)

$$[S_{sc}, \mathbb{N}_{in}] = 0 \tag{14}$$

Let us try to implement the idea of a relativistic particle pair interaction with the simple Ansatz (assuming a situation of selfconjugate particles) in on-shell rapidity variables

$$\begin{aligned} Z(\theta)Z(\theta') &= S(\theta-\theta')Z(\theta')Z(\theta) \\ Z(\theta)Z^*(\theta') &= S^{-1}(\theta-\theta')Z^*(\theta')Z(\theta) + \delta(\theta-\theta') \end{aligned} \tag{15}$$

with the star-structure determining the remaining commutation relations and the unitarity of S with $S^{-1}(\theta) = \bar{S}(\theta) = S(-\theta)$[10] etc. Together with $Z(\theta)\Omega = 0$ we can express all correlation functions of the would be PFG's $F(\hat{f})$'s in terms of S's and the computation of correlation functions proceeds as for free fields namely by commuting the annihilation operators Z to the right vacuum e.g.

$$(\Omega, Z(\theta_4)Z(\theta_3)Z^*(\theta_2)Z^*(\theta_1)\Omega) = S(\theta_2-\theta_3)\delta(\theta_3-\theta_1)\delta(\theta_4-\theta_2) + ... \tag{16}$$

where the dots stand for pure δ-function contributions without S. One easily sees that we could also have started from the following formula which represents $Z^*Z^*\Omega$ state vectors in terms of corresponding free field terms

$$Z^*(\theta_2)Z^*(\theta_1)\Omega = \chi_{21}a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{12}S(\theta_2-\theta_1)a^*(\theta_1)a^*(\theta_2)\Omega \tag{17}$$

Here the symbol $\chi_{P(1)...P(n)}$ denotes the characteristic function of the region $\theta_{P(1)} > ... > \theta_{P(n)}$. It is easy to see that the S-dependent terms in the inner product together with

$$\begin{aligned} \{\chi_{12}S(\theta_2-\theta_1) + \chi_{21}\bar{S}(\theta_3-\theta_4)\} &\quad \delta(\theta_3-\theta_1)\delta(\theta_4-\theta_2) \\ = S(\theta_2-\theta_1) &\quad \delta(\theta_3-\theta_1)\delta(\theta_4-\theta_2) \end{aligned} \tag{18}$$

[10] The modular interpretation requires the S in (15) to be identified with the two-particle S-matrix. Consistency forces the time-dependent scattering theory for local fields or operators from double cone localized algebras (wedge localization is not sufficient for the derivation of scattering theory) to reproduce this factorizing S-matrix.

lead to an ordering free result which agrees with (16) In fact if we had started with a more general two-particle interaction Ansatz by allowing the structure of the second equation in (15) to be different $S^{-1} \to T$, the consistency with (17) would immediatly bring us back to $T = S^{-1}$.

The formula for the 4-point function suggests the possibility to replace the algebraic Ansatz by the following formula for multi-Z^* state vectors

$$\begin{aligned} & Z^*(\theta_n)...Z^*(\theta_1)\Omega \\ = & \sum_{perm} \chi_{P(n)...P(1)}(\prod_{transp} S)a^*(\theta_{P(n)})...a^*(\theta_{P(1)})\Omega \end{aligned} \tag{19}$$

where the product of S-factors in the bracket contains one S for each transposition. The associativity of the $Z's$ i.e. the Yang-Baxter relation for matrix-valued $S's$ insures the consistency of the formula. We call $\theta_{P(1)} > ... > \theta_{P(n)}$ the natural order of the multi-Z^* state vector. From the state characterization (19) one can derive the algebraic definition. The generalizations of PFG's beyond factorizing systems in the next section are done on the level of the Z^*-state formulae with a suitable substitute for the bracket in (19) instead of (15).

The F's in (12) are most conveniently written in terms of rapidity variables using the following more appropriate path notation

$$F(\hat{f}) = \frac{1}{\sqrt{2\pi}} \int_C Z(\theta) f(\theta), \; supp\hat{f} \in W \tag{20}$$

where C is a path consisting of the upper/lower rim of a $i\pi$-strip with the real θ-axis being the upper boundary. Whereas the on-shell value of the Fourier transform $f(\theta)$ of $\hat{f}$ is analytic in this strip, the relation $Z(\theta - i\pi) := Z^*(\theta)$ is an abbreviation (since operators by themselves are never analytic in spacetime labels!) which however inside expectation values becomes coherent in notation with their meromorphic properties.

We want to show that the F's are PFG's and a for a proof (as a result of modular theory) we only have to check the KMS property for the F-correlation functions with the modular generator being the infinitesimal boost K. The fact that in contrast to the one-sided spectrum of the Hamiltonians in the Gibbs formula, the spectrum of the boost K is two-sided is encouraging. The desired KMS-property for the wedge reads

$$\left\langle F(\hat{f}_n)...F(\hat{f}_1)\right\rangle = \left\langle F(\hat{f}_{n-1})...F(\hat{f}_2)F(\hat{f}_n^{2\pi i})\right\rangle \tag{21}$$

where the superscript $2\pi i$ indicates the imaginary rapidity translation from the lower to the upper rim of the KMS strip.

A rather straightforward calculation based on the previously explained rules for the Z's yields the following result

Theorem 6 *([9][34]) the KMS-thermal aspect of the wedge algebra generated by the PFG's is equivalent* *to the crossing symmetry of the S-matrix*

$$\mathcal{A}(W) := alg\left\{F(\hat{f}); supp\hat{f} \in W\right\} \Leftrightarrow S(\theta) = S(i\pi - \theta)$$

Furthermore the possible crossing symmetric poles in the physical strip of S will be converted into intermediate composite particle states in the GNS Hilbertspace associated with the state defined by the correlations on the $\mathcal{A}(W)$-algebra. The latter commutes with its geometric opposite $\mathcal{A}(W^{opp})$ in case of $\mathcal{A}(W^{opp}) = \mathcal{A}(W)' = AdJ\mathcal{A}(W)$. A sufficient condition for this is the existence of a parity transformation whose action on $\mathcal{A}(W)$ equals the commutant $\mathcal{A}(W)'$.

We sketch the proof for the 4-point function of $F's$ which may be obtained as the scalar product of two-particle state vectors (*c.t.* denotes the F-contaction terms)

$$F(\hat{f}_2)F(\hat{f}_1)\Omega = \int\int f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)Z^*(\theta_1)Z^*(\theta_2)\Omega + c.t. \tag{22}$$

$$\begin{aligned} = & \int\int f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)\{\chi_{12}a^*(\theta_1)a^*(\theta_2)\Omega + \\ & +\chi_{21}S(\theta_2 - \theta_1)a^*(\theta_2)a^*(\theta_1)\Omega\} + c\Omega \end{aligned} \tag{23}$$

and the analogous formula for the bra-vector. The formula needs some explanation. The symbol χ with the permutation subscript denotes as before the characteristic function associated with the permuted rapidity order. The order for the free creation operators a^* is governed by particle statistics. For each transposition starting from the natural order (19) one obtains an S factor[11]. The Yang-Baxter relation assures that the various ways of doing this are consistent. For the inner product the S-dependent terms are. Finally the terms proportional to the vacuum are contraction terms corresponding to the δ-function in (15). For the S-dependent terms in the inner product we obtain

$$\begin{aligned} & \int\int f_4(\theta_2)f_3(\theta_1)\{\chi_{21}S(\theta_2 - \theta_1) + \chi_{12}\bar{S}(\theta_1 - \theta_2)\}f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)d\theta_1 d\theta_2 \\ = & \int\int f_4(\theta_2)f_4(\theta_1)S(\theta_2 - \theta_1)f_2(\theta_2 - i\pi)f_1(\theta_1 - i\pi)d\theta_1 d\theta_2 \end{aligned} \tag{24}$$

The analogous computation for KMS crossed term in (29) gives

$$\int\int f_{2'}(\theta_1)f_2(\theta_2)S(\theta_1 - \theta_2)f_1(\theta_1 - i\pi)f_{1'}(\theta_2 - i\pi + 2\pi i)d\theta_1 d\theta_2 \tag{25}$$

This formula makes only sense if the $F(f)$ operators are restricted in such a way that the $2\pi i$ translation on them is well-defined, i.e. for wave functions f which are analytic in a strip of size $2\pi i$. It is well-known that the KMS condition does not hold on all operators of the algebra but rather on a dense set of suitably

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[11] The notation has used the statistics in order to bring the product of incoming fields a_{in} into the natural order say 1...n. The ordering of the $Z's$ encodes the θ-ordering and not the particle statistics. It is connected with the different boundary values of state vectors and expectation values in θ- space in approaching the physical boundary from the analytic region. This is analogous to the association of the n! n-point x-space correlation functions with different boundary values of one analytic "master function" in the Wightman theory.

defined analytic elements [12]. The S-independent terms which we have not written down are identical to terms in the 4-point function of free fields They separately satisfy the KMS property. What remains is to show the identity of (24) and (25). This is done by $\theta_2 \to \theta_2 - i\pi$ contour-shift in (25) without picking up terms frm infinity. Using the denseness of the wave funcions one finally obtains

$$S(\theta_2 - \theta_1) = S(\theta_1 - \theta_2 + i\pi) \tag{26}$$

which is the famous crossing symmetry or the $z \longleftrightarrow -z$ reflection symmetry around the point $z_0 = \frac{1}{2}i\pi$.

In physical terms we may say that the wedge structure of factorizing models is that of a kind of relativistic quantum mechanic. This continues to be true if the crossing symmetric S-matrix has poles in the physical strip. In that case the above contour shift would violate the KMS property unless one modifies the multi-Z^* state vector formula (19) by the inclusion of bound states. For the case n=2 (17) this means

$$\begin{aligned} Z^*(\theta_2)Z^*(\theta_1)\Omega &= (Z^*(\theta_2)Z^*(\theta_1)\Omega)^{scat} + \\ &+ |\theta, b\rangle \langle \theta, b\, | Z^*(\theta - i\theta_b) Z^*(\theta + i\theta_b) |\, \Omega\rangle \end{aligned} \tag{27}$$

The bracket with the superscript scat denotes the previous contribution (17), whereas the second line denotes the bound state contribution. The validity of the KMS property demands the presence of this term and determines the coefficient; here θ_b is the imaginary rapidity related to the bound state mass. For a detailed treatment which includes the bound state problem we refer to a forthcoming paper. We emphasize again that it is the representation of the F-correlations in terms of the S-matrix and the KMS property of these correlation functions, which via the GNS construction converts the poles in the (possibly matrix-valued) function S into the extension of the Fock space of the $a's$ by additional free field operators. In this way the poles in numerical functions are converted into the enlargment of Fock space in such a way that a few $Z's$ can describe many more particles. One may call the Z to be "fundamental" and the introduce new Z_b and $F_b's$; the latter will however be operators which are already associated with the original F-algebra. What needs an extension is the wedge algebra of incoming fields. It is very important to note that this quantum mechanical picture is converted into LQP *with vacuum polarization as soon as we e.g. go to double cone localization*, this will be shown in the sequel. The extension of the above proof beyond 4-point functions is left to the reader.

