ISSN 0029 - 3865

CBPF-NF-055/88

COVARIANT FORMULATION OF COULOMB EXCITATION IN HEAVY ION COLLISIONS AT ULTRA-RELATIVISTIC ENERGIES*

by

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*Supported by FAPESP, Brasil

ABSTRACT

Derivation of Coulomb excitation cross sections at relativistic energies is presented for the case where both projectile and target are considered as extended objects. Cross sections for projectile and/or target excitations are calculated as a function of bombarding energy in the context of a covariant theory. Several systems are analyzed and compared with available data.

Key-words: Ultra-relativistic heavy ion reactions; RIO effect; Geometrical cross section.

1. Introduction

The study of Relativistic Heavy Ion Collisions (RHIC) has recently attracted much attention of the physics community.. This is mainly due to the general expectation of observing the quark-gluon plasma and the possibility of studying properties of hot and dense nuclear matter which is supposed to take place mostly in head-on collision processes.

However, one should not forget another very basic aspect of the RHIC which is not necessarily connected to new states of matter, nor with highly excited (unknown) nuclear states. Their scattering processes at relativistic energies offers us for the first time the opportunity of studying the interplay of kinematical and/or dynamical effects of relativity and the effects of quantum mechanical correlations in extended objects.

For such purpose, it is most appropriate to choose the peripheral processes rather than central collisions for the simple reason that the nucleus has a better chance to survive from violent alterations in its final state. Among the peripheral processes, Coulomb Excitation has an obvious advantage that its dynamical agent (electromagnetic interaction) is well known. In fact, unexpectedly large electromagnetic reaction cross sections with respect to the usual theories have been recently reported[1)] at ultra-relativistic energies ($\cong$ 200 Gev/A). Therefore, the conventional excitation mechanism caused by the relativistic electromagnetic field of a point-like charge does not seem to suffice to explain the data. One of the processes which should be studied in order to clarify this puzzle is the inclusion of nuclear structure effects in the nucleus previously treated as a point particle. Such study demands a relativistically covariant formulation of the Coulomb excitation processes for heavy ions treating both nuclei as extended objects.

A first step in this direction was presented in Ref.(2), and the general form of the cross section was derived (Eq.II-24 in (2)). This formula was applied to a very special and simple situation where only the projectile nucleus is excited while the target remains in its ground state. It is the aim of this paper to present more complete analysis of the cross section formula for the Relativistic Coulomb Excitation process, including the excitaiton of both target and projectile as well as the nuclear absorption processes. Results for various reactions for the excitation of giant dipole ressonance in one nucleus only (single excitation) or both nuclei (double excitation) are presented. The energy dependence of these are compared. In section II, we derive a complete expression for general Coulomb excitation ressonance cross sections. Effects of nuclear absorption on the pure Coulomb excitation cross section are discussed in Sec III. The evaluation of current matrix element using a macroscopic model and sum-rules is done in section IV. Results of application of our formalism to various reactions is presented in section V. Concluding remarks are given in section VI.

II. Coulomb Excitation Cross Section

The most general expression for the Coulomb excitation within first order perturbation theory (one photon exchange between projectile and target nuclei) for small momentum transfers is given in CM system by[2)]

$$\frac{d\sigma}{d\Omega} = 4\,\alpha^2 (Z_A\ Z_B)^2 \left(\frac{M_A\ M_B}{\sqrt{s}}\right)^2 \sum_f \frac{\left| F_B^{\tau}\ (q_B)\ \Lambda_{\tau\iota}\ F_A^{\iota}\ (q_A)\right|^2}{q^4}, \qquad (1)$$

where α is the fine structure constant, $\sqrt{s}$ is the total center-of-mass energy, Z's and M's are atomic and mass numbers of each nucleus, respectively. q is the four-momentum transfer, q_A (q_B) is its spatial part in the proper system of the nucleus A (B) (See Appendix I). F's are Fourier transform of transition current matrix elements given by

$$\begin{aligned} F_A^{\iota}\ (q_A;\lambda_A,m_A) &= \frac{1}{eZ_A} \int d^3r\ \langle E_A^*\ ;\lambda_A\ ,m_A|\ j_A^{\iota}(r)\,|A\ g.s.\rangle\ e^{iq_A r} \\ &= \langle E_A^*\ ;\lambda_A\ ,m_A|\ j_A^{\iota}(q_A)\,|A\ g.s.\rangle\ , \end{aligned} \qquad (2)$$

and

$$\begin{aligned} F_B^{\tau}\ (q_B;\lambda_B,m_B) &= \frac{1}{eZ_B} \int d^3r\ \langle E_B^*\ ;\lambda_B\ ,m_B|\ j_B^{\tau}(r)\,|B\ g.s.\rangle\ e^{iq_B r} \\ &= \langle E_B^*\ ;\lambda_B\ ,m_B|\ j_B^{\tau}(q_B)\,|B\ g.s.\rangle\ . \end{aligned} \qquad (3)$$

In Eqs(2) and (3) currents and nuclear states are represented in their proper system, and $|E;\lambda,m\rangle$ stands for the final state of nucleus A or B with angular momentum λ whose projection in the incident beam direction (e_z') is m . $\sum_f$ means the summation over these final states. The Lorentz transformation matrix is given as

$$\Lambda_{\tau\iota} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \beta\gamma & 0 & 0 & -\gamma \end{pmatrix} , \qquad (4)$$

where β is the velocity of nucleus A in the rest frame of B,and γ is the associated Lorentz factor ($\gamma=1/\sqrt{1-\beta^2}$).

