# Quark fragmentation into vector and pseudoscalar mesons at LEP 

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#### Abstract

Some data on the ratio of vector to vector + pseudoscalar mesons, $V /(V+P)$, and the probability of helicity 0 vector states, $\rho_{00}(V)$, are now available from LEP. A possible relation between such two quantities and their interpretation in terms of polarized fragmentation functions are discussed; numerical estimates are given for the relative occupancies of $K^{0}$ and $K^{0 *}, D$ and $D^{*}, B$ and $B^{*}$ states.


Key-words: Quark fragmentation; Polarization; Meson production; LEP.

## 1 - Introduction

The inclusive production of hadrons in $e^{+} e^{-}$annihilations at LEP, $e^{-} e^{+} \rightarrow h+X$, allows to study the fragmentation properties of quarks; such fragmentation process is recently receiving much attention both theoretically [1]-[4] and experimentally [5], [6]. It yields information on basic non perturbative aspects of strong interactions or, in case of heavy quarks, may test existing perturbative computations of fragmentation functions.

We consider here some spin dependence of particular fragmentation processes, namely the production of vector and pseudoscalar mesons, their relative abundance and the probability for the vector meson to be in a zero helicity state. These quantities can be related to polarized and unpolarized quark fragmentation functions and their measurement supplies basic information on the hadronization process of a quark; simple statistical models for such a process are discussed.

The quarks produced in $e^{+} e^{-}$annihilation at LEP are longitudinally polarized with probability $\rho_{ \pm \pm}(q)$ of being in a $\pm$ helicity state; the off-diagonal elements of the quark helicity density matrix $\rho_{ \pm \mp}(q)$ can be neglected. The probability for a hadron $h$ produced in the fragmentation of such a quark $q$ to be in a helicity $\lambda_{h}$ state is then given by:

$$
\begin{equation*}
\rho_{\lambda_{h} \lambda_{h}}(h)=\frac{1}{\sum_{q} D_{q}^{h}} \sum_{q}\left[D_{q,+}^{h, \lambda_{h}} \rho_{++}(q)+D_{q,-}^{h, \lambda_{h}} \rho_{--}(q)\right], \tag{1}
\end{equation*}
$$

where $D_{q, \lambda_{q}}^{h, \lambda_{h}}$ is the polarized fragmentation function of a quark $q$ with helicity $\lambda_{q}$ into a hadron $h$ with helicity $\lambda_{h}$. The unpolarized fragmentation function is

$$
\begin{equation*}
D_{q}^{h}=\frac{1}{2} \sum_{\lambda_{h}, \lambda_{q}} D_{q, \lambda_{q}}^{h, \lambda_{h}}=\sum_{\lambda_{h}} D_{q}^{h, \lambda_{h}} \tag{2}
\end{equation*}
$$

Eq. (1) holds in a probabilistic scheme, with a single quark independent fragmentation; in Refs. [7] and [8] it has been shown that for diagonal matrix elements this is a good approximation and that the coherent fragmentation of the quark, which takes into account its interaction with the $\bar{q}$, only induces corrections of $\mathrm{O}\left(p_{T} /(x \sqrt{s})\right)^{2}$, where $p_{T}$ is the transverse momentum of the hadron relative to the jet axis and $x$ is the fraction of the quark energy carried by the hadron.

Let us first consider final hadrons which predominantly originate from the fragmentation of only one type of quark, like mesons containing one heavy flavour, $D, D^{*}, B, B^{*}$ and possibly $K, K^{*}$ at large $x$. In such a case one has:

$$
\begin{align*}
\rho_{11} & =\frac{1}{D_{q}^{V}}\left[D_{q,+}^{V, 1} \rho_{++}(q)+D_{q,-}^{V, 1} \rho_{--}(q)\right] \\
\rho_{00} & =\frac{1}{D_{q}^{V}} D_{q}^{V, 0}  \tag{3}\\
\rho_{-1-1} & =\frac{1}{D_{q}^{V}}\left[D_{q,+}^{V,-1} \rho_{++}(q)+D_{q,-}^{V,-1} \rho_{--}(q)\right]
\end{align*}
$$