The KMS computation can be immediately extended to "formfactors" i.e. mixed correlation functions containing in addition to F's one generic operator $A \in \mathcal{A}(W)$ so that the previous calculation results from the specialization $A = 1$. This is so because the connected parts of the mixed correlation function is related to the various (n, m) formfactors (1) obtained by the different ways of distributing n+m particles in and out states. These formfactors are described by different boundary values of one analytic master function which is in turn related to the various forward/backward on shell values which appear in one mixed A-F correlation function. We may start from the correlation function with one A to the left and say n F's to the right and write the KMS condition as

$$\left\langle AF(\hat{f}_n)...F(\hat{f}_2)F(\hat{f}_1)\right\rangle = \left\langle F(\hat{f}_1^{2\pi i})AF(\hat{f}_n)...F(\hat{f}_2)\right\rangle \tag{28}$$

The n-fold application of the F's to the vacuum on the left hand side creates besides an n-particle term involving n operators Z^* to the vacuum (or KMS reference state vector) Ω also contributions from a lower number of $Z^{*\prime}s$ together with $Z - Z^*$ contractions. As with free fields, the n-particle contribution can be isolated by Wick-ordering[12]

$$\left\langle A : F(\hat{f}_n)...F(\hat{f}_2)F(\hat{f}_1) :\right\rangle = \left\langle F(\hat{f}_1^{2\pi i})A : F(\hat{f}_n)...F(\hat{f}_2) :\right\rangle \tag{29}$$

Rewritten in terms of A-formfactors the n-particle scattering contribution (using the denseness of the $f(\theta)$) reads as

$$\begin{aligned} &\langle \Omega, AZ^*(\theta_n)...Z^*(\theta_2)Z^*(\theta_1 - 2\pi i)\Omega\rangle \\ = &\langle \Omega, Z(\theta_1 + i\pi)AZ^*(\theta_n)...Z^*(\theta_2)\Omega\rangle \\ = &\langle Z^*(\theta_1 - i\pi)\Omega, AZ^*(\theta_n)...Z^*(\theta_2)Z^*(\theta)\Omega\rangle \end{aligned} \tag{30}$$

Here the notation suffers from the usual sloppiness of physicist: the analytic continuation by $2\pi i$ refers to the correlation function and not to the operators. For the natural order of rapidities $\theta_n > .. > \theta_1$ this yields the following crossing relation

$$\begin{aligned} &\langle \Omega, Aa_{in}^*(\theta_n)...a_{in}^*(\theta_2)a_{in}^*(\theta_1 - \pi i)\Omega\rangle \\ = &\langle a_{out}^*(\theta_1)\Omega, Aa_{in}^*(\theta_n)...a_{in}^*(\theta_2)\Omega\rangle \end{aligned} \tag{31}$$

The out scattering notation on the bra-vectors becomes only relevant upon iteration of the KMS condition since the bra $Z's$ have the opposite natural order. The above KMS relation (29) contains additional information about bound states and scattering states with a lower number of particles. The generalization to the case of antiparticles≠particles is straightforward. More generally we see that the connected part of the mixed matrix elements

$$\langle a_{out}^*(\theta_k)...a_{out}^*(\theta_1)\Omega, Aa_{in}^*(\theta_n)...a_{in}^*(\theta_{k-1})\Omega\rangle \tag{32}$$

is related to $\langle \Omega, AZ^*(\theta_n)...Z^*(\theta_2)Z^*(\theta_1)\Omega\rangle$ by analytic continuation which a posteriori justifies the use of the name formfactors in connection with the mixed A-F correlation functions.

The upshot of this is that such an A must be of the form

$$A = \sum \frac{1}{n!}\int_C ... \int_C a_n(\theta_1, ...\theta_n) : Z(\theta_1)...Z(\theta_n) : \tag{33}$$

where the a_n have a simple relation to the various formfactors of A (including bound states) whose different in-out distributions of momenta correspond to the different contributions to the integral from

[12]Note that as a result of the commutation relation (15), the change of order within the Wick-ordered products will produce rapidity dependent factors

the upper/lower rim of the strip bounded by C, which are related by crossing. The transcription of the a_n coefficient functions into physical formfactors (32) complicates the notation, since in the presence of bound states there is a larger number of Fock space particle creation operators than PFG wedge generators F. It is comforting to know that the wedge generators despite their lack of vacuum polarization clouds nevertheless contain the full (bound state) particle content. The wedge algebra structure for factorizing models is like a relativistic QM, but as soon as one sharpens the localization beyond wedge localization the field theoretic vacuum structure will destroy this simple picture and replace it with the appearance of the characteristic virtual particle structure which separates local quantum physics from quantum mechanics.

In order to see by what mechanism the quantum mechanical picture is lost in the next step of localization, let us consider the construction of the double cone algebras as a relative commutants of of shifted wedge (shiftes by a inside the standard wedge)

$$\begin{aligned} \mathcal{A}(C_a) &: = \mathcal{A}(W_a)' \cap \mathcal{A}(W) \\ C_a &= W_a^{opp} \cap W \end{aligned} \tag{34}$$

For $A \in \mathcal{A}(C_a) \subset \mathcal{A}(W)$ and $F_a(\hat{f}_i) \in \mathcal{A}(W_a) \subset \mathcal{A}(W)$ the KMS condition for the W-localization reads as before except that whenever a $F_a(\hat{f}_i)$ is crossed to the left side of A we may commute it back to the right side since $\left[\mathcal{A}(C_a), F_a(\hat{f}_i)\right] = 0$. The resulting relations e.g.

$$\begin{aligned} &\left\langle AF_a(\hat{f}_1) : F_a(\hat{f}_n)...F_a(\hat{f}_2) :\right\rangle \\ = &\left\langle A : F_a(\hat{f}_n)...F_a(\hat{f}_2)F_a(\hat{f}_1^{2\pi i}) :\right\rangle \end{aligned} \tag{35}$$

Note that the $F_a(\hat{f}_1)$ in the first line is outside the Wick-ordering. Since it does neither act on the bra nor the ket vacuum, it contains both frequency parts. The creation part can be combined with the other F's under one common Wick-ordering whereas the annihilation part via contraction with one of the Wick-ordered F's will give an expectation value of one A with $(n-2)$ F's. Using the density of the f's and going to rapidity space we obtain ([11]) the so-called kinematical pole relation

$$Res_{\theta_{12}=i\pi} \left\langle AZ^*(\theta_n)...Z^*(\theta_2)Z^*(\theta_1)\right\rangle = 2i\mathbf{C}_{12} \left\langle AZ^*(\theta_n)...Z^*(\theta_3)\right\rangle (1 - S_{1n}...S_{13}) \tag{36}$$

Here the product of two-particle S-matrices results from commuting the $Z(\theta_1)$ to the right so that it stands to the left of $Z^*(\theta_2)$ whereas the charge congugation matrix $\mathbf{C}$ only appears if we relax our assumption of selfcongugacy. This relation appears for the first time in Smirnov's axiomatic approach [13] as one of his recipes and more recently was derived as a consequence of the LSZ formalism adapted to the factorizing model situation [14]. In the present approach it has an apparently very different origin: it is together with the Z-F algebra structure a consequence of the wedge localization of the generators $F(\hat{f})$ and the sharpened double cone locality (34) of A. The existence problem for the QFT associated with an admissable S-matrix (unitary, crossing symmetric, with correct physical residua at one-particle poles) of a factorizing theory is the nontriviality of the relative commutant algebra i.e. $\mathcal{A}(C_a) \neq \mathbf{C} \cdot 1$. Intuitively the operators in double cone algebras are expected to behave similar to pointlike fields applied to the vacuum; namely one expects the full interacting polarization cloud structure. For the case at hand this is in fact a consequence of the above kinematical pole formula since this leads to a recursion which for nontrivial two-particle S-matrices is inconsistent with a finite number of terms in (33) unless the operator A is a composite of a free field.

The determination of a relative commutant or an intersection of wedge algebras is even in the context of factorizing models not an easy matter. We expect that the use of the following "holographic" structure significantly simplifies this problem. We first perform a lightlike translation of the wedge into itself by letting it slide along the upper light ray by the amount given by the lightlike vector a_+. We obtain an inclusion of algebras and an associated relative commutant

$$\mathcal{A}(W_{a_\pm}) \subset \mathcal{A}(W) \qquad (37)$$
$$\mathcal{A}(W_{a_\pm})' \cap \mathcal{A}(W)$$

The intuitive picture is that the relative commutant lives on the $a_\pm$ part of the upper/lower light ray, since this is the only region inside W which is spacelike to the interior of the respective shifted wedges. This relative commutant subalgebra is a lightray part of the above double cone algebra, and it is easier to handle. One only has to take a generic operator in the wedge algebra which formally can be written as a power series (33) in the generators and [9][34] find those operators which commute with the shifted F's

$$[A, U(e_+)F(f)U^*(e_+)] = 0 \qquad (38)$$

Since the shifted F's are linear expressions in the Z's, the nth order polynomial contribution to the commutator comes from only two adjacent terms in A namely from a_{n+1} and a_{n-1} which correspond to the annihilation/creation term in F. The size of the shift gives rise to a Paley-Wiener behavior in imaginary direction, whereas the relation between a_{n+1} and a_{n-1} is identical to (36), so we do not learn anything new beyond what was already observed with the KMS technique (35). However as will be explained in section 5, the net obtained from the algebra

$$\mathcal{A}_\pm := \cup_{a_\pm} \mathcal{A}(C_{a\pm}) \qquad (39)$$

is a chiral conformal net on the respective subspace $H_\pm = \overline{\mathcal{A}_\pm \Omega}$. If our initial algebra were conformal d=1+1, the total space would factorize $H = H_+ \bar{\otimes} H_- = \overline{(\mathcal{A}_+ \bar{\otimes} \mathcal{A}_-)\,\Omega}$, and we would recover the well-known fact that two-dimensional local theory factorizes into the two light ray theories. If the theory is massive, we expect $H = \overline{\mathcal{A}_+ \Omega}$ i.e. the Hilbert space obtained from one horizon already contains all state vectors. This would correspond to the difference in classical propagation of characteristic massless/massive data in d=1+1 where it is known that although for the massless case one needs the characteristic data on the two light rays, the massive case only requires one light ray. In fact there exists a rigorous proof that this classical behavior carries over to free quantum fields: with the exception of m=0 massless theories, in all other cases (including light-front data for higher dimensional m=0 situations) the vacuum is cyclic with respect to one light front $H = \overline{\mathcal{A}_{\pm}\Omega}$ [15]. The proof is representation theoretic and holds for all cases except the d=1+1 massless case. Hence in the case of interaction free algebras the holographic light front reduction which has d-1 dimensions always fulfills for d>2 the Reeh-Schlieder property, where for d=1+1 only massive theories obey holographic cyclicity. In order to recover the wedge algebra from the holographic restriction one needs the opposite translation with $U(a_-)$ i.e. $\mathcal{A}(W) = \cup_{a_-<0} AdU(a_-)\mathcal{A}_+$. For the nontriviality of the net associated with $\mathcal{A}(W)$ it is sufficient to show that the associated chiral conformal theory is nontrivial. In order to achieve this, one has to convert the bilinear forms (33) in the Z-basis which fulfil the recursion relation into genuine operators on the one-dimensional light ray. This is outside the scope of this paper.

