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The BV Quantisation of Superparticles Type I and II.

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ABSTRACT

This letter discusses the BRST cohomology of superparticles type I and II. It was used an extended super-space to construct $S0(9,1)$ superparticle actions that lead to super-wave functions whose spinor components satisfy $S0(9,1)$ covariant constraints. Their BRST charges were found by using BV methods, since the models present a large number of symmetries and only close on-shell. It is shown that the zero ghost-number cohomology class of both models reproduce the same spectrum as that of N=1 ten dimensional super-Yang-Mills theory.

Key-Words: Superparticles; Covariant quantisation; Gauge theories; Superspace.

Discussions of the mechanics of particles with spin shows that these can be described by either a particle theory with local world-line supersymmetry [1], or by a local fermionic symmetry [2]. This was generalised to superpace, to obtain a number of *spinning* superparticle theories satisfying certain constraints whose spectrum were precisely those of the ten-dimensional supersymmetric Yang-Mills theory. The quantum mechanics of a free superparticle in a ten-dimensional space-time is of interest because of its close relationship to ten-dimensional super Yang-Mills theory, and this corresponds to the massless sector of type I superstring theory. The $SO(9,1)$ covariant superfield formulation of super Yang-Mills theory which reduces to $SO(8)$ formulation can be obtained by either an $SO(9,1)$ vector or spinor superfields. The superfields are chosen to satisfy either rotational quadratic ("Type I") or linear ("Type II") constraints that restricts their field content to the physical propagating fields. The constraints are imposed by an explicit projection operator, constructed out of super-covariant derivatives, acting on unconstrained superfields. They were explicitly given on its $SO(8)$ form in [3], and presented on its $SO(9,1)$ form in [2]. Although, it is well known the difficult covariant quantisation of superparticle models due to the large number of symmetries that they present[4], there are by now several formulations which can be covariantly quantized. These superparticle theories with spectra coinciding with that of the ten dimensional super Yang-Mills are constructed by adding appropriate Lagrange multiplier terms to certain superparticle actions, some of them leading to type I or II constraints.

This letter compares the BRST spectrum of type I and II superparticle models arising from using the BV covariant quantisation techniques[5]. The zero ghost-number BRST cohomology class of these models gives the same physical spectrum as that of D=10, N=1 super-Yang-Mills theory. It is also shown that both quantum actions leads to free theories. We begin by briefly reviewing the description of ten-dimensional type I superparticle models [6]. A spinor wavefunction can be obtained either from a spinning particle with local world-line supersymmetry, or from a particle action with local fermionic symmetry. In [2], it was seen that super Yang-Mills theory in ten-dimensions is described by precisely such wavefunctions subject to certain extra super-covariant constraints. The quantum mechanics of the type I superparticle theory was given in [6]. This superparticle action is formulated in an extended ten-dimensional superspace with coordinates $(x^\mu, \theta_A, \phi^A)$ where θ_A and ϕ^A are anti-commuting Majorana-Weyl spinors *. The action is given by adding

$$S_0 = \int d\tau \left[p_\mu \dot{x}^\mu + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi}\right], \tag{1}$$

and

$$S_1 = \int d\tau \left[-\frac{1}{2}ep^2 + i\psi \not{p} d + i\Lambda \not{p}\hat{\phi} + \frac{1}{2}(d\chi d - 2\hat{\phi}\Gamma^\mu \chi \Gamma_\mu \not{p}\phi) + \frac{1}{2}\hat{\phi}\Upsilon\hat{\phi}\right] \tag{2}$$

* A Majorana spinor Ψ corresponds to a pair of Majorana-Weyl spinors, Ψ_A and Ψ^A. The 32×32 matrices $C\gamma^\mu$ (where C is the charge conjugation matrix) are block diagonal with 16×16 blocks $\gamma^{\mu AB}$, $\gamma^\mu{}_{AB}$ which are symmetric and satisfy $\gamma^{\mu AB}\gamma^\nu{}_{BC} + \gamma^{\nu AB}\gamma^\mu{}_{BC} = 2\eta^{\mu\nu}\delta^A_C$. In this notation the supercoordinates has components θ_A, $\theta\gamma^\mu\dot{\theta} = \theta_A\gamma^\mu{}_{AB}\dot{\theta}_B$, $\not{p}_{AB} = p^\mu\gamma^\mu{}_{AB}$, etc.