Note that this formulation shows explicitly the effects of relativistic kinematics in the relative motion combined with conventional nuclear transition matrix elements. This is only possible due to the special physical situation where final states are reached through small momentum transfers. This allows for the separation of the center of mass motion in each nucleus, as extensively discussed in Ref.(2).

To evaluate the current matrix elements, it is convenient to use the coordinate system whose z-direction is taken to be the direction of momentum transfer seen by the nucleus. Let us introduce a Cartesian basis vectors, { e_x , e_y , e_z}, where e_z = q / |q|, whereas the original basis vectors are denoted by {e'_x , e'_y , e'_z }. They are related to each other by

$$\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} =$$

$$= \begin{pmatrix} (\cos\theta-1)\cos^2\varphi + 1 & (\cos\theta-1)\sin\varphi\cos\varphi & -\sin\theta\cos\varphi \\ (\cos\theta-1)\sin\varphi\cos\varphi & (\cos\theta-1)\sin^2\varphi + 1 & -\sin\theta\sin\varphi \\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{pmatrix} \begin{pmatrix} e'_x \\ e'_y \\ e'_z \end{pmatrix} ,$$

where θ and φ are polar angles of e_z with respect to the basis vectors $\{e'_x, e'_y, e'_z\}$. The corresponding spherical basis vectors, $\{e_{+1}, e_0, e_{-1}\}$ and $\{e'_{+1}, e'_0, e'_{-1}\}$ are related each other by

$$\begin{pmatrix} e_{+1} \\ e_0 \\ e_{-1} \end{pmatrix} = \begin{pmatrix} e'_{+1} \\ e'_0 \\ e'_{-1} \end{pmatrix} \mathbb{D}^{(1)}(\varphi,\theta,-\varphi) \quad , \tag{5}$$

where $\mathbb{D}^{(1)}$ is the usual rotation matrix for $j=1$. Using these relations we can express the four-vector current in terms of spherical component with respect to the momentum transfer direction (e_z) as

$$\begin{pmatrix} \rho \\ j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \mathbb{U}\,\mathbb{D}^T & \\ 0 & & & \end{pmatrix} \begin{pmatrix} \rho \\ j_{+1} \\ j_0 \\ j_{-1} \end{pmatrix} \quad , \tag{6}$$

where the 3x3 matrix $\mathbb{U}$ transforms spherical vectors into Cartesian ones.

Now let us denote the matrix element in Eq.(1) by

$$\mathcal{M}_{m_B m_A} = F_B^{\,T}(q_B;\lambda_B,m_B)\;\Lambda_{T\iota}\;F_A^{\,\iota}(q_A;\lambda_A,m_A) \quad . \tag{7}$$

To make use of selection rules of the angular momentum states in the current matrix elements, it is more convenient to express nuclear states and current operators in a spherical basis taking the z-direction to be that of the momentum transfer q. When the ground state of the nucleus has zero angular momentum, the selection rules for current matrix elements are

$$
\begin{aligned}
&\langle E^*;\ \lambda,\mu|\rho|0,0\rangle = \langle\rho\rangle\ \delta_{\mu,0}\ ,\\
&\langle E^*;\ \lambda,\mu|j_{+1}|0,0\rangle = \langle j_1\rangle\ \delta_{\mu,1}\ ,\\
&\langle E^*;\ \lambda,\mu|j_0|0,0\rangle = \langle j_0\rangle\ \delta_{\mu,0}\ ,\\
&\langle E^*;\ \lambda,\mu|j_{-1}|0,0\rangle = \langle j_{-1}\rangle\ \delta_{\mu,-1} = \langle j_1\rangle\ \delta_{\mu,-1}\ ,
\end{aligned}
\tag{8}
$$

where μ refers the projection of angular momentum in the direction of q. The last equality $\langle j_1\rangle = \langle j_{-1}\rangle$ comes from time-reversal invariance.

The angular momentum eigenstates $|\lambda,m\rangle$ whose quantization axis is e_z' are related to $|\lambda,\mu\rangle$ as

$$
|\lambda,m\rangle = \sum_{\lambda} |\lambda,\mu\rangle\ \ D^{(\lambda)^{-1}}_{\lambda,m}(\varphi,\theta,-\varphi)\ , \tag{9}
$$

we may express the matrix element $\langle\lambda,m|j_\iota|0,0\rangle$ in matrix form with respect to indices m and ι as

$$
(\ j_{m\iota}) = \bar{D}^{(\lambda)} \begin{pmatrix} 0 & \langle j_{+1}\rangle & c & c \\ \langle\rho\rangle & 0 & \langle j_0\rangle & c \\ 0 & 0 & c & \langle j_{-1}\rangle \end{pmatrix}, \tag{10}
$$

where the $\bar{D}^{(\lambda)} = D^{(\lambda)}$ is a $(2\lambda+1)x3$ matrix: for $\lambda = 0$, one should use $\bar{\mathbb{D}}^{(0)} = (1\ 0\ 0)$, for $\lambda = 1$, $\bar{\mathbb{D}}^{(1)} = \mathbb{D}^{(1)}$, and for $\lambda > 1$, it is given by the $(2\lambda+1)\times 3$ sub-matrix of $\mathbb{D}^{(\lambda)}$ formed by its three central columns.