Hence the modular approach leads to a dichotomy of *real particle creation* (absent in factorizing models) in the PFG's and in the aspect of wedge localization, versus the full QFT *virtual particle structure* of the vacuum[13] if tested with more sharply localized operators. In some sense the wedge is the best compromise between the particle/field point of view. In this and only in this sense the particle-field dualism (as a generalization of the particel wave dualism of QM) applies to QFT. Since it is left invariant by an appropriate L-boost, the algebra contains enough operators in order to resolve at least vacuum and one-particle states which cannot be resolved from the remaining states in any algebra with a lesser localization. In the next section we will argue that this is not a freak of factorizing models, whereas in a later section we will reveal the less pedestrian aspects of light cone subalgebras and holography. As we have argued on the basis of the previous pedestrian approach, the holography aspect will be important in the modular construction of QFT's because it delegates certain properties of a rather complicated theory to those of (in general several) simpler theories.

It is worthwhile to emphasize two aspects which already are visible from this pedestrian considerations. One is the notion of "quantum localization" as compared to the more classical localization in terms of test function smearing of pointlike fields. The wedge localization of the PFG's cannot be improved by choosing smaller supports of test functions inside the wedge; the only possibility is to intersect algebras. In that case the old generators become useless e.g. in the description of the double cone algebras, the latter has new generators. Related to this is that the short distance behavior looses its dominating role. If one does not use field-coordinatizations it is not even clear what one means by "the short distance behavior of a theory", short distance behavior of what object? There is no short distance problem of PFG's since they have some natural cutoff (to the extend that the use of such words which are filled with preaasigned öld meaning is reasonable in the new context). *Intersection of algebras does not give rise to short distance problems* in the standard sense of this word. An explicit construction of pontlike field coordinates from algebraic nets is presently only available for chiral conformal theories [16]. It produces

[13]The deeper understanding of the virtual vacuum structure (or the particle content of say state vectors obtained by application of a double cone localized operator to the vacuum) is presumably hidden in the modular groups of double cone algebras.

fields of arbitrary high operator dimension, and as a result of its group theoretical techniques it also does not suffer from short distance problems. This feature of the modular approach clearly has an interesting but non-understood relation to similar claims in string theory, which also does not seem to be threatened by ultraviolet problems.

3 PFG's in Presence of Real Particle Creation

For models with real particle creation it is not immediatly clear how to construct PFG's. In order to get some clue we first look at d=1+1 theories which do not have any transversal extension to wedges. Furthermore we assume that there is only one kind of particle which corresponds to the previous assumption concerning the absence of poles in the two-particle S-matrix for factorizing models.

>From the previous discussion we take the idea that we should look for a relation between the ordering of rapidities and the action of the scattering operator. We fix the state vector of n $Z^{*\prime}s$ applied to the vacuum for the natural order to be an incoming n-particle state. The totally mirrored order should then be a vector obtained by applying the full S-matrix to the incoming n-particle vector. If the particles are bosons, the order in the incoming operators on which S is applied does not matter.

But what should we do do for the remaining permutations for the remaining permutations? We should end up with a prescription which for factorizing systems agrees with the old one. For two Z^* there is no problem; the formula looks as before (17), except that the application of the S-operator to the two-particle in-vector has components to all n-particle multiparticle vectors for $n \geq 2$

$$Z^*(\theta_2)Z^*(\theta_1)\Omega \sim \chi_{21}a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{21}Sa^*(\theta_1)a^*(\theta_2)\Omega \tag{40}$$
$$\langle a^*(\theta_n)...a^*(\theta_3)\Omega Sa^*(\theta_1)a^*(\theta_2)\rangle \neq 0,\ n \geq 4$$

The 4-point F-correlation function has the same form as for the previous factorizing case if one replaces (25) by

$$\int\int f_1'(\theta_1')f_2'(\theta_2') \left\langle \theta_2', \theta_1' \left| S \right| \theta_2, \theta_1 \right\rangle \bar{f}_2(\theta_2)\bar{f}_1(\theta_1) \tag{41}$$

Some thinking reveals that subsequent applications of S-matrices on tensor factors of the n-particle tensor product vectors only makes sense for nonoverlapping situations. The action of the S-matrix on one tensor factor is associated with the mirror pertutation of that tensor factor $12...k \rightarrow k...21$ since intuitively speaking one only obtains the full k-particle scattering if the incoming velocities (or rapidities) are such that all particles meet kinematically which only happens if the order of incoming velocities is the mirrored natural order. Mathematically we should write each permutation as the nonoverlapping product of "mirror permutations" The smallest mirror permutations are transpositions of adjacent factors. An example for an overlapping product is the product of two such transpositions which have one element in common e.g $123 \rightarrow 132 \rightarrow 312$; there is no meaning interms of a subsequent tensor S-matrix action. However the composition $123 \to 213 \to 312$ has a meaningful S-matrix counterpart: namely $S \cdot S_{12}a^*(\theta_1)a^*(\theta_2)a^*(\theta_3)\Omega$ where S_{12} leaves the third tensor factor unchanged i.e. is the Fock space vector $(Sa^*(\theta_1)a^*(\theta_2)\Omega) \otimes a^*(\theta_3)\Omega$ on which the subsequent action of S (which corresponds to the mirror permutation of all 3 objects) is well defined. In general if one mirror permutation is completely inside a larger one the scattering correspondence which is consistent with the tensor product structure of Fock space. On the other hand for overlapping products pf mirror permutations the association to scattering data becomes meaningless, where overlapping means that part of each mirror permutation is outside of the other. Fortunately, as it is easy to see, there is precisely one representation in terms of nonoverlapping mirror permutations. This leads to a unique representation of multi Z^*-state vectors in terms of scattering data. On the other hand if we were to write each mirror permutation as a product of (necessarily overlapping) transpositions, we loose the uniqueness and we then need the Yang-Baxter structure in order to maintain consistency; in this case we return to the modular setting of factorizing models in the previous section.

Let us elaborate this in a pedestrian fashion by writing explicit formulas for n=3,4. For n=3 the Z^*-state vector is a sum of 3!=6 terms

$$
\begin{aligned}
&Z^*(\theta_3)Z^*(\theta_2)Z^*(\theta_1)\Omega \sim \\
&\quad \tilde{}\chi_{321}a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{312}S_{21}a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega \\
&\quad + \chi_{231}S_{32}a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{123}S_{321}a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega \\
&\quad + \chi_{132}S_{321} \cdot S_{23}^*a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega + \chi_{213}S_{321} \cdot S_{12}^*a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega
\end{aligned}
$$

Here χ denotes again the characteristic function of the respective orders and $S_{..}$ acts on the respective tensor factor with the remaining particle being a spectator. As in the two-particle case, this action creates a vector with a complicated incoming particle content having components to all particle numbers. The normalization constant in front is the same as for the statistics permutation i.e. as in the case S=1. Note that the last two terms correspond to nested mirror permutations and, as will be seen below, results in the appearance of "nondiagonal inclusive processes" terms in the F-correlation functions which generalize the diagonal inclusive processes which result from the summation over final states in cross sections.

The inner products of these vectors with themselves contribute to the 6-point F-correlation function. The integrand of one of those contributions describes the already mentioned nondiaogonal inclusive contribution

$$
\left\langle \theta_3', \theta_2', \theta_1' \left| S \cdot S_{12}^* \right| \theta_3, \theta_2, \theta_1 \right\rangle \tag{42}
$$

In a graphical scattering representation particle 1 and 2 would scatter first and produce arbitrarily many (subject to the conservation laws for the total energy-momentum) particles which together with the third incoming particle (which hitherto was only a spectator) enter an additional scattering process of which only the 3-particle outgoing component is separated out by the matrix element in (42). The dot means summation over all admissable intermediate states and could be represented by e.g. a heavy line in the graphical representation in order to distinguish it from the one-particle lines.

There are alltogether six S-independent diagonal term in the inner product which result from the inner product of a bra-component with the ket component having a characteristic function χ with identical subscripts. Thanks to the unitarity of the S-matrix products compensate and in the remaining sum involving products of χ and matrix-elements of the unit operator the sum over products of χ add up to one and hence the $\chi's$ drop out. The terms which contain S acting only on two-particle tensor factors are the same as for two-particle scattering. Terms containing the 3-particle S-matrix with one χ function have a corresponding term with the hermitian adjoint S^* and the mirrored χ. The 6-point correlation function also contains a contribution from the inner product of a 4-$Z^*(\theta)$ vector with a 2-$Z^*(\theta)$ vector which originate from terms where the first Z^* in $Z^*Z^*Z^*\Omega$ has been replaced by a $Z(\theta) = Z^*(\theta + i\pi)$. In the inner product we simply transport Z from the ket to the bra and in this way one obtains a $2 \to 4$ inner product which apart from analytic continuation is related to $3 \to 3$ by crossing. In vectors with more than 3 F's there are also terms where annihilators appear between creators. We will not present the disentangling of such terms. The presence of such terms is important for the KMS property since the crossing of one particle formally links the $2 \to 4$ contribution to the analytically continued $3 \to 3$ contribution. The structure of the 4-$Z^*(\theta)$ vector will be described below. Since the 6-point function of the $F's$ also involves As a meas of safety we will call the crossing symmetry obtained from KMS the "*modular crossing property*" simply to allow for the possibility that the standard crossing symmetry to the extend that is has been defined in the literature may not completely agree with the crossing derived from KMS. Here we have used a self-conjugate model for explanatory purpose, the adjustment to the particle$\neq$antiparticle case is easily done. in terms of converting external legs in scattering graphs by crossing incoming particle legs into antiparticle outgoing legs continued to the negative mass shell. Note that in the 6-point function the on-shell energy-momentun conservation allows the crossing of a single Z, whereas for n=4 this was only possible with a crossing of a pair of Z's.

The present analytic understanding in terms of the KMS analyticity of the wedge localization replaces the mysterious "maximal analyticity". The present modular approach also shows that the old bootstrap program contained a lot of physically sound structures; it mainly failed (in the sence of becoming sterile) because of its ideological "cleansing" rage against off-shell concepts as quantum fields. In the present modular context it appears the most successful among all failed theories of this century.

For 4- Z^* state vector there is the new possibility of having two two-particle S's acting on two nonoverlapping pairs of in-particles, before the action of either the identity or the full S-matrix is applied. We will not write down all 24 contributions for the different possible θ-orderings. Rather we will list only the six classes of nonoverlapping mirror permutation structures by one of their representatives using a recursive way of writing.

$$\begin{gathered} a^*(\theta_4)a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega \\ S\cdot a^*(\theta_4)a^*(\theta_3)a^*(\theta_2)a^*(\theta_1)\Omega \\ (Z^*(\theta_4)Z^*(\theta_3)Z^*(\theta_2)\Omega)'\times a^*(\theta_1)\Omega \\ (Z^*(\theta_4)Z^*(\theta_3)\Omega)'\times (Z^*(\theta_2)Z^*(\theta_1)\Omega)' \\ S\cdot (Z^*(\theta_4)Z^*(\theta_3)Z^*(\theta_2)\Omega)'\times a^*(\theta_1)\Omega \\ S\cdot (Z^*(\theta_4)Z^*(\theta_3)\Omega)'\times (Z^*(\theta_2)Z^*(\theta_1)\Omega)' \end{gathered} \tag{43}$$

The first, second, fourth and last class contains only one vector, the third and fifth class contain $10 = 2\times(6-1)$ elements (the dash on the bracket denotes omission of the identity contribution which was already taken into account in the first term). These 24 contributions correspond to the 24 different θ-orderings of 4 Z^*-creation operators applied to the vacuum. Again as in the n=3 case, the ordering prescription for the θ-integration drops out in the calculation of the 24×24 inner products which contribute to the 8-point function.