where p_μ is the momentum conjugate to the space-time coordinate x^μ, while $\hat{\theta} = d - \not{p}\theta$ and $\hat{\phi}$ are the corresponding momentum conjugate to the spinor coordinates θ and ϕ, respectively. The gauge fields e, ψ, Λ, χ and Υ correspond to local symmetries, and impose the following constraints

$$\begin{aligned} p^2 =& 0, \qquad \not{p}d = 0, \qquad \not{p}\hat{\phi} = 0, \\ \hat{\phi}\hat{\phi} =& 0, \qquad dd + 2\hat{\phi}\Gamma\Gamma\not{p}\phi = 0. \end{aligned} \tag{3}$$

The physical states are described by a superspace wavefunction satisfying $SO(9,1)$ covariant constraints[2]

$$\begin{aligned} p^2\Psi_A = 0, \qquad \not{p}_{AC}D^C\Psi_B = 0, \qquad \not{p}^{AB}\Psi_B = 0, \\ D^AD^B\Psi_C + 8(\Gamma^\mu)^{E[A}(\Gamma_\mu\not{p})^{B]}{}_C\Psi_E = 0. \end{aligned} \tag{4}$$

which leaves a superfield $\Psi_a(x^i, \theta^{\dot{a}})$ satisfying a quadratic projection condition and describes precisely the $SO(8)$ constraints of ten dimensional super Yang-Mills theory. The covariant quantisation of this superparticle was briefly discussed in [6] in the gauge $e = 1$ with the other gauge fields set to zero. Covariant quantisation requires the methods of Batalin and Vilkovisky [5] since the gauge algebra only closes on shell, and requires an infinite number of ghost fields as the symmetries are infinitely reducible. Following the BV formalism, it leads to a gauge-fixed quantum action which, after field redefinitions and integrating out all non-propagating fields, takes the form [6]

$$S_Q = \int d\tau \Big[p_\mu \dot{x}^\mu - \frac{1}{2}p^2 + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi} + \hat{c}\dot{c} + \hat{\kappa}\dot{\kappa} + i\hat{v}\dot{v} + i\hat{\rho}\dot{\rho} + \hat{\zeta}\dot{\zeta}\Big], \tag{5}$$

where $\hat{\theta} = d - \not{p}\theta - 4\hat{c}\kappa$. This S_Q proves to be invariant under BRST transformations generated by the BRST charge

$$\begin{aligned} Q =& \frac{1}{2}cp^2 - Tr(\hat{\rho}\rho\not{p}\rho) + \frac{1}{2}id\rho d + d\not{p}\kappa - 2\hat{c}\kappa\not{p}\kappa - 4\hat{c}d\rho\kappa + 4i\hat{c}\hat{\phi}\Gamma^\mu\rho\Gamma_\mu\zeta \\ &+ 4iTr(\hat{v}\Gamma^\mu\rho\Gamma_\mu\not{p}v) - 2i\hat{\phi}\Gamma^\mu\rho\Gamma_\mu\not{p}\phi + \hat{\phi}\not{p}\zeta + \frac{1}{2}i\hat{\phi}v\hat{\phi}. \end{aligned} \tag{6}$$

The quantum action defines a free field theory and so easy to quantize by imposing canonical commutation relations on the operators corresponding to the variables $(p_\mu, x^\mu, e, \hat{\theta}, \theta, \hat{\phi}, \phi, \hat{c}, c, \hat{\kappa}, \kappa, \hat{v}, v, \hat{\rho}, \rho, \hat{\zeta}, \zeta)$. It proves useful to choose a Fock space representation for the ghost and define a ghost vaccum $|0>$ which is annihilated by each of the antighosts $(\hat{\kappa}|0>= 0, \hat{c}|0>= 0, \hat{v}|0>= 0, \hat{\rho}|0>= 0, \hat{\zeta}|0>= 0)$. It also proves useful to define a *twisted* ghost vacuum $|0>_g$, where for each ghost g in the subscript, that ghost is an annihilation operator and the corresponding anti-ghost is a creation operator. The physical states on both twisted and untwisted Fock space should be the same, as they are *dual* representations of the same spectrum. It is then viewed the superspace coordinates x^μ ,$\hat{\theta}_A$ and ϕ^A as hermitian coordinates while $p_\mu = -i\partial/\partial x^\mu$, $\hat{\theta}_A = \partial/\partial\theta_A$

and $\hat{\phi}^A = \partial/\partial\phi^A$. A state of the form $\Phi(x,\theta,\phi)M|\Omega>$ with super-wavefunction Φ, a monomial M and $|\Omega>$ is considered. The zero ghost-number cohomology class is given by Φ, while M is constructed out of ghost and anti-ghost fields and $|\Omega>$ is one of the ghost ground state. It was found then that the ghost-independent state $\Phi(x,\theta,\phi)|0>$ gives the physical spectrum consisting of eight bosons and eight fermions which form the $D = 10$ super Yang-Mills multiplet together with the zero-momentum ground state which is a supersymmetry singlet.