Denoting M the whole set of elements $M_{m_B m_A}$ as a $(2\lambda_B+1)\cdot(2\lambda_A+1)$ matrix , we get, after inserting Eqs(6) and (10) into Eq.(7),

$$M = \bar{\mathbb{D}}^{(\lambda_B)}(\hat{q}_B) \begin{pmatrix} 0 & \langle j_{+1}\rangle & 0 & 0 \\ \langle\rho\rangle & 0 & \langle j_0\rangle & 0 \\ 0 & 0 & 0 & \langle j_{-1}\rangle \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \mathbb{D}^{(1)\dagger}(\hat{q}_B) & \\ 0 & & & \end{pmatrix} \times$$

$$\times \begin{pmatrix} \gamma & 0 & -\beta\gamma & 0 \\ 0 & 0 & 0 & 1 \\ \beta\gamma & 0 & -\gamma & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \times \tag{11}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \mathbb{D}^{(1)*}(\hat{q}_A) & \\ 0 & & & \end{pmatrix} \begin{pmatrix} c & \langle\rho\rangle & 0 \\ \langle j_{+1}\rangle & 0 & 0 \\ c & \langle j_0\rangle & 0 \\ c & 0 & \langle j_{-1}\rangle \end{pmatrix} \bar{\mathbb{D}}^{(\lambda_A)^T}(\hat{q}_A) .$$

The argument of $\mathbb{D}$, $\hat{q}$ symbolically represents the Euler angles associated to the rotation required to transform e_z' to e_z in each nucleus.

The center-of-mass differential cross section of the Coulomb excitation process of

$$|A\ gs;0,0\rangle \longrightarrow |E_A^*;\lambda_A,m_A\rangle ,$$

$$|B\ gs;0,0\rangle \longrightarrow |E_B^*;\lambda_B,m_B\rangle ,$$

is then given by

$$\frac{d\sigma}{d\Omega}\bigg|_{m_B\ m_A} = F(s)\ |\ M_{m_A\ m_B}|^2 / q^4 , \tag{12}$$

where

$$F(s) = 4\ \alpha^2 (Z_A Z_B)^2 \left(\frac{M_A\ M_B}{\sqrt{s}}\right)^2 . \tag{13}$$

For a given multipolarity λ, the inclusive differential cross section with respect to angular momentum projection states is expressed as

$$\frac{d\sigma}{d\Omega} = F(s)\quad Tr\ [\ M^\dagger M\ ]\ /\ q^4 . \tag{14}$$

If there exists more than one final state for an observed process, Eq.(14) should be summed up over these states.

In the case where the nucleus B stays unexcited, we get

$$\frac{d\sigma}{d\Omega}\bigg|_{m=\pm1}^{single} = \frac{F(s)}{2}\ |\langle\rho_B\rangle|^2\gamma^2$$

$$\cdot \sin^2\theta_A\ |\ [\langle\rho_A\rangle - \beta\cos\theta_A\ \langle j_0^A\rangle] + \beta\cos\theta_A\ \langle j_1^A\rangle\ |^2 , \tag{15-a}$$

$$\frac{d\sigma}{d\Omega}\Bigg|_{m=0}^{single} = F(s)\ |\langle\rho_B\rangle|^2\gamma^2 \cdot$$

$$\left|\ \cos\theta_A\{\ \langle\rho_A\rangle - \beta\cos\theta_A\ \langle j_0^A\rangle\ \} - \frac{1}{2}\ \beta\sin^2\theta_A\ \langle j_1^A\rangle\ \right|^2 , \qquad (15\text{-b})$$

where θ_A, φ_A represent the polar angle of q_A in the rest frameof A. It is important to stress that Eq.(15) can also be obtained in the context of Plane Wave Born Approximation where the unexcited nucleus B is replaced by a point-like charge[3]. Thus, for this type of process, the covariant formulation presented here is nothing but the momentum representation of the above quoted treatment. However, Eq.(14) shows a highly nontrivial interplay between relativistic kinematics and nuclear transition matrix elements. This effect has never been studied before, neither experimentally nor theoretically.

III. Nuclear Absorption Effects

From the field theoretical point of view adopted in Ref.(2), it is a very complicated matter to include nuclear absorption effects. One possible way to estimate such effects at least phenomenologically is to introduce a cut-off in the relative transversal coordinates in the following manner : First, we write the total cross section in terms of an integral over the transversal momentum transfer,

$$\sigma_{tot} = \int d^2q_\perp\ \ F(s)\ \ Tr[\ M(q_\perp)^\dagger\ M(q_\perp)\ ]\ /\ q^4\ . \qquad (16)$$