The representation of permutations in terms of nonoverlapping mirror permutations and their use in the construction of the rapidity space integrands of the correlation functions of PFG's $F(\hat{f})$ can be generalized to arbitrary n, a task which we will leave to the reader. At the end we obtain a system of correlations fulfilling positivity (as a result that all representations of state vectors take place in a Fock space and are defined in terms of subsequent applications of unitaties) and therefore defining a state on a *-algebra generated by the $F(\hat{f})$.We will not continue with our pedestrian computations to general n but be satisfied with the 6-point function. A proof of the existence of the PFG's in the nonintegrable case clearly requires a more streamlined formalism than the one used in this paper. A forteriori the higher dimensional case will not be mentioned. It is not clear presently whether one should try to construct higher dimensional wedge algebras directly or via holographic techniques via chiral conformal QFT's.

For non-factorizing theories the interest in the modular localization approach is (besides the improvement in the understanding the structure of interacting QFT) the possible existence of an on-shell perturbation theory of local nets avoiding the use of the nonintrinsic field coordinatizations. This is a the revival of the perturbative version of the old dream to construct an S-matrix just using crossing symmetry in addition to unitarity and no pointlike fields. The old S-matrix bootstrap program admittedly did not get far, but now we could formulate a similar but structurally richer problem as a perturbative approach to correlation functions of the on-shell PFG's. Modular theory has given us a lot of insight and nobody nowadays would try to cleanse the Einstein causality and locality concepts from the stage or claim that this way one can obtain a "TOE" (minus quantum gravity) as it was done in the 60^{ies} (notably by G. Chew). To the contrary, the local off-shell observable algebras would be in the center of interest and there is even no cleansing of field coordinatizations for ideological reasons but just a pragmatical avoidance. In particular the sharpening of localization beyond wedges is done by algebraic intersections of wedge algebras rather than by cut-off or test function manipulations on field coordinates.

The successful d=1+1 bootstrap-formfactor program for factorizing models yields S-matrices and formfactors which for models with a continuous coupling are analytic around g=0. A good illustration is the Sine-Gordon theory [14]. The more local off-shell quantities however (i.e. pointlike field operators or operators from algebras belonging to bounded regions) are radically different since they involve virtual particle polarization clouds which formally may be represented by infinite series in the on shell F's similar to the factorizing d=1+1 case of the previous section. The analytic status of these quantities (i.e. localized operators and their correlation functions) is presently not known; it may well turn out that they are only Borel summable or (in the general non-factorizable case) worse. The on-shell/off-shell dichotomy of the modular approach for the first time allows to determine more precisely if the cause of the possible nonanalyticity at g=0 are the polarization clouds.

A solution of these problems, even if limited to some new kind of perturbation theory (perturbation theory of wedge algebras and their intersections) should also shed some light on the question of how to handle theories involving higher spin particles, which in the standard off-shell causal perturbation theory lead to short distance non-renormalizability. A very good illustration of what I mean is the causal perturbation of massive spin=1 vectormesons. Here the coupling of covariant fields obtained by covariantizing the Wigner particle representation theory in the sense of the previous section will not be renormalizable in the sense of short distance power counting. In the standard perturbative approach the indefinite metric ghosts are used to lower the operator dimension of the interaction densities (free field polynomials) $W(x)$, which as a result of the free vectormeson dimension $\dim A_\mu = 2$, are at least 5, down to the value 4 permitted by the renormalization requirements in a d=1+3 causal perturbative approach [20]. Since the ghosts are removed at the end, the situation is akin to a catalyzer in chemistry: they do not appear in the original question and are absent in the final result (without leaving any intrinsic trace behind). In theoretical physics the presence of such catalyzers should be understood as indicating that the theory wants to be analyzed on a deeper level of local quantum physics i.e. further away from quantization and quasiclassics. Indeed in the present on-shell modular approach the short distance operator reason for introducing such ghosts would not be there and the remaining question is again whether the modular program allows for a perturbative analytically managable formulation.

4 The AQFT Framework

After our pedestrian presentation of the wedge algebra approach it is time to be more systematic and precise. For noninteracting free system the conversion of the rather pedestrian spatial nets of real subspaces of the Wigner space of momentum space (m.,s) wave functions into a interaction-free net in Fock space produces with the following three properties which continue to hold in the presence of interactions. They have been explained in many articles [22] and a book [1] and my main task here is their adjustment to the main problems of this survey.

1. *A net of local* (C*- or von Neumann) *operator algebras* indexed by classical spacetime regions $\mathcal{O}$

$$\mathcal{O} \rightarrow \mathcal{A}(\mathcal{O})$$

Without loss of generality the regions $\mathcal{O}$ maybe restricted to the Poincaré covariant family of general double cones and the range of this map may be described in terms of a concrete operator algebra in Hilbert space for which the vacuum representation π_0 may be taken i.e. $\mathcal{A}(\mathcal{O}) \equiv \pi_0(\mathcal{A}(\mathcal{O}))$. The geometrical and physical coherence properties as isotony: $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$ for $\mathcal{O}_1 \subset \mathcal{O}_2$ and Einstein causality: $\mathcal{A}(\mathcal{O}^{'}) \subset \mathcal{A}(\mathcal{O})'$ are then evident coherence requirements. Here we use the standard notation of AQFT: the dash superscript on the region denotes the causal disjoint and on the von Neumann algebra it stands for the commutant within $B(H)$ where H is the ambient space (here the representation space of the vacuum representation). Einstein causality can be interpreted as an a priori knowledge about some with $\mathcal{A}(\mathcal{O})$ commensurable observables in the sense of von Neumann. This causality property suggests the question if complete knowledge about commensurability $\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})'$ is possible. It turns out that this is indeed the generic behavior of vacuum nets called Haag duality. The cases of violation of this duality are of particular interest since they can be related to a very fundamental intrinsic characterization of spontaneous symmetry breaking, thus vastly generalizing the Nambu-Goldstone mechanism which was abstracted from quantization.

2. *Poincaré covariance and spectral properties.*

$$g \in \mathcal{P} \rightarrow \alpha_g \textit{ automorphism}$$
$$\alpha_g(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\mathcal{O})$$

is unitarily implements in the vacuum representation

$$\begin{aligned} U(g)AU^*(g) &= \alpha_g(A) \\ A &\in \mathcal{A}(\mathcal{O}) \end{aligned}$$

The unitaries for the translations have energy-momentum generators which fulfil the relativistic spectrum (positive energy) condition, symbolically $specU(a) \in V^1$ (the forward light cone)

3. The phase space structure of local quantum physics or the "nuclearity property".

Remark 7 *The precise fomulation of the third property is somewhat involved and will be presented after the following remarks on the first two structural properties. Since in the formulation of the net one may work without loss of generality with von Neumann algebras [1], the first question is what type in the Murray-von Neumann-Connes-Haagerup classification occurs. There is a very precise answer for* *wedges (which may be considered as double cones at infinity). As a result of the existence of a one-sided translation into a wedge, the wedge algebras $\mathcal{A}(W)$ turn out to be a factor of type III_1. This implies in particular that the algebra has properties which take it far away from the structure of QM (factors of type I_∞). Such algebras do not have pure states or minimal projectors, rather all faithful states on such algebras are thermal i.e. obey the KMS condition which makes them similar to states appearing in CST with bifurcated horizons as in Hawking-Unruh situations (but more "quantum".i.e. without the classical geometric Killing vector aspects of horizons). Also in the case of the wedge and double cone algebras the modular flow near the boundary becomes asymptotically geometric and Killing-like (in the wedge case it is even globally geometric). The origin of the thermal aspects are primarily on the local quantum physics side and not on the CST gravity side; the black hole has a natural localizing horizons (or one created by the cosmological theater of the Dear Lord), whereas the horizons of e.g. localizing double cones algebras are constructs of the human mind which serve to test the content of LQP.*

The nuclearity requirement results from the idea to obtain a local quantum physical counterpart of the phase space of QM in a box. The famous *finite number of degrees of freedom law per unit cell of QM phase space* results from limiting the discrete box spectrum by a cut-off in energy. As first suggested by Haag and Swieca [1], the corresponding LQP counterpart, based on the causally closed double cone analogue of the quantization box in Schrödinger QM, points into the direction of a "weakly" infinite number; according to their estimates this set of state vectors was compact in Hilbert space. Subsequent refinements of techniques revealed that this set is slightly smaller namely "nuclear", and exact calculations with interaction-free theories demonstrated that the LQP situation also cannot be better than nuclear.

The best way to understand this issue is to follow the motivating footsteps of Haag and Swieca. They, as many other physicists at that time (and as contemporary philosophers of nature), were attracted by the intriguing consequences of the of the so-called Reeh-Schlieder property of QFT

$$\begin{aligned} \overline{\mathcal{P}(\mathcal{O})\Omega} &= H,\ \textit{cyclicity of}\ \Omega \\ A \in \mathcal{P}(\mathcal{O}),\ A\Omega &= 0 \Longrightarrow A = 0\ \textit{i.e.}\ \Omega\ \textit{separating} \end{aligned} \tag{44}$$

which either holds for the polynomial algebras of fields or for operator algebras $\mathcal{A}(\mathcal{O})$. The first property, namely the denseness of states created from the vacuum by operators from arbitrarily small localization regions (e.g. a state describing a particle behind the moon[14] and an antiparticle on the earth can be approximated inside a laboratory of arbitrary small size and duration) is totally unexpected from the global viewpoint of general QT. In the algebraic formulation this can be shown to be dual to the second one (in the sense of passing to the commutant), in which case the cyclicity passes to the separating property of Ω with respect to $\mathcal{A}(\mathcal{O}')$. Referring to its use, the separating property is often called the *state*

[14] This weird aspect should not be held against QFT but rather be taken as indicating that localization by a piece of hardware in a laboratory is also limited by an arbitrary large but finite energy, i.e. is a "phase space localization" (see subsequent discussion). In QM one obtains genuine localized subspaces without energy limitations.

vector-field relation. The mathematical terminology is to say that the pair ($\mathcal{A}(\mathcal{O})$,Ω) is "standard". The large enough commutant required by the latter property is guarantied by causality (the existence of a nontrivial $\mathcal{O}'$) and shows that causality is again responsible for the unexpected property.

Of course the claim that somebody causally separated from us may provide us nevertheless with a dense set of states is somewhat queer if one thinks of the tensor factorization properties of ordinary Schrödinger QM with respect to an inside/outside separation via a subsystem box.

If the naive interpretation of cyclicity/separability in the Reeh-Schlieder theorem leaves us with a feeling of science fiction (and as already mentioned, also has attracted a lot of attention in philosophical quarters), the challenge for a theoretical physicist is to find an argument why, for all practical purposes, the situation nevertheless remains similar to QM. This amounts to the fruitful question namely which among the dense set of state vectors can be really produced with a controllable expenditure (of energy); a problem from which Haag and Swieca started their investigation. In QM this question is not that interesting and urgent, since the localization at a given time via support properties of wave functions leads to a tensor product factorization of inside/outside so that the inside state vectors are evidently never dense in the whole space and the "particle behind the moon paradox" does not occur.