and

$$
\begin{equation*}
P_{V} \equiv \frac{V}{V+P}=\frac{D_{q}^{V}}{D_{q}^{V}+D_{q}^{P}}, \tag{4}
\end{equation*}
$$

where $V$ stands for vector and $P$ for pseudoscalar mesons. Both above and in the sequel we exploit the parity invariance of the fragmentation process, i.e.:

$$
\begin{align*}
D_{q,+}^{V, 0} & =D_{q,-}^{V, 0}=D_{q}^{V, 0} \\
D_{q,+}^{V, 1}+D_{q,+}^{V,-1} & =D_{q,-}^{V, 1}+D_{q,-}^{V,-1}=D_{q}^{V, 1}+D_{q}^{V,-1} \tag{5}
\end{align*}
$$

Notice that for valence quarks and meson wave functions with zero orbital angular momentum one expects

$$
\begin{equation*}
D_{q,-}^{V, 1}=D_{q,+}^{V,-1}=0 . \tag{6}
\end{equation*}
$$

Only $\rho_{00}(V)$ - via observation of the angular distribution of the vector meson two body decays - and the ratio $V /(P+V)$ can be measured; let us discuss what can be learnt about the quark hadronization process from such measurements. These two observables do not depend on the quark polarization state, but only on the spin and helicity nature of the final meson.

We define:

$$
\begin{equation*}
\frac{D_{q}^{V, 1}+D_{q}^{V,-1}}{D_{q}^{V, 0}} \equiv \gamma_{q}^{V} ; \quad \frac{D_{q}^{P}}{D_{q}^{V, 0}} \equiv \beta_{q}^{V} . \tag{7}
\end{equation*}
$$

Such quantities might depend on $x$. From Eqs. (2)-(5) we have:

$$
\begin{equation*}
\rho_{00}(V)=\frac{1}{1+\gamma_{q}^{V}} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{V}=\frac{1+\gamma_{q}^{V}}{1+\gamma_{q}^{V}+\beta_{q}^{V}}, \tag{9}
\end{equation*}
$$

which implies

$$
\begin{equation*}
P_{V}=\frac{1}{1+\beta_{q}^{V} \rho_{00}} \tag{10}
\end{equation*}
$$

Notice that in pure statistical spin counting one has

$$
\begin{equation*}
\gamma_{q}^{V}=2, \quad \beta_{q}^{V}=1 \tag{11}
\end{equation*}
$$

so that

$$
\begin{equation*}
\rho_{00}=\frac{1}{3}, \quad P_{V}=\frac{3}{4}, \tag{12}
\end{equation*}
$$

and the vector meson alignment

$$
\begin{equation*}
A=\frac{1}{2}\left(3 \rho_{00}-1\right) \tag{13}
\end{equation*}
$$

is zero.
Eqs. (8) and (9) show the relation between the measurable quantities $\rho_{00}(V)$ and $P_{V}$ and the parameters $\gamma_{q}^{V}$ and $\beta_{q}^{V}$, whose physical meaning is clear through the definitions (7). Eq. (10) gives a relation between the probability for a quark to fragment into a vector meson and the probability for the vector meson to have zero helicity, in terms of the parameter $\beta_{q}^{V}$ : an increase of $\rho_{00}$ implies a decrease of $P_{V}$ and vice versa.

There are some models in the literature which predict a definite relation between $P_{V}$ and $\rho_{00}$. In Ref. [9] one assumes $\beta=1$ so that $P_{V}=1 /\left(1+\rho_{00}\right)$, while in Ref. [10] one has $\gamma=1+\beta$ and $P_{V}=1 /\left(2-2 \rho_{00}\right)$.