Now introducing a Fourier transform

$$f(b) = \frac{1}{2\pi} \int d^2q \, \frac{\sqrt{F(s)\, M(q_\perp)}}{q^2} \, e^{iq\cdot b} . \quad (17)$$

where $M(q_\perp)$ is the abreviation of

$$M(q_\perp) = \sqrt{\frac{d^2\Omega}{dq_\perp^2}}\, M(q) = \sqrt{\frac{1}{p\sqrt{p^2 - q_\perp^2}}}\, M(q) , \quad (18)$$

where p is the CM momentum of the projectile. We can rewrite the total cross section as

$$\sigma_{tot} = \int d^2b \; Tr[\, f(b)^\dagger f(b)\,] . \quad (19)$$

We may interprete the above expression as the impact parameter representation of the cross section. In order to estimate the nuclear absorption effects in a simple way, we then attribute those parts in the integral coming from all values of $|b|$ smaller than the sum of the radii of the projectile and target to the absorption coming from nuclear interactions. Thus, introducing the cut-off parameter b_{min}, we assume the pure Coulomb excitation cross section σ^{Coulex} as

$$\sigma^{Coulex} = \int_{b_{min}}^{\infty} d^2b \; Tr[\, f(b)^\dagger f(b)\,] . \quad (20)$$

Substituting Eq.(17) in the above, we have

$$\sigma^{Coulex} = \frac{F(s)}{(2\pi)^2} \int_{b_{min}}^{\infty} d^2b \int d^2q_\perp \int d^2q_\perp'$$

$$Tr \left[\frac{1}{q^2} M(q_\perp)^\dagger \; e^{-ib(q_\perp - q_\perp')} \; M(q_\perp') \frac{1}{q'^2} \right] . \qquad (21)$$

Using the explicit representations of rotation matrices, we may work out all the integrals in azimuthal angles term by term as well as the integral in b. The final form of the total cross section is

$$\sigma^{Coulex} = \sigma_{tot} - \sigma_{abs} , \qquad (22)$$

with

$$\sigma_{abs} = \sum_{m=-\lambda_B}^{\lambda_B} \sum_{m'=-\lambda_A}^{\lambda_A} \int q_\perp dq_\perp \int q_\perp' dq_\perp' \frac{F(s)}{q^2(s,q_\perp) \; q^2(s,q_\perp')}$$

$$M^*_{mm'}(q_\perp) \; \Delta_{mm'}(q_\perp,q_\perp') \; M_{mm'}(q_\perp') , \qquad (23)$$

with

$$\Delta_{mm'}(q_\perp,q_\perp') = \left(\frac{1}{2\pi}\right)^2 \int_{b_{min}}^{\infty} d^2b \int d\varphi \int d\varphi' \; e^{-ib(q_\perp - q_\perp') + i(m+m')(\varphi-\varphi')}$$

$$= \frac{2\pi \; b_{max}}{q_\perp^2 - q_\perp'^2} \left[q_\perp \; J_{1+m+m'}(q_\perp b_{max}) \; J_{m+m'}(q_\perp' b_{max}) - \right.$$

$$\left. - \; q_\perp' J_{m+m'}(q_\perp b_{max}) \; J_{1+m+m'}(q_\perp' b_{max}) \right] , \qquad (24)$$

where J's are cilindrical Bessel functions and $M_{mm'}(q_\perp)$'s should be calculated from Eq.(11), setting $\varphi_B = \pi$ and $\varphi_A = 0$, whereas the four-momentum transfer q should be expressed in terms of $\sqrt{s}$ and $q_\perp$.

Again for the case that nucleus B is not excited, we have

$$\sigma^{abs}_{single} = \int q_\perp dq_\perp \int q'_\perp dq'_\perp \frac{F(s)\ |\langle\rho_B\rangle|^2 \gamma^2}{q^2(s,q_\perp)\ q^2(s,q'_\perp)}$$

$$\Big[A_0(q_\perp)\ A_0(q'_\perp)\ \Delta_{00}(q_\perp,q_\perp')\ +\ A_1(q_\perp)\ A_1(q'_\perp)\Delta_{11}(q_\perp,q_\perp')\Big] , \qquad (25)$$

with

$$A_0 = \left|\frac{\partial\ \Omega}{\partial\ q_\perp}\right|^{1/2} \Big[\cos\theta_A[\langle\rho_A\rangle - \beta\cos\theta_A\ \langle j_0^A\rangle] - \beta\sin^2\theta_A\ \langle j_1^A\rangle\Big] ,$$

$$(26)$$

$$A_1 = \left|\frac{\partial\ \Omega}{\partial\ q_\perp}\right|^{1/2} \sin\theta_A \Big[\ [\langle\rho_A\rangle - \beta\ \cos\theta_A\ \langle j_0^A\rangle] + \beta\cos\theta_A\ \langle j_1^A\rangle\ \Big] ,$$

where

$$\sin\,\theta_A = \frac{q_\perp}{|q_A|} . \qquad (27)$$

IV. Giant Ressonance

For Coulomb excitation processes in relativistic heavy ion collisions, the most important contribution to the cross section is likely to come from the giant ressonance excitation. In such case, a macroscopic model such as hydrodynamical model is useful to evaluate the relevant current matrix elements. Suzuki and Rowe have derived a sum rule for current matrix elements which corresponds to the incompressible fluid model[4)]. In their model, the giant ressonance state is represented by a unique level of given multipolarity and the transition matrix element of the current from the ground state to this level is assumed to be of the form

$$\langle gs|\ j\ (r)|\ E^*,\lambda\mu\rangle = \frac{N}{2i\ m}\ \rho_o(r)\ \nabla\ \{\ r^\lambda\ Y_{\lambda\mu}\ \}\ , \tag{28}$$