Later we will see that most of the very important physical and geometrical informations are encoded into features of dense domains, in fact the aforementioned modular theory is explaining this deep relation between operator domains of the Tomita S's and spacetime geometry. The individuality of the various S-operators is only the difference in domains, they always do the same thing in their domains namely map $A\Omega$ to $A^*\Omega$ for all $A \in \mathcal{A}(\mathcal{O})$.

For the case at hand the reconciliation of the paradoxical aspect [17] of the Reeh-Schlieder theorem with common sense has led to the discovery of the physical relevance of *localization with respect to phase space in LQP*, i.e. the understanding of the *size of degrees of freedom* in the set: (notation $\mathbf{H} = \int EdP_E$)

$$P_E\mathcal{A}(\mathcal{O})\Omega \textit{ is compact} \tag{45}$$

$$P_E\mathcal{A}(\mathcal{O})\Omega \textit{ or } e^{-\beta\mathbf{H}}\mathcal{A}(\mathcal{O})\Omega \textit{ is nuclear} \tag{46}$$

The first property was introduces way back by Haag and Swieca [1] whereas the second more refined statement (and similar nuclearity statements involving modular operators of local regions instead of the global hamiltonian) which is saturated by QFT and easier to use, is a later result of Buchholz and Wichmann [21]. It should be emphasized that the LQP degrees of freedom counting of Haag-Swieca, which gives an infinite but still compact set of localized states is different from the QM finiteness of degrees of freedom per phase used in entropy calculations of string theory.

The map $\mathcal{A}(\mathcal{O}) \rightarrow e^{-\beta H}\mathcal{A}(\mathcal{O})\Omega$ is only nuclear if the mass spectrum of LQP is not too accumulative in finite mass intervals e.g. in particular infinite towers of equal mass particles are excluded (which then would cause the strange appearance of a maximal "Hagedorn" temperature). The nuclearity assures that a QFT, which was given in terms of its vacuum representation, also exists in a thermal state. An associated nuclearity index turns out to be the counterpart of the quantum mechanical Gibbs partition function [22] [1] and behaves in an entirely analogous way.

The peculiarities of the above degrees-of freedom-counting are very much related to one of the oldest "exotic" and at the same time characteristic aspects of QFT, namely vacuum polarization. As first observed by Heisenberg, the partial charge:

$$Q_V = \int_V j_0(x) d^3x = \infty \tag{47}$$

diverges as a result of uncontrolled vacuum particle/antiparticle fluctuations near the boundary. For the free field current it is easy to see that a better definition involving test functions, which smoothens the behavior near the boundary and takes into account the fact that the current is a 4-dim distribution which has no restriction to equal times, leads to a finite expression. The algebraic counterpart is the so called "split property", namely the statement [1] that if one leaves between say the double cone (the inside of a "relativistic box") observable algebra $\mathcal{A}(\mathcal{O})$ and its causal disjoint (its relativistic outside) $\mathcal{A}(\mathcal{O}')$ a "collar" (geometrical picture of the relative commutant) $\mathcal{O}_1' \cap \mathcal{O}$, i.e.

$$\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}_1), \;\; \mathcal{O} \ll \mathcal{O}_1 \,, \; properly \tag{48}$$

then it is possible to construct in a canonical way a type I tensor factor $\mathcal{N}$ which extends in a "fuzzy" manner into the collar $\mathcal{A}(\mathcal{O})' \cap \mathcal{A}(\mathcal{O}_1)$ i.e. $\mathcal{A}(\mathcal{O}) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_1)$. With respect to $\mathcal{N}$ the Hilbert space factorizes i.e. as in QM there are states with no fluctuations (or no entanglement) for the "smoothened" operators in $\mathcal{N}$. Whereas the original vacuum will be entangled from the box point of view, there also exists a disentangled product vacuum on $\mathcal{N}$. The algebraic analogue of Heisenberg's smoothening of the boundary is the construction of a this factorization of the vacuum with respect to a suitably constructed type I factor algebra which uses the collar extension of $\mathcal{A}(\mathcal{O})$. It turns out that there is a canonical, i.e. mathematically distinguished factorization, which lends itself to define a natural "localizing map" Φ and which has given valuable insight into an intrinsic LQP version of Noether's theorem [1], i.e. one which does not rely on any parallelism to classical structures as is the case with quantization. It is this "split inclusion" which allows to bring back the familiar structure of QM since type I factors allow for pure states, tensor product factorization, entanglement and all the other properties at the heart of standard quantum theory and the measurement process. However despite all the efforts to return to structures known from QM, the original vacuum retains its thermal (entanglement) properties with respect to all localized algebras, even with respect to the "fuzzy" localized $\mathcal{N}$.

Let us collect in the following some useful mathematical definitions and formulas for "standard split inclusions" [23]

Definition 8 *An inclusion $\Lambda = (\mathcal{A}, \mathcal{B}, \Omega)$ of factors is called standard split if the collar $\mathcal{A}' \cap \mathcal{B}$ as well as $\mathcal{A}, \mathcal{B}$ together with Ω are standard in the previous sense, and if in addition it is possible to place a type I_∞ factor $\mathcal{N}$ between $\mathcal{A}$ and $\mathcal{B}$.*

In this situation there exists a canonical isomorphism of $\mathcal{A} \vee \mathcal{B}'$ to the tensor product $\mathcal{A}\bar{\otimes}\mathcal{B}'$ which is implemented by a unitary $U(\Lambda) : H_\Lambda \to H_1\bar{\otimes}H_2$ (the "localizing map") with

$$U(\Lambda)(AB')U^*(\Lambda) = A\bar{\otimes}B' \qquad (49)$$
$$A \in \mathcal{A},\ B' \in \mathcal{B}'$$

This map permits to define a canonical intermediate type I factor $\mathcal{N}_\Lambda$ (which may differ from the $\mathcal{N}$ in the definition)

$$\mathcal{N}_\Lambda := U^*(\Lambda)B(H_1) \otimes \mathbf{1}U(\Lambda) \subset \mathcal{B} \subset B(H_\Lambda) \qquad (50)$$

It is possible to give an explicit formula for this canonical intermediate algebra in terms of the modular conjugation $J = U^*(\Lambda)J_\mathcal{A} \otimes J_\mathcal{B}U(\Lambda)$ of the collar algebra $(\mathcal{A}' \cap \mathcal{B}, \Omega)$ [23]

$$\mathcal{N}_\Lambda = \mathcal{A} \vee J\mathcal{A}J = \mathcal{B} \wedge J\mathcal{B}J \qquad (51)$$

The tensor product representation gives the following equivalent tensor product representation formulae for the various algebras

$$\begin{aligned} \mathcal{A} &\sim \mathcal{A} \otimes \mathbf{1} \\ \mathcal{B}' &\sim \mathbf{1} \otimes \mathcal{B}' \\ N_\Lambda &\sim B(H_\Lambda) \otimes \mathbf{1} \end{aligned} \qquad (52)$$

As explained in [23], the uniqueness of $U(\Lambda)$ and N_Λ is achieved with the help of the "natural cones" $\mathcal{P}_\Omega(\mathcal{A} \vee \mathcal{B}')$ and $\mathcal{P}_{\Omega\otimes\Omega}(\mathcal{A} \otimes \mathcal{B}')$. These are cones in Hilbert space whose position in H_Λ together with their facial subcone structures preempt the full algebra structure on a spatial level. The corresponding marvelous theorem of Connes [24] goes far beyond the previously mentioned state vector/field relation of the Reeh-Schlieder theorem.

Returning to our physical problem, we have succeeded to find the right analogue of the QM box. Contrary to the causally closed local type III algebras with their sharp light cone boundaries ("quantum horizons"), the "fuzzy box" type I factor N_Λ permits all the structures we know from QM: pure states, inside/outside tensor factorization, (dis)entanglement etc. In fact the vacuum is highly entangled in the tensor product description, the modular group of the state $\omega \mid_{\mathcal{A}\bar{\otimes}\mathcal{B}'}$represented in the tensor product cone $P_{\Omega\otimes\Omega}(\mathcal{A}\bar{\otimes}\mathcal{B}')$ is not the tensorproduct of those of $\mathcal{A}$ and $\mathcal{B}'$, whereas the modular conjugation J acts on the tensor product cone as $J_\mathcal{A}\bar{\otimes}J_\mathcal{B}$ (since the restriction $\omega \mid_{\mathcal{A}\bar{\otimes}\mathcal{B}'}$is faithful). Note also that the restriction of the product state $\omega \otimes \omega$ to $\mathcal{B}$ or $\mathcal{B}'$ is not faithful resp. cyclic on the corresponding vectors and therefore the application of those algebras to the representative vectors $\eta_{\omega\otimes\omega}$ yields projectors (e.g. $P_\Lambda = U^*(\Lambda)B(H_1)\bar{\otimes}1U(\Lambda)$).

Since the fuzzy box algebra N_Λ is type I, we are allowed to use the usual trace formalism based on the density matrix description, i.e. the vacuum state can be written as a density matrix ρ_Ω on $\mathcal{N}_\Lambda$ which leads to a well-defined von Neumann entropy. We know that if we restrict to the collar subalgebra or to $\mathcal{A}$, we have for $A \in \mathcal{A}$

$$(\Omega, A\Omega) = tr\rho_\Lambda A \quad (53)$$

$$S(\rho_\Lambda) = -tr\rho_\Lambda log\rho_\Lambda \quad (54)$$

but this is not sufficient to determine ρ_Λ which is needed for the von Neumann entropy of of the fuzzy box $S(\rho_\Lambda)$. If we would be able to compute the unitary representer $\Delta^{it}_{\mathcal{N}_\Lambda}$ of the modular group of the pair $(\mathcal{N}_\Lambda, \Omega)$ then we know also ρ_Λ since the modular operator of a type I factor is known to be related to an unnormalized density matrix $\check{\rho}_\Lambda$ with $\rho_\Lambda = \frac{1}{tr\check{\rho}_\Lambda}\check{\rho}_\Lambda$ through the tensor product formula on $H_1\bar{\otimes}H_2$

$$\Delta = \check{\rho}_\Lambda \bar{\otimes} \check{\rho}_\Lambda^{-1}$$

For chiral conformal theories on the line one can carry the analysis a bit further. Choose two intervals of length $2a > 2b$ symmetrically around the origin. Thanks scale invariance together with the canonicity of the construction of the fuzzy interval algebra N_Λ, the scaled family $2\lambda a > 2\lambda b$ with collar size $\lambda(a-b)$ has the same entropy independent of λ. Hence if we keep the collar size fixed say at the value ε and consider intervals of increasing length $L = \lambda a$, the limiting behavior for L$\rightarrow \infty$ is the same as for an interval of unit length with shrinking collar size ε. >From this observation we learn that small collar size can be interpreted as large interval and hence the finite coefficient of the leading ε^{-1} behavior is nothing but the entropy per unit of length for a very long interval. The coefficient depends on the characteristics of the conformal matter content (e.g. for the minimal models with central term c<1, the coefficient is expected to be computable in terms of c only) and will not be computed here.

Returning to the general case, the behavior of the above nuclearity index in the limit of $\mathcal{O} \rightarrow \mathbb{R}^d$ [1] suggests that the fuzzy box entropy is proportional to the volume of the box. Therefore it comes as a surprise that in fact the entropy in fact is only proportional to the area of th horizon (boundary of causal completion) of the localization region in the limit of large localization regions. A computation from first principles is feasible with the help of the holographic property which we will explain in section 6. But in contradistinction to the entropy picture coming from from black hole physics, the area behavior of localization entropy is not limited to curved spacetime situations with bifurcated Killing horizons but rather represents a general behavior in local quantum physics for horizons obtained by causal completion of localization regions. If these horizons have in addition classical Killing symmetry properties then one expects that this entropy can also be seen on the side of the curved spacetime metric. This would be the scenario for a quantum physical understanding of the Bekenstein aerea law from the LQP point of view. We will return to this question in the last section.