In case of more than one flavour contributing to the final mesons, like for $\pi, \rho$, the expressions of $\rho_{00}$ and $P_{V}$ become:

$$
\begin{equation*}
\rho_{00}(V)=\frac{\sum_{q} D_{q}^{V, 0}}{\sum_{q} D_{q}^{V, 0}\left(1+\gamma_{q}^{V}\right)} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{V}=\frac{\sum_{q} D_{q}^{V, 0}\left(1+\gamma_{q}^{V}\right)}{\sum_{q} D_{q}^{V, 0}\left(1+\gamma_{q}^{V}+\beta_{q}^{V}\right)}, \tag{15}
\end{equation*}
$$

while Eq. (10) changes into

$$
\begin{equation*}
P_{V}=\frac{\sum_{q} D_{q}^{V, 0}}{\sum_{q} D_{q}^{V, 0}\left(1+\beta_{q}^{V} \rho_{00}\right)} \tag{16}
\end{equation*}
$$

One recovers the simple results (8)-(10) not only in case of a single flavour contributing, but also in the case in which $\gamma_{q}^{V}$ and $\beta_{q}^{V}$ do not depend on the flavour $q$. This might be the case for valence quarks.

## 2 - Numerical values and their interpretation

To estimate the numerical values of the parameters $\gamma$ and $\beta$ we use data on $K, K^{0 *}$, $D, D^{*}$ and $B, B^{*}$ production obtained by the OPAL Collaboration [6].

$$
\begin{align*}
& \rho_{00}\left(K^{*}\right)=0.550 \pm 0.050, \quad P_{V}(K)=0.750 \pm 0.102, \quad \text { for } \quad x>0.5  \tag{17}\\
& \rho_{00}\left(D^{*}\right)=0.400 \pm 0.020, \quad P_{V}(D)=0.570 \pm 0.060, \quad \text { for } \quad x>0.2  \tag{18}\\
& \rho_{00}\left(B^{*}\right)=0.360 \pm 0.092, \quad P_{V}(B)=0.760 \pm 0.090, \quad \text { for } x>0.33 \tag{19}
\end{align*}
$$

where $x$ is the energy fraction of the mesons. The contributions from weak decays were subtracted [11], where appropriate. This, via Eqs. (8)-(10), yields:

$$
\begin{align*}
\gamma_{s}^{K^{*}} & =0.82 \pm 0.22, & & \beta_{s}^{D^{*}}=0.58 \pm 0.52  \tag{20}\\
\gamma_{c}^{D^{*}} & =1.50 \pm 0.15, & & \beta_{c}^{D^{*}}=1.89 \pm 0.62  \tag{21}\\
\gamma_{b}^{B^{*}} & =1.78 \pm 0.80, & & \beta_{b}^{B^{*}}=0.88 \pm 0.72 \tag{22}
\end{align*}
$$

The parameters $\gamma$ and $\beta$ have a natural definition in terms of fragmentation functions, but the experimental results can also be interpreted in terms of the relative occupancies of all vector and pseudovector states [5]: let us denote by $P_{S}^{\lambda}$ the probability that the fragmenting quark produces a meson with spin $S$ and helicity $\lambda$ and let us consider only the production of vector and pseudovector mesons. Then we have:

$$
\begin{equation*}
\rho_{00}(V)=\frac{P_{1}^{0}}{P_{1}^{ \pm 1}+P_{1}^{0}} \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
P_{V}=P_{1}^{ \pm 1}+P_{1}^{0} \tag{24}
\end{equation*}
$$

with

$$
\begin{equation*}
P_{1}^{ \pm 1}=P_{1}^{1}+P_{1}^{-1}, \quad P_{1}^{ \pm 1}+P_{1}^{0}+P_{0}^{0}=1 \tag{25}
\end{equation*}
$$