where $\rho_o(r)$ is the ground state density distribution, m nucleon mass and N is the normalization constant related to the sum rule value S as,

$$N = \sqrt{\ E^*\ /\ S\ }\ , \tag{29}$$

with

$$S = \frac{3\lambda\ A\ R^{2\lambda-2}}{8\pi\ m}\ , \tag{30}$$

The Fourier transform of Eq.(29) gives the required matrix element. After some algebra, we get

$$\langle\rho\rangle = \frac{N\sqrt{4\pi\ (2\lambda+1)}\ \lambda\ \int dr\ r^{\lambda+1}\ \ \rho_o(r)\ j_{\lambda-1}(qr)}{2m\ i^\lambda\ \ E^*\ /\ |q|}\ , \tag{31}$$

where $j_{\lambda+1}(qr)$ are spherical Bessel functions, and

$$\langle j_o\rangle = \frac{E^*}{|q|}\ \langle\rho\rangle\ , \tag{32}$$

and

$$\langle j_1\rangle = \left[\ \frac{\lambda\ +\ 1}{2\ \lambda}\ \right]^{1/2}\ \langle j_o\rangle\ . \tag{33}$$

Note that in this model, magnetic current is neglected. It is known that for giant ressonances, the contribution of magnetic current is small. For very high energies ($\gamma>>1$), and low momentum transfer limit ($qR<<1$), we may approximate the nuclear matrix element eq. (31) as

$$\langle\rho\rangle = \sqrt{\frac{A\ E^*}{2\ m}} \quad \frac{|q_A|}{E^*} \ . \tag{34}$$

Furthermore, the four-momentum transfer squared is given by

$$q^2 \cong -\left[\left(\frac{E^*}{\beta\gamma}\right)^2 + q_\perp^2\right] \ . \tag{35}$$

Note that, in this limit, $M_0(q_\perp) \simeq 0$, $M_1(q_\perp)\cong(\gamma/p)\sqrt{\frac{A\ E^*}{2\ m}}\ q_\perp e^{-i\varphi}$, and we can carry out integrals in $q_\perp$ and $q_\perp'$ as well as over the impact parameter b. We obtain, for example for the dipole single excitation,

$$\sigma^{Coulex}_{single} = \pi\ \left(\frac{Z_P\ \alpha}{m}\right)^2 \frac{N_T Z_T}{A^{2/3}}\ \ln[(\delta/\xi)+1]\ , \tag{36}$$

where $\xi=E^*b/\beta\gamma$ and δ=0.681085...

VI. Applications

Experimentally, it is not an easy matter to measure the pure Coulomb excitation cross section. One of methods proposed is to measure the cross section of the process,

$$H.I\ +\ {}^{A}Z\ \longrightarrow\ {}^{A-1}Z\ +\ \textit{anything}\ , \tag{34}$$

attributing this process of one-neutron emission to the giant ressonance excitation of the target nucleus[4]. Of course, other possible processes than Coulomb excitation which leads to final states in Eq(34), such as neutron stripping reactions, should be evaluated. However, such contributions can be empirically subtracted using the fatorization hypothesis of limiting fragmentation[5].

Another possible way of estimating the Coulomb Excitation cross section which is currently adopted is to subtract the geometrical cross section from the total reaction cross section,

$$\sigma^{Coulex} \cong \sigma^{Reaction} - \sigma^{Geom} . \qquad (35)$$

We are aware of the limitations of these methods, but in this paper we shall be concerned only with the theoretical Coulomb excitation cross section. They will be compared to the above experimental "Coulomb excitation cross section". In Table I, we calculate σ^{Coulex} for several reactions only the dipole excitation process.

As pointed out before, the single excitation cross section coincides with the PWBA calculation of Bertulani and Baur[3](Figs.1,2 and 3). In Fig.1, we also show the double excitation cross section which is found to be very small. The energy dependence of these two processes is different. This result indicates that the relative contribution of nuclear structure effects of the incident nucleus on the excitation of the target decreases with bombarding energy. Our calculation performed in an entirely covariant framework confirms quantitatively the currently adopted image of the Coulomb excitation as being caused by the relativistic field of a point particle (at least for $\gamma>>1$). It is thus also confirmed that the measured Coulomb dissociation[1] for the system in Fig.2 can not be explained in this context.

VI. Concluding Remarks

In this paper, we presented a covariant formulation of the Coulomb excitation process which allows for the inclusion of both relativistic and nuclear structure effects. In the particular case of the double excitation of projectile and target, we obtain a highly nontrivial structure combining both effects. However, numerical results show that this contribution is too small to explain the excess cross section relative to the estimated cross section of usual single excitation by the relativistic field of a point-like particle of charge Z.

Some authors claim that eletromagnetic excitation processes other than giant ressonaces, for example photomesonic or baryon ressonance may explain this discrepancy[6,7]. However recent measurements of total projectile fragmentation cross section as a function of the target atomic number seems lower than the typical Coulomb Z^2-dependence[6]. This can be taken as an evidence of somewhat different excitation mechanism than Coulomb excitation. Recently it was sugested the possibility of an increase of nuclear cross section as a function of incident energy[8]. A theoretical investigation of such mechanism in the same framework as the one presented here is now under progress.

ACKNOWLEDGEMENTS: We acknowledge stimulating discussions with C.A.Bertulani.