5 Modular Inclusions and Intersections, Holography

One of the oldest alternative proposals for canonical (equal time) quantizations is the so called light ray or light front quantization. The trouble with it is that it apparantly inherits some the short distance diseases from the canonical quantization The latter is known to only makes sense for superrenormalizable interactions but not for strictly renormalizable ones which lead to infinite multiplicative renormalizations. Let us ignore this and look at some additional problems which canonical quantization does not have. If one considers it as a quantization procedure, one looses the connection with local QFT; in fact in none of the papers on light cone quantization it is spelled out how to return to a local QFT. This problem of light front restricted QFT was recently looked at (for interaction free theories) by Dimock [25].

Our modular inclusion techniques in section 2 suggested that for massive (and massless for $d \neq 1+1$) theories the wedge algebra and the chiral light front algebra are identical

$$A(W) = A(\mathbb{R}_{>}) \tag{55}$$

We already mentioned there that the chiral algebra really should be thought of as the transversally unresolved light front algebra but the use of a light front notation like $A(\mathbb{R}^{d-1}_{>})$ could suggest the wrong idea that one knows the whole light front net. If we just refer to the global algebras and not to their local (sub)net structure, then all three objects are equal and their could be no confusion. The equality can be shown to hold for all algebras which have PFG generators which includes free fields and according to our arguments in the third section should encompass all QFT's with a mass gap (which have a LSZ/Haag-Ruelle scattering theory). The rigorous construction of a chiral net for $\mathcal{A}(\mathbb{R}_{>})$ indicated in section 3 will now be presented in more detail within its natural setting of modular inclusions [18].

One first defines an abstract modular inclusion in the setting of von Neumann algebras. There are several types of inclusions which have received mathematical attention[15]. An inclusion of two factors $\mathcal{N} \subset \mathcal{M}$ is called (+ halfsided) modular if the modular group $\Delta^{it}_{\mathcal{M}}$ for $t<0$ transforms $\mathcal{N}$ into itself (compression of N)

$$Ad\Delta^{it}_{\mathcal{M}}\mathcal{N} \subset \mathcal{N} \tag{56}$$

We assume that $\cup_t Ad\Delta^{it}_{\mathcal{M}}\mathcal{N}$ is dense in $\mathcal{M}$ (or that $\cap_t\Delta^{it}_{\mathcal{M}}\mathcal{N} = \mathbb{C}\cdot 1$). This means in particular that the two modular groups $\Delta^{it}_{\mathcal{M}}$ and $\Delta^{it}_{\mathcal{N}}$ generate a two parametric group of (translations, dilations) in which the translations have positive energy. Let us now look at the relative commutant as done in the appendix of [26].

Let $(\mathcal{N} \subset \mathcal{M}, \Omega)$ be modular with non trivial relative commutant. Then look at the subspace generated by relative commutant $H_{red} \equiv \overline{(\mathcal{N}' \cap \mathcal{M})\Omega} \subset H$. The modular groups to $\mathcal{N}$ and $\mathcal{M}$ leave invariant this subspace: $\Delta^{it}_{\mathcal{M}}, t<0$ maps $\mathcal{N}' \cap \mathcal{M}$ into itself by the inclusion being modular. Look at the orthogonal

[15] In addition to the split inclusion of the previous section, there are the famous Jones inclusions whose charateristic property is the existence of conditional expectations. Their domain in particle physics is charge fusion and internal symmetry.

complement of H_{red} in H. This orthogonal complement is mapped into itself by $\Delta^{it}_{\mathcal{M}}$ for positive t. Let ψ be in that subspace, then

$$\langle\psi, \Delta^{it}_{\mathcal{M}}(\mathcal{N}' \cap \mathcal{M})\Omega\rangle = 0 \; for \; t > 0. \tag{57}$$

Analyticity in t then gives the vanishing for all t.

Due to Takesaki's theorem [4] we can restrict $\mathcal{M}$ to H_{red} using a conditional expectation to this subspace defined in terms of the projector P onto H_{red}. Then

$$E(\mathcal{N}' \cap \mathcal{M}) \subset \mathcal{M}|_{\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}} = E(\mathcal{M}) \tag{58}$$

$$E(\cdot) = P \cdot P \tag{59}$$

is a modular inclusion on the subspace H_{red}. $\mathcal{N}$ also restricts to that subspace and this restriction is obviously in the relative commutant of $E(\mathcal{N}' \cap \mathcal{M}) \subset E(\mathcal{M})$. Moreover using arguments as above it is easy to see that the restriction is cyclic w.r.t. Ω on this subspace. Therefore we arrive at a reduced modular "standard inclusion"

$$(E(\mathcal{N}) \subset E(\mathcal{M}), \Omega) \tag{60}$$

Standard modular inclusions are however isomorphic to chiral conformal field theories [18].

This theorem and its extension to modular intersections leads to a wealth of physical applications in QFT, in particular in connection with "hidden symmetries" symmetries which are of purely modular origin and have no interpretation in terms of quantized Noether currents [11][26]. The modular techniques unravel structures which cannot (or have not) be seen in terms of field coordinatizations. Holography and problems of degrees of freedom counting (phase space in LQP) as well as the issue of localization entropy are other examples.

Let us briefly look again at applications to d=1+1 massive theories. It is clear that in this case we should use the two modular inclusions which are obtained by sliding the (right hand) wedge inside itself. Hence we chose $\mathcal{M} = \mathcal{A}(W)$ and $\mathcal{N} = \mathcal{A}(W_{a_+})$ or $\mathcal{N} = \mathcal{A}(W_{a_-})$ where $W_{a\pm}$ denote the two upper/lower light like translated wedges $W_{a\pm} \subset W$. As explained in section 2 following ([15]) and above, we do not expect the appearance of a nontrivial subspace (i.e. we expect $P = 1$) in the action of the relative commutants onto the vacuum

$$\mathcal{A}(I(0, a_\pm)) \equiv \mathcal{A}(W_{a_\pm})' \cap \mathcal{A}(W) \tag{61}$$

$$\overline{\mathcal{A}(I(0, a_\pm)\Omega} = H$$

where the notation indicates that the localization of $\mathcal{A}(I(0, a_\pm))$ is thought of as the piece of the upper/lower light ray interval between the origin and the endpoint $a_\pm$. By viewing this relative commutant as a lightlike limiting case of a spacelike shift of W into itself (and using Haag duality), on obtains the interval $I_\pm$ as a limit of a double cone.

>From the standardness of the inclusion one obtains according to the previous dicussion an associated conformal net on the line with the following formula for the chiral conformal algebra on the half line

$$\mathcal{A}_{\pm}(R_{>}) \equiv \bigcup_{t\geq 0} Ad\Delta_{W}^{it}\left(\mathcal{A}(I(0,a_{\pm}))\right) \subseteq \mathcal{A}(W), \tag{62}$$

We expect the equality sign to hold

$$\mathcal{A}_{\pm}(R_{>}) = \mathcal{A}(W) \tag{63}$$

but our argument was tied to the existence of PFG's since as a result of their massshell structure

$$F(\hat{f}) = \int Z(\theta)f(\theta)d\theta = F_{res}(\hat{f}_{res}) \tag{64}$$

where the notation *res* indicates the corresponding generators in light ray theory which are identical in rapidity space and only differ in their x-space appearance. This is a significant stengthening of the cyclicity property $\overline{\mathcal{A}_{\pm}(R_{>})\Omega} = \overline{\mathcal{A}(W)\Omega}$ for the characteristic data on one light ray. The argument is word for word the same in higher spacetime dimensions since the appearance of transversal components (which have no influence on the localization) in addition to θ do not modify the argument. One would think that the inference of PFG generators can even be disposed of and the equality should follow from the standard causal shadow property of QFT in the form

$$\mathcal{A}(W) = \mathcal{A}(R_{>}^{(\alpha)}) \tag{65}$$

where $R_{>}^{(\alpha)}$ is a spacelike positive halfline with inclination α with respect to the x-axis. The idea is that if this relation would remain continuous for $R_{>}^{(\alpha)}$ approaching the light ray ($\alpha = 45\,^{\circ}$) which then leads to the desired equality. We believe that the relation (65) which will be called "characteristic shadow property" is a general consequence of the causal shadow property (the identity of $\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$ where $\mathcal{O}''$ is the causal completion of the convex spacelike region $\mathcal{O}$) in any spacetime dimension.

Theories with the charateristic shadow property are the objects of the light ray folklore. The present conceptually more concise approach explains why the light ray quantization in the presence of interactions is basically nonlocal which significantly restricts its unqualified physical usefulness. The reason is that also the halfline algebra is equal to the wedge algebra (since all rays of forward light cone propagation which pass through the upper/lower half light ray $R_{>}$ have passes or will pass through W), the locality on the light ray cannot be propagated into the wedge (the strips inside the wedge subtended from an interval I on the light ray by the action of the opposite light ray translation are for massive theories not outside the propagation region of the complement of I). Only for the halfline itself one obtains a 2-dimensional shadow region namely the wedge region. If one uses both light cones then it is possible to reconstruct a causal d=1+1 net by intersections. This construction uses the the two-dimensional translation group on the wedge and the ensuing double cone relative commutants. Note that in order to achieve this with halflines of light rays one needs the relative position of the two halfline chiral light ray algebras relative to each other in the common space H. In fact one shifted right light ray chiral algebra together with its parity reflected image is equal to the union of two opposite spacelike separated wedge algebras. The reflected light ray algebra may also be replaced by a subalgebra on the left hand extension of the original ray. As a result of the fact that the beginning of this left hand extension is not in the complete shadow of the reflected ray and therefore part of the subalgebra is not geometric but fuzzy. A similar conclusion is valid for intersections of light ray algebras with their parity reflected images in which case the inner of the double cone except its light cone boundaries is fuzzy. This agrees with the qualitative behavior one expects for the modular group in a massive theory for the union or the intersection of two appropriately shifted opposite wedges. This underlines that if one uses several chiral conformal algebras in order to desribe the net of a massive theory the important part of the information of regaining the massive theory from its conformal "holographic pieces" is encoded in the relative positions of these algebras. The light ray algebra belonging to a wedge differs from that of a chiral conformal QFT only in that the former has two independent translations instead of one. The mass spectrum of a d=1+1 massive theory is contained in these two translations. With just one light ray and two translations acting differently on the one ray one can already reconstruct the full d=1+1 net. Later we will see that this is enough to understand the localization entropy which turns out to have the surface behavior first observed in the context of classical localization behind black hole horizons by Bekenstein.