In terms of previous parameters

$$
\begin{equation*}
P_{1}^{ \pm 1}=\frac{\gamma}{1+\gamma+\beta}, \quad P_{1}^{0}=\frac{1}{1+\gamma+\beta}, \quad P_{0}^{0}=\frac{\beta}{1+\gamma+\beta} . \tag{26}
\end{equation*}
$$

The data (17)-(19) give:

$$
\begin{array}{lll}
P_{1}^{ \pm 1}\left(K^{*}\right)=0.34 \pm 0.06 & P_{1}^{0}\left(K^{*}\right)=0.41 \pm 0.07 & P_{0}^{0}(K)=0.25 \pm 0.10 \\
P_{1}^{ \pm 1}\left(D^{*}\right)=0.34 \pm 0.04 & P_{1}^{0}\left(D^{*}\right)=0.23 \pm 0.03 & P_{0}^{0}(D)=0.43 \pm 0.06 \\
P_{1}^{ \pm 1}\left(B^{*}\right)=0.49 \pm 0.09 & P_{1}^{0}\left(B^{*}\right)=0.27 \pm 0.08 & P_{0}^{0}(B)=0.24 \pm 0.09 \tag{29}
\end{array}
$$

in agreement with Eqs. (20)-(22) and (26). Simple spin counting would give $P_{1}^{ \pm 1}=0.5$ and $P_{1}^{0}=P_{0}^{0}=0.25$.

## 3 - Comments and conclusions

We have shown how the simultaneous measurements of the ratio of vector to vector + pseudovector mesons and $\rho_{00}(V)$ supply basic information on the fragmentation of quarks; such information does not depend on the helicity of the quark, but on the spin and helicity of the final meson. Some data are available for $K, D$ and $B$ mesons and in some cases show clear deviations from simple statistical spin counting and predictions of other models. Such information could be of crucial importance for the correct formulation of quark fragmentation Monte Carlo programs, which at the moment widely assume simple relative statistical probabilities.

Let us consider our results, Eqs. (20)-(22) and (27)-(29). For strange mesons the data agree with spin counting in the amount of $K$ versus $K^{*}$, but, among vector mesons, helicity zero states seem to be favoured; these are in absolute the most abundantly produced, $P_{1}^{0}\left(K^{*}\right)=0.41$. For charmed mesons both values of $\gamma_{c}^{D^{*}}$ and $\beta_{c}^{D^{*}}$ differ from the spin counting values (11), suggesting a prevalence of pseudoscalar states, $P_{0}^{0}(D)=0.43$, and, among vector mesons, of helicity 0 states; they also disagree with the models of Refs. [9] and [10], which, respectively, predict $\beta=1$ and $\gamma=1+\beta$. The heavy $b$-mesons, instead, are produced in good agreement with statistical spin counting rules, as one expects.

Data from LEP1 are still being analyzed. For this kind of study it is important that the production rates for the scalar and vector mesons and the results on spin alignment are presented on the same footings, i.e., with the same cuts on the scaled energies and a consistent subtraction of the contributions from weak decays. It will be interesting to determine the parameters $\gamma$ and $\beta$ and the relative spin state occupancies $P_{S}^{\lambda}$ for other mesons, like $\pi$ and $\rho$. There might be some common features to all light quark fragmentation processes, leading to the same vector and pseudovector state occupancy probabilities for $\pi, \rho$ and $K, K^{*}$.

It would also be worthwhile to study the dependence of these probabilties on the scaled energy of the mesons. Also, the degree of universality of quark fragmentation could be tested by studying the same quantities discussed here in other processes, like meson production in lepton-hadron or photon-hadron interactions; or in $\gamma-\gamma$ and hadronhadron collisions. It would also be interesting to compare data on the production of spin $1 / 2$ and spin $3 / 2$ baryons. Some non perturbative aspects of strong interactions can only be tackled by gathering experimental information and looking for patterns and regularities which might allow the formulation of correct phenomenological models.

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