APPENDIX I. Kinematical Relations

We consider a reaction,

A + B -> A′ + B′

where the total incident kinetic energy of projectile A is denoted as T_{in}. The total CM energy $\sqrt{s}$ is

$$\sqrt{s} = \sqrt{2\,(M_A+T_{in})\,M_B + M_A^2 + M_B^2} \tag{A1}$$

The Lorentz factor of the relative motion γ is

$$\gamma = \frac{s - M_A^2 - M_B^2}{2\sqrt{s}} \tag{A2}$$

Energies of projectile and target nucleus in CM system are

$$E_A = \frac{s - M_A^2 + M_B^2}{2\sqrt{s}} \tag{A3}$$

$$E_B = \frac{s - M_B^2 + M_A^2}{2\sqrt{s}} \tag{A4}$$

where the CM momentum of the projectile p is

$$p = \sqrt{E_A^2 - M_A^2} \tag{A5}$$

Lorentz factors of nuclei in the CM system are then

$$\gamma_A = E_A / M_A$$
$$\beta\gamma_A = p / M_A$$

$$\gamma_B = E_B / M_B$$
$$\beta\gamma_B = -p / M_B$$

The four-momentum transfer q of nucleus A is

$$q = \begin{pmatrix} \delta E \\ q_\perp \cos\varphi_A \\ q_\perp \sin\varphi_A \\ \dfrac{\delta p(2p+\delta p)-q_\perp^2}{p\sqrt{p^2 - q_\perp^2}} \end{pmatrix}$$

where

$$\delta E = E_A^* / \gamma_A + p\, \delta p / E_A$$

and

$$\delta p = - \frac{E_A^* / \gamma_A + E_B^* / \gamma_B}{p \dfrac{E_A E_B}{E_A + E_B}} .$$

FIGURE CAPTIONS

Figure 1: Single(full) and double(dashed) eletromagnetic excitation cross sections for the system $^{208}Pb(^{208}Pb,X)^{207}Pb$ versus projectile energy in the lab, in the energy range up to 200 GeV/Nucleon, as calculated from eqs. 22 and 25.

Figure 2: Single electromagnetic excitation for the system $^{208}Pb(^{32}S,X)^{207}Pb$ as a function of incident energy.

Figure 3: Same as Fig.2, here for the system $^{197}Au(^{16}O,X)^{196}Au$.

Table I: Numerical results for the polarizations of the excited nucleus final state for various systems. P_T and P_L are transverse and longitudinal polarizations respectivelly (with respect to the momentum transfer direction).

Table II: Single eletromagnetic excitation cross section for various systems. σ_{teo} are calculated from eqs. 22 and 25, σ_{exp} are measured cross sections as reported in ref. 5.