Because of the transversal extension, the holography in terms of one-dimensional chiral conformal theories is more complicated for higher dimension. There one needs a family of chiral conformal theories which is obtained from "modular intersections". Rather than associating the chiral conformal theorie with a light ray, it is more appropriate to associate it with the transverse space of the wedge which contains the light cone (light front instead of light ray). Such a family is obtained by applying L-boosts to the standard wedge W which tilt W around one of its defining light rays, so that the transversal degeneracy of the modular inclusion is destroyed. In this way one obtains a fan-like ordered family of wedges corresponding to a family of chiral conformal theories whose relative position within the original Hilbert space contains all the informations which are necessary in order to reconstruct the original (massive) theory. A detailed and rigorous account will be given in a future paper. Here we will only mention some analogies to the above light ray situation. The process of tilting by applying a family of boost transformations which leave the common light ray invariant is described by unitary transformations of one chiral conformal theory into another. Each single one, according to the higher dimensional characteristic shadow property, is equal to a wedge algebra. Knowing the position of a finite number of such chiral conformal theories with respect to each other (the number increases with increasing spacetime dimensions), determines the relative position of a finite number of wedge algebras which accoding to the previous remarks is sufficient to reconstruct the original net (the blow-up property in [26][2]). As previously mentioned, in the d=1+1 case the second light ray can be thought of as obtained from the first one by a unitary parity reflection (assuming that the theory is parity invariant). The terminology "scanning by a finite family of chiral conformal theories" is more appropriate for the construction of higher dimensional theories [34][26]. It has been shown elsewhere [34] that the modular inclusion for two wedges gives rise to two reflected eight-parametric subgroups of the 10-parametric Poincare group which contain a two parametric transversal Galileian subgroup of the type found by formal light front quantization arguments [33]. All these considerations show the primordial role of the chiral conformal QFT as a building block for the higher dimensional QFT's.

There is another much more special kind of holography in which an isomorphism of a massive QFT in d+1 dimensions to a conformal d-dimensional theory is in the focus of interest. This isomorphism appears in Rehren's solution [7] of Maldacena's conjecture [27][28] about a holographic relation of quantum matter in a (d+1)-dimensional Anti de Sitter spacetime with that in a d-dimensional conformal QFT. This Maldacena-Witten holography has not been observed outside the anti de Sitter spacetime and since it is an isomorphism to a conformal theory, the degrees of freedom are not really reduced in the sense of 't Hooft' [29], as it was the case in the previous holography via light ray reduction. The M-W holography is apparently of importance within the development of string theory, in fact the protagonists believe that it contains information about a possible message about quantum gravity of string theory. Within the present AQFT setting its main interest is that it requires the field-coordinatization free point of view in its strongest form: whereas in most problems of QFT there exist appropriate field coordinatizations which often facilitate calculations, the M-W isomorphism defined in rigorous terms by Rehren is not pointlike and has no description in terms of fields outside its algebraic version. In contradistinction to the light ray holography which happens at the causality horizon (light front boundary) of modular localization (or its classical Killing counterpart in case of black holes) the AdS holography takes place at the boundary at infinity! This is a result of the weird Einstein causality structure of the AdS curved spacetime which lacks the property of global hyperbolicity. At this point of causality properties one is nolens volens driven into the AQFT conceptual framework of nets with isomophism between algebras which are labeled by geometrically maps between regions. The understanding of this phenomenon is enhanced if one starts from an d+2 dimensional spacetime with two time like directions on which the conformal symmetry group SO(d,2) acts linearly which was already useful in handling the conformal compactification which is necessary in order to give a meaning to the nonlinear fractional action on d-dimensional Minkowski space coordinates. Since the AdS reading of conformal QFT is very similar[16], let us have a brief look. The SO(d,2) group is associated with the d+2 dim. metric

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} & & \\ & -1 & \\ & & +1 \end{pmatrix} \tag{66}$$

For the conformal compactification one considers the d+1 dim. submanifold

$$M_{d+1} = \{\xi = (\boldsymbol{\xi}, \xi_5, \xi_6) \in \mathbb{R}^{d+2} : \xi^2 = 0, \xi_6 > 0\} \tag{67}$$

[16]The AdS manifold is the mass hyperboloid in the linear representation space of SO(d,2) factorized by the total reflection in the ambient space. The mass hyperboloid shares the same asymptotic part with the M_{d+1} the light cone surface.

i.e. the d+1 dim. surface of a d+2 dim. forward light cone. The parametrization $\xi = (sin\tau, \vec{\xi}, \xi_5, cos\tau)$ together with the requirement of $\mathbf{e} = (\vec{\xi}, \xi_5)$ being a unit vector describes a parametrization of the rays on M_{d+1}. This parametrization yields the compactified Minkowski space

$$\begin{aligned} t &= \frac{sin\tau}{\xi_5 + cos\tau} \\ \vec{x} &= \frac{\vec{\xi}}{\xi_5 + cos\tau} \end{aligned} \tag{68}$$

gives the compactification formula in terms of a periodic time τ and the fact that is a unit vector. In fact one obtains the well-known periodic embedding of Minkowski space M into $\tilde{M} = S^{d-1} \times \mathbb{R}$ which is known to have a causal structure.. For a single copy we have

$$M = \{(\mathbf{e}, \tau) : cos\tau + \xi_5 > 0, -\pi < \tau < \pi\} \tag{69}$$

In the relation to AdS one does not use the surface of the light cone in (67), but rather a hyperboloid in the forward light cone. We leave the verification that this causes more havoc with Einstein causality than the previous lightlike parametrization to the reader. The causality problems comes under control as soon as one gives up the idea of a map between points and uses instead the wedge/double cone spacetime indexing of the nets of AQFT following Rehren's paper [7]. The AdS-conformal isomorphism uses a boundary at infinity rather than the lightray in the previous light ray holography/scanning. From our constructive use of holography there is not much to be gained (outside the indicated pedagogical lesson) by this isomorphism since the higher conformal field theories are as difficult as their non-conformal counterparts.

At this point it may be helpful to remind the reader that the resolution of the Einstein causality paradox in conformal QFT which consists in noting that "would be" conformally invariant theories as the e.g. massless Thirring model, where charged fields violate Huygens principle and hence lead to apparent causality confusions (through the possibility of linking the timelike region in M by global conformal transformations via lightlike infinity to spacelike events), led (already 10 years before the famous BPZ paper[17]) to the appearance of the conformal blocks in a decomposition theory with respect to the center of the global conformal group [30]. With other words, genuinely interacting local fields tend be reducible under the global conformal group and decompose into nonlocal but irreducible conformal blocks. The knowledge of these irreducible components is equivalent to a the description of a field on the many sheeted covering which is local in the sense of the causality structure of the covering space. According to the expectations in [5][7] there should be a similar decomposition theory with respect to nets on AdS with a certain subclass of interacting theories being given in terms of conformal block objects by inverting the Rehren isomorphism [2].

[17] But not yet to the discovery of minimal models! Only the d=1+1 exponential Bose fields with abelian braid group statistics were known during the 70^{ies}.

6 The Entropy Problem in LQP

The presentation of thermal aspects of modular localization versus the heat bath setting would be incomplete without the incorporation of entropy. In fact in the case of a black hole metric with its classical Killing symmetry, the analogy of the behavior of the black hole surfaces with the entropy of heat bath systems first observed by Bekenstein was the basis of Hawking's great discovery about the thermal behavior of quantum matter enclosed behind black hole horizons. In this setting it is not really necessary to directly confront the problem of entropical behavior of enclosed quantum matter, rather one may understand a large amount of facts about black holes via the classical metric and analogies with thermodynamics. This elegant hiding of quantum matter behind the size of the black hole surface led 't Hooft in more recent times to formulate a new quantum principle: the holographic principle. The setting was that of quantum matter in QFT behind a bifurcate Killing horizon. In fact as will be shown in the following, the holographic aspect in the description of general QFT of the previous section combined with the remarks on localization entropy of the fuzzy box in section 4 will lead to an understanding of the Bekenstein area law of a suitably defined entropy for large double cone localizations. In order to see this, we choose a symmetric double cone $\mathcal{O}_a$ with radius a around the origin (for reasons of simplicity in d=1+2). Its commutant algebra which accoding to Haag duality is $\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})'$ can be represented as the union of all wedges obtained from a standard wedge translated by a into the x-direction $\mathcal{A}(W_a)$ by a spatial rotation $Rot(\vartheta)$ in the x-y plane. We have

$$\begin{aligned} A(O_a') &= alg\left\{\bigcup_{\vartheta} AdRot(\vartheta)\mathcal{A}(W_a)\right\} \\ &= alg\left\{\bigcup_{\vartheta} AdRot(\vartheta)\mathcal{A}_+(R_a)\right\} \end{aligned} \tag{70}$$

where $R_a = [a,\infty]$denotes the halfline indexing for the chiral theory and where we used the characteristic shadow property for the wedge algebra $\mathcal{A}(W_a)$. The unitary modular group for this situation commutes with the rotations since the latter leaves the algebra as well as the reference state Ω invariant. On the rotation invariant subalgebra which as a result of the compactness can be obtained via a conditional expectation with the projector onto the rotation invariant subspace, the modular acts the modular group on the halfline. Hence the localization entropy for the rotational invariant subalgebra in a large radialy fuzzy box behaves as

$$S_{inv}(\rho) = -tr\rho ln\rho \sim 2\pi \times L = area \tag{71}$$

Here ρ is the density matrix for the fuzzy radial box subalgebra of rotationally invariant operators. The result follows from the entropy in section 4 for a large interval in a chiral conformal theory by mapping the radial problem into the chiral one. The generalization to d>1+2 is straightforward. We believe that the area law for the entropy remains valid without the restriction to a subalgebra of rotationally invariant elements but have presently no proof. Note that the shape of the spacetime regions is restricted by the requirement that a horizon of a localization region $\mathcal{O}$ must be the boundary of its causal completion $\partial\mathcal{O}''$.

The divergence of localization entropy was of course expected from the firmly established hyperfinite von Neumann type III_1 nature of (sharply) local algebras. This kind of ultraviolet divergence is intrinsic and cannot be disposed of. Under the previous conditions the scale invariance of the light ray (or light front) chiral theory allows to convert the inverse collar size of the fuzzy box into the area law (at the expense of loosing a normalization constant) at least for very large localization regions. The resulting area law was not expected and comes as a surprise. All this does not require any knowledge of quantum gravity.

On the other hand the present notion of localized matter entropy has no obvious relation to the classical notion of entropy a la Bekenstein which one expects to see on the side of the classical metric. In fact it appears that this gravitational aspect of entropy cannot be reconciled with that of the quantum matter content behind horizon. The solution of this problem (if there is one) may contain indications which point into the direction of that elusive quantum gravity. To see this, one should notice that the above chiral matter entropy is a quantity which refers to an equivalence class of theories. Namely all theories which in the holographic reduction on the light ray (the bifurcated horizon) lead to the same chiral theory will be members of this class. If the horizons are Killing horizons in curved spacetime, the class contains families of metric which coalesce on the horizon. This then would bring us close to a situation discussed by Carlip [31], although the details in particular the treatment of boundaries would be different[18]. As far as I can see this is the only conservative idea of how possibly the elusive quantum gravity could leave an imprint on CST+quantum matter enclosed behind a horizon. The string explanation may be more elegant, but one is asked to accept a large number of prescriptions and assumptions which have no good physical interpretations and have not been confronted with those successful principles on which QFT has had its greatest triumph.