The general state of zero ghost-number and momentum p_μ which is annhilated by the BRST charge (6) satisfies the following conditions

$$\begin{aligned} p^2\Phi = 0, \qquad \not{p}d\Phi = 0, \qquad \not{p}\hat{\phi}\Phi = 0, \\ \hat{\phi}\hat{\phi}\ \Phi = 0, \qquad (d\ d - 8\hat{\phi}\Gamma^\mu\Gamma_\mu\not{p}\phi)\ \Phi = 0. \end{aligned} \tag{7}$$

These conditions (7) imply that the only non-vanishing parts of $\Phi(x,\theta,\phi)$ are $\Psi_0(x,\theta)$ and $\Psi_A(x,\theta)$. However, Ψ_0 is trivial unless $p_\mu = 0$. Thus the only non trivial part of Φ is Ψ_A and satisfies precisely the covariant constraints (4), which lead to the D=10, N=1 super-Yang-Mills multiplet. The monomial M is constructed out of ghost and anti-ghost, and to satisfy $Q^2 = 0$, states of non-trivial ghost dependence should include terms that involve $\Gamma^\mu\rho\Gamma_\mu$, which after removing the bispinor parameters lead to the identity of $\Gamma's$ to cancel in D=10, like in super-Yang-Mills theories appears a term proportional to $\epsilon\psi^3$ which vanishes if supersymmetry is to hold [7].

I shall now describe a ten-dimensional type II superparticle model with a spinor super-wavefunction satisfying the following $S0(9,1)$ covariant linear constraints

$$\begin{aligned} p^2\Psi_A = 0, \qquad \not{p}_{AC}D^C\Psi_B = 0, \qquad \not{p}^{AB}\Psi_B = 0, \\ (\Gamma^{\mu\nu\rho\sigma})_A{}^B D^A\Psi_B = 0, \end{aligned} \tag{8}$$

which proves to be equivalent to constraints (4). This model is also formulated in an extended ten-dimensional superspace with coordinates $(x^\mu, \theta_A, \phi^A)$ where θ_A and ϕ^A are anticommuting Majorana-Weyl spinors, and to describe super Yang-Mills we wish to impose the extra constraints $d^A(\Gamma^{\mu\nu\rho\sigma})^B_A = 0$ and $\hat{\phi}\hat{\phi} = 0$ which can be done by adding appropriate lagrange multiplier terms (See Ref. 2 for further details).

The type II superparticle action is then given by the sum of [2]

$$S_0 = \int d\tau\Big[p_\mu\dot{x}^\mu + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi}\Big], \tag{9}$$

and

$$S_1 = \int d\tau\Big[-\frac{1}{2}ep^2 + i\psi\not{p}d + i\varphi\not{p}\hat{\phi} + id\not{\Lambda}\hat{\phi} - i\beta(\phi\hat{\phi} - 1) + \frac{1}{2}\hat{\phi}\omega\hat{\phi}\Big] \tag{10}$$

where, as usual, p_μ is the momentum conjugate to the space-time coordinate x^μ, d^A is a spinor introduced so that the Grassmann coordinate θ_A has a conjugate momentum $\hat{\theta}^A = d^A - \not{p}^{AB}\theta_B$, ϕ^A is a new spinor coordinate and $\hat{\phi}_A$ is its conjugate momentum. The fields e, ψ^A, φ_A, $\Lambda_{\mu\nu\rho\sigma}$, β and $\omega^{AB} = -\omega^{BA}$ are all Lagrange multipliers, which are also gauge fields for corresponding local symmetries and impose the following constraints

$$\begin{aligned} p^2 =&0, \qquad \not{p} d = 0, \qquad \not{p}\hat{\phi} = 0, \\ \hat{\phi}\hat{\phi} = 0, \qquad &\phi\hat{\phi} - 1 = 0, \qquad d(\Gamma^{\mu\nu\rho\sigma})\hat{\phi} = 0. \end{aligned} \tag{11}$$