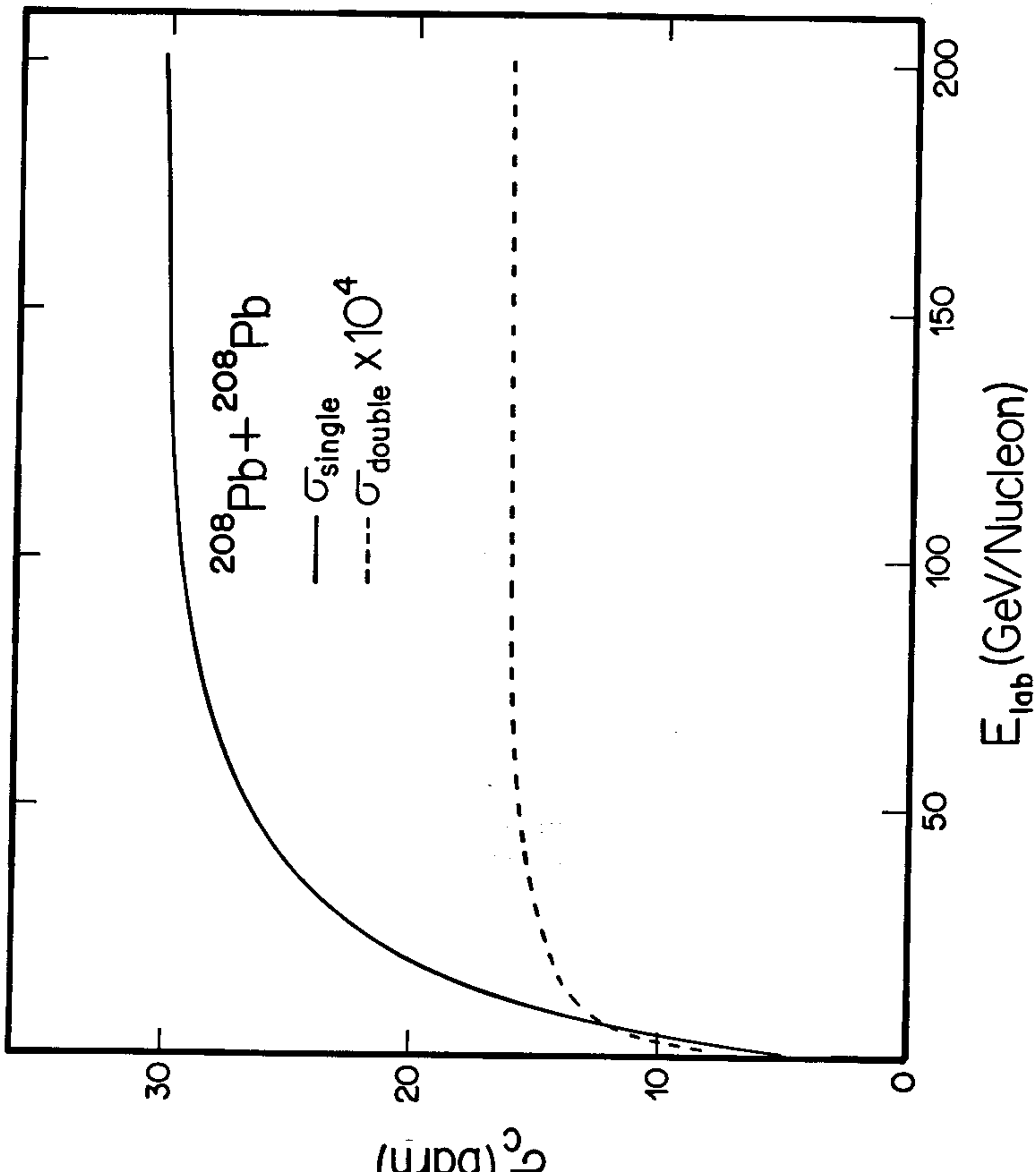
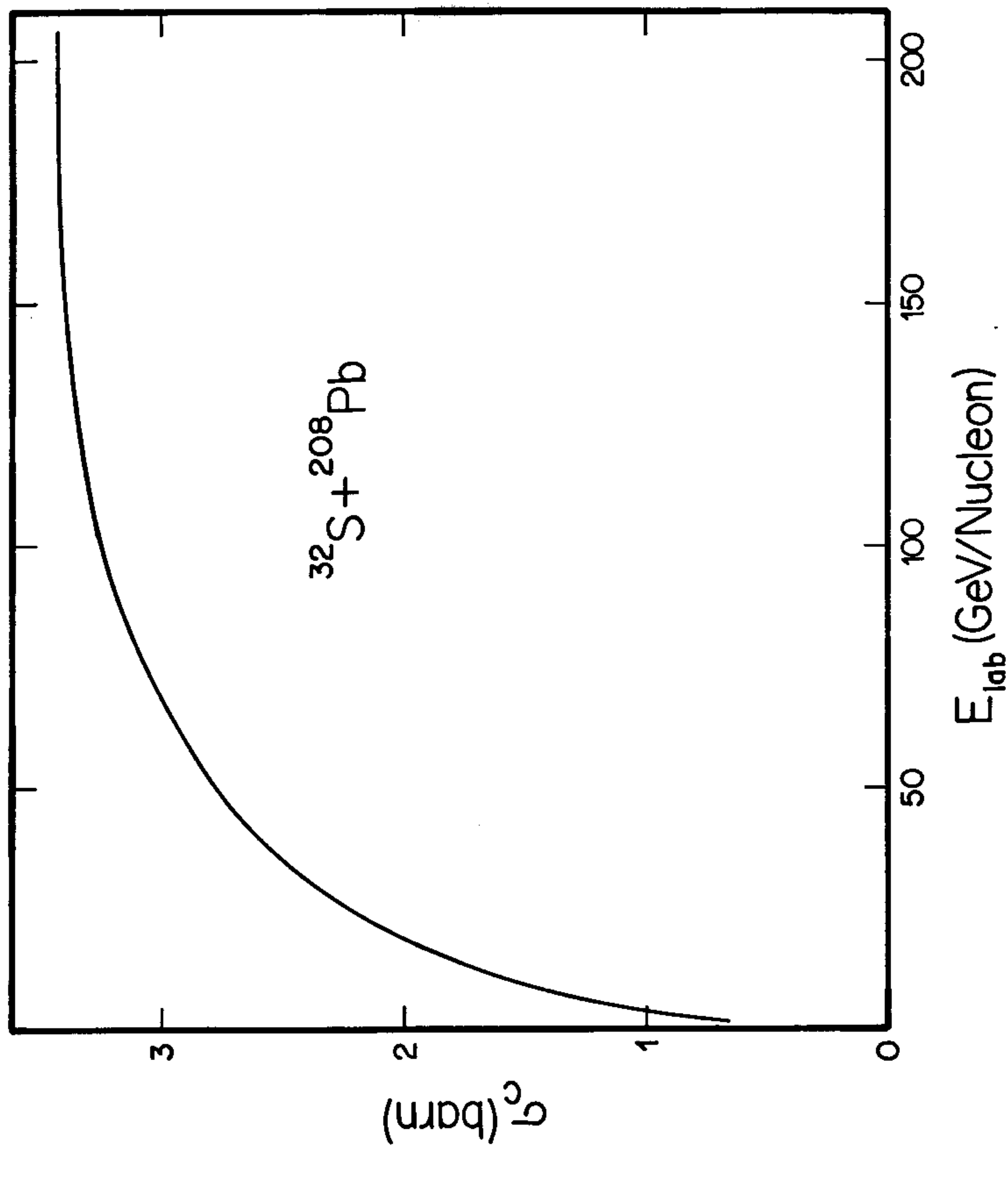

$^{208}Pb+^{208}Pb$
— σ_{single}
---- $\sigma_{double} \times 10^4$
E_{lab} (GeV/Nucleon)
σ_c (barn)
0
10
20
30
50
100
150
200