7 Comparison with String Theory

As mentioned in the introduction, historically string theory originated from the attempt to understand and implement the issue of crossing symmetry of the S-matrix. To be more specific, the dual model which was at the cradle of string theory, used a special form of crossing called duality. The idea behind this did not come from a conceptual spacetime analysis of QFT, but was as a kind of momentum space engineering of an entirely phenomenological ideas in those days. For the two-particle S-matrix it consisted in demanding that the sum over intermediate one-particle states in the s-channel agrees with the similar sum over t-channel intermediate particle states. In view of the fact that scattering theory demands the presence of multi-particle cuts it is really hard to understand why a phenomenological idea which neglected important physical structure enjoyed such past popularity. In fact after Veneziano's construction of

[18] In particular it is unclear how the ultraviolet divergent part of the localized matter entropy gets lost on the fluctuating metric side (or why only the finite "fuzzy" box part appears on the metric side).

the dual model, it became clear that it had to be unitarized which, as a result of the infinite particle towers permitted a string theoretic interpretation in the sense of the mass spectrum (but not in terms of localization).

This unitarization invalidated, as expected, on the one hand the original naive duality requirement in terms of particles[19], but on the other hand gave birth to string theory. It led to an interesting new kind of systematics which has the form of a perturbation theory with respect to the genus of Riemann surfaces in a complex auxiliary parameters which generalized a real α-representation parameter used in QED by Schwinger and which nicely complied with a string interpretation. But it should be stressed that the word "string" of string theory has a very different meaning from string-localized objects in the formal intuitive sense of Mandelstam. They were as explained below "spectral strings" and the fact that these strings could only exist in very high-dimensional spaces had to be discovered by mathematical consistency considerations and was not the result of apriori physical intuition. Also its remedy, the dimensional reduction by a Klein-Kaluza conversion of spacetime dimensions into internal symmetry degrees of freedom was anything else than natural with respect to full local quantum physics; it is entirely of (semi)classical origin and its consistency with local quantum physics is hard to check (and never was checked) beyond the semiclassical pictures.

Another problematic point is the intrinsic meaning of "stringyness" in form of an infinite tower of particles with an oscillator-like mass spectrum. As long as mass spectra do not accumulate (by increase of multiplicities) too densely, they are compatible with the phase space structure of QFT and lead to reasonable thermal behaviour, i.e. the pathological situation of a finite Hagedorn-temperature can probably be avoided in string theory. But if this is the case, what then is the intrinsic difference with (nonperturbative) QFT and the meaning of stringyness? Extended objects can also exist in QFT build on perfect local observables; in fact the superselection theory even demands their existence as carriers of e.g. braidgroup statistics in d=1+2 dimensions.

The most important problem for any kind of relativistic quantum physics is causality since it is inexorably linked to the interpretation of the theory. There exists no physically interpretable framework without Einstein causality of observables and localization of fields/operators and all attempts to overcome causality up to now have failed [17]. Even if the positive energy condition is fulfilled and the mass spectrum contains one-particle states as it happens in lowest order of string perturbation, the interpretability is still far from being secured. In order to e.g. resolve the interpretation of the continuum of the mass spectrum in terms of multi-particle scattering states, one needs cluster properties which have their origin in locality in conjunction with spectral properties. The diagonalization of a mass operator and the determination of its discrete eigenstates is physically quite void unless one has an idea of how this operator can be

[19] QFT does not realize duality in the sense of equality of s and t channels on the level of particles, but rather on the level of superselected charge sectors. The simplest illustration of this is chiral conformal QFT which may be interpreted as the solution of the nonlinear Schwinger-Dyson equations with the imposition of the charge duality which linearizes this system of equations for correlation functions.

expressed in terms of localizable variables. Unfortunately the ascend of the string-theoretical formalism popularity went hand in hand with a loss of feeling for the importance of conceptual properties as causality. Hence it is not known if the word "string" only refers to the oscillator-like mass spectrum or whether it has something to do with the (target)spacetime localization. The key to this problem, so it is said since almost 20 years, lies in a "string field theory". How can one interpret something when the key to interpretation is missing? The only paper(s) which have tried to shed some light on this question were conspicuously ignored by string theorist ([25] and two references therein). Therefore the relation of string theory to QFT, and hence also the intrinsic meaning of the word "stringyness" as a characteristic distinguishing property of string theory versus (general, not just perturbative) QFT remain obscure. Here also the standard string textbook analogy to some alleged relativistic quantum mechanics in terms of a square root action (the line element in Minkowski space) in a functional integral does not help even if one is willing to overlook that there is no theory of interactions of this kind (this is why we are doing field theory after all!), the tricks and the massaging which such expressions require in order to arrive at Schwinger's α-representation for the free propagator stand no comparision to the lucidity of Wigner's representation approach to free particles. What is an analogy good for which after a closer look goes in the opposite direction?

Compared with the complicated historical path from S-matrix theory to modern string theory, the arguments which led to the present modular approach in QFT are at least logically quite straightforward. The modular concepts serve to close the gap between the crossing symmetric S-matrix framework and the causal off-shell field theory (without the inference of the old duality idea which had to be abandoned in the unitarization of the dual model anyhow). The main resulting message is the inexorable manner in which QFT links crossing symmetry and (Hawking-Unruh) localization-thermality (KMS-properties) with Einstein causality.

An attractive aspect of string theory is of course its ultraviolet finiteness which is attributed to the extended nature of strings. As we showed in this paper, the wedge localization approach is also ultraviolet finite. This is a result of the construction of double cone algebras via intersecting (naturally cut-off) wedge algebras and the avoidance of pointlike fields in intermediate computational steps. The existence problem of the modular approach is then tied up with the nontriviality ($\neq \mathbf{C}1$) of the compactly localized algebras. Even at the risk of repetition we emphasize again that the wedge localization construction clarifies the elusive aspects of crossing symmetry in terms of well-known and very basic thermal KMS properties of modular localization. The thermal aspects are independent of the origin of localization; they are present for classical horizons defined in terms of Killing vector fields as one encounters them in curved space time (black hole physics) as well as in Minkowski spacetime particle physics for matter enclosed behind the lightlike bounding surface of a causally completed region. String theory lacks the local quantum aspects and produces thermal behaviour via Killing symmetry and differential geometry. It does not know thermalization via localization.

Both the modular wedge localization approach as well as string theory attribute a basic significance to chiral conformal theory and both know the notion of holography. But the use and the physical interpretation of these concepts is quite different. Whereas in AQFT chiral conformal theories are the building blocks of holographic images of higher dimensional theories and therefore are positioned in the same Minkowski space, string theory places the chiral conformal data into an auxiliary source space and identifies the physical space as the target space of the fields in which they take their values. Related to this is in fact the notorious difficulty of defining a string field theory and an associated locality concept.

Another point of difference is the question of physical "naturalness" of supersymmetry. String theory requires SUSY together with the high spacetime dimension for reasons of consistency. AQFT on the other hand, although having made important contributions to the understanding of supersymmetry [1], nevertheless views supersymmetry with a certain amount of physical reservations for several reasons. AQFT has not been able to attribute a clearcut physical aim to this symmetry. It does not play any role in the extraction of internal symmetries from the superselection theory of AQFT [32], nor does it seem to play an essential role in the rigorous construction of low-dimensional models[20] (e.g. the tricritical Ising model); it seems to be present for its own sake, like an accidental symmetry. Indeed its behavior under thermalization by a heat bath [35] (collapse instead of spontaneous breaking of symmetry) lends additional weight to this suspicion. In the present modular localization framework which by its very nature is ultraviolet-finite, one even looses the ultraviolet finiteness argument which is usually given in its favor. In order to shed some further light on this confusing situation it would be nice to have an explicit look at the lowest nontrivial order of gauge invariant correlation functions in N=4 supersymmetric gauge theories. Nontrivial conformal invariant 4-dim. correlations are of course sensational, even in lowest order perturbation theory (in particular gauge invariant low order 4-point conformal functions). In the literature one only finds calculations on beta-functions and (in contrast to QED) no lowest order gauge invariant correlation functions.

Perhaps the biggest difference is the attitude towards the elusive quantum gravity. It is well known that the occurance of all spins including the spin=2 in string theory was interpreted in favor of messages about quantum gravity i.e. the string tension coupling strength was identified by fiat with the gravity coupling on the basis of this s=2 appearance and the differential geometric (sigma-model) reading was used as an additional argument. In fact this was the reason why the original interpretation in terms of strong interaction physics was abandoned and replaced by an interpretation as a "theory of everything" including quantum gravity. The recent Maldacena-Witten conjecture [27][28] on the AdS-conformal correspondence created considerable exitement as a result of this interpretation and its recent extensions. But its rigorous solution in terms of an algebraic isomorphism [7] could not confirm this quantum gravity aspect of the conjecture. We have seen that the present modular localization approach applies to localizations behind

[20] The various families of conformal or factorizing models are classified by their charge superselection rules. The knowledge that the rotational eigenstates or particles which carry these charges have a supersymmetric rotational or mass spectrum is not needed for their construction.

natural CST horizons with classical Killing symmetries as well as to "quantum" horizons of causally completed Minkowski space regions. If there is any message about QG at all, then as indicated in the previous section, it is buried in the interplay of the family of CST regions which have the same quantum matter behavior near the horizon and the family of spacetime metrics which coalesce if restricted to the horizon.

The logic of string theory is that of differential geometry and topology whereas the present approach follows entirely the logic of local quantum physics of which modular theory is the most important part. The reason why differential geometric methods have gained so much more popularity with physicist than those of local quantum physics can be found in the use of euclidean field theory and functional methods of the 60^{ies}. These deep structural discoveries which related certain families of (noncommutative) real time local quantum theories with certain types of (commutative) classical statistical mechanics and whose practical use for QFT was very much restricted to quasiclassical and perturbative computations, became gradually enriched with differential geometry and topology in the 70^{ies} and 80^{ies}. The esthetical appeal of the latter as well as the subconscious desire of many physicists for a more intuitive classical realm (with added fluctuations) contributed to the unfortunately very widespread identification of QFT with those commutative structures. In QM according to Bohr and Heisenberg, the position and momentum are not attributes of the electron but rather properties of the factualization in a measurement. In very special situations one may apply quasiclassical (WKB) ideas i.e. fluctuations around a classical solution. In the transition from QM to local quantum physics (from typeI to typeIII observable algebras) this tendency towards the Noncommutative is even enhanced. In string theory on observes the opposite tendency because there the classical picture corrected by some fluctuations is the standard picture of string theorists about their subject. It is amazing to see how particle physics via string theory falls back into a pre Heisenberg time. It is doubtful that an added pinch of noncommutative geometry can correct this situation.

If this paper convinced one or the other reader that the only safe way is to return to important cross roads of the past and to solve some of the left crucial problems, then its purpose has been accomplished. In all cases of rigorous physically successful treatment of low dimensional exactly soluable models, the natural real time noncommutative structure (and not the commutative actions and Lagrangians) played the important role. Even in soluable models of low dimensional statistical mechanics, particularily the computation of critical indices, it was the conversion into an auxiliary noncommutative real time problem (the transfer matrix in case of lattice systems) which finally led to a solution and rarely the other way around.

It is true that the first attempt at an on-shell theory in form of the S-matrix bootstrap of the 60^{ies} failed. But its shortcoming was not in the basic on-shell ideas as crossing itself, but rather its anti QFT stance in the name of some purity principle (concerning what constitutes a particle observable). As far as failed particle theories of this century are concerned, it was amazingly successful; many particle physics concepts originated in the tail of that framework. The present modular localization approach tries to pick up that heritage and enrich them with deep additional structures which preempt the all important Einstein causality already in the on-shell wedge algebra and its PFG generators.

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