Physical states are described by a superspace wavefunction satisfying $SO(9,1)$ covariant linear constraints (8), which leaves a superfield $\Psi_a(x^i, \theta^{\dot{a}})$ satisfying a linear projection condition which is precisely the $SO(8)$ constraint of the ten dimensional super Yang-Mills theory. The free quantum action is given by

$$S_Q = \int d\tau \Big\{ p_\mu \dot{x}^\mu + i\hat{\theta}\dot{\theta} + i\hat{\phi}\dot{\phi} + \frac{1}{2}p^2 + \hat{c}\dot{c} + \hat{\kappa}\dot{\kappa} + \hat{\zeta}\dot{\zeta} + \hat{\not{\Sigma}}\dot{\not{\Sigma}} + \hat{\eta}\dot{\eta} + \hat{v}\dot{v} \Big\}. \tag{12}$$

where $\hat{\theta} = d - \not{p}\theta - 4i\hat{c}\kappa_1$. This quantum action is invariant under modified BRST transformations which are generated by the following conserved ($\dot{Q}_{BRST} = 0$) and nilpotent ($Q^2_{BRST} = 0$) BRST charge

$$\begin{aligned} Q_{BRST} =& \frac{1}{2}cp^2 + 2id\not{p}\kappa + 2i\hat{\phi}\not{p}\zeta - 2id\hat{\phi}\not{\Sigma} - \hat{\phi}v\hat{\phi} - \hat{\phi}\eta\phi + \hat{\zeta}\zeta\eta + \hat{\not{\Sigma}}\not{\Sigma}\eta \\ &+ \hat{\eta}\eta\eta + 2\hat{v}v\eta - 2\hat{c}\hat{\theta}\kappa_2 - 2i\hat{c}\kappa\not{p}\kappa + i\hat{\kappa}\not{p}\kappa_2 + 4\hat{c}\hat{\zeta}\kappa_2\not{\Sigma} + 2i\hat{\kappa}\hat{c}\kappa_3 + 2i\hat{v}\not{\Sigma}\not{p}\not{\Sigma}. \end{aligned} \tag{13}$$

The quantum action (12) is free and the superparticle type II can again be quantized canonically by replacing each of the fields by an operator and imposing canonical (anti-) commutation relations on the conjugate pairs (p_μ, x^μ), $(\hat{\theta}, \theta)$, $(\hat{\phi}, \phi)$, $(\hat{c}, c)$, $(\hat{\kappa}_1, \kappa_1)$, $(\hat{\zeta}_1, \zeta_1)$, $(\hat{\not{\Sigma}}_1, \not{\Sigma}_1)$, $(\hat{\eta}_1, \eta_1)$ and $(\hat{v}_1, v_1)$. The cohomology classes can be classified according to their total ghost number and the physical states are taken to be the cohomology class of some definite ghost number. We consider two distinct Fock space representations of the ghost system, the *untwisted* one in which the ghost ground state $|0>$ is annihilated by each of the antighosts ($\hat{\kappa}_1|0>= 0, \hat{c}|0>= 0, \hat{v}_1|0>= 0, \hat{\eta}_1|0>= 0, \hat{\not{\Sigma}}_1|0>= 0, \hat{\zeta}_1|0>= 0$), and the *twisted* one in which antighosts are creation operators and ghosts are annihilation operators. It is viewed x, θ and ϕ as hermitian coordinates while $p_\mu = -i\partial/\partial x^\mu$, $\hat{\theta}_A = \partial/\partial\theta_A$ and $\hat{\phi}^A = \partial/\partial\phi^A$, and consider again states of the form $\Phi(x, \theta, \phi)\ M\ |\Omega>$ with super-wave function Φ. M is some monomial constructed out of ghosts and anti-ghost and $|\Omega>$ is one of the ghost ground state. A general state of zero-ghost number and momentum p_μ which is annhilated by the BRST charge (13) satisfies

$$\begin{aligned} p^2\Phi = 0, \qquad &\not{p}d\Phi = 0, \qquad \not{p}\hat{\phi}\Phi = 0, \\ \hat{\phi}\hat{\phi}\Phi = 0, \qquad &(\phi\hat{\phi} - 1)\Phi = 0, \qquad d\Gamma^{\mu\nu\rho\sigma}\hat{\phi}\Phi = 0. \end{aligned} \tag{14}$$

These implies that the only non-vanishing parts of $\Phi(x, \theta, \phi)$ are $\Psi_0(x, \theta)$ and $\Psi_A(x, \theta)$. However, $(\phi\hat{\phi} - 1)\Phi = 0$ implies that Ψ_0 is trivial. The only non-trivial part of the super-wave function is Ψ_A which precisely satisfies the covariant constraints (11) leading to the D=10, $N = 1$ super-Yang-Mills multiplet. The monomial M is constructed out of ghost and anti-ghosts and to satisfy $Q^2 = 0$, states of non-trivial ghost dependence include terms that involve products of $\Gamma's$, like the term $\not{\Sigma}\not{p}\not{\Sigma}$ which after removing the multi-tensor bosonic parameters leads also to an identity of $\Gamma's$ in $D = 10$, and needed to avoid the propagation of these bosonic ghost fields. Lets recall that the covariant quantisation of the $SSP2$ superparticle model lead to the appearance of multispinor ghosts and rouse the status of the BRST cohomology of that model [8].

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