Fig. 1



$^{32}S+^{208}Pb$
E_{lab} (GeV/Nucleon)
σ_c (barn)
0
1
2
3
50
100
150
200

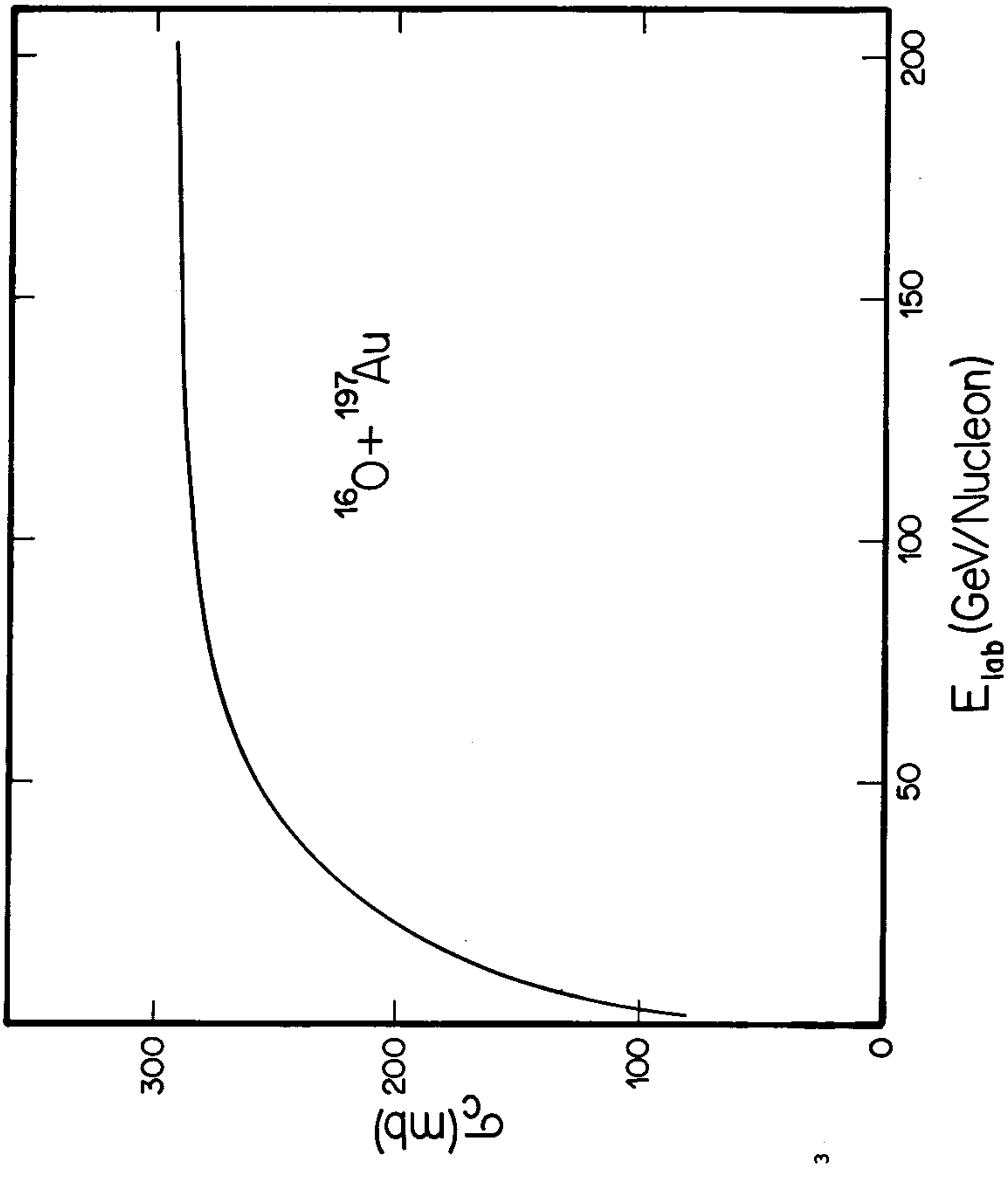

300
200
100
0
σ_r(mb)
$^{16}O + ^{197}Au$
50
100
150
200
E_{lab} (GeV/Nucleon)

Fig. 3

RHI	^{59}Co (RHI, X) ^{58}Co		^{89}Y (RHI, X) ^{88}Y		^{197}Au (RHI, X) ^{196}Au	
	P_T	P_L	P_T	P_L	P_T	P_L
^{12}C (2.1 GeV/nucleon)	0,97	0,03	0,97	0,03	0,97	0,03
^{20}Ne (2.1 GeV/nucleon)	0,97	0,03	0,97	0,03	0,97	0,03
^{40}Ar (1.8 GeV/nucleon)	0,95	0,04	0,95	0,04	0,95	0,04
^{56}Fe (1.7 GeV/nucleon)	0,954	0,046	0,954	0,046	0,954	0,046

TABLE I

RHI	^{59}Co (RHI, X) ^{58}Co		^{89}Y (RHI, X) ^{88}Y		^{197}Au (RHI, X) ^{196}Au	
	$\sigma_{teo.}$	$\sigma_{exp.}$	$\sigma_{teo.}$	$\sigma_{exp.}$	$\sigma_{teo.}$	$\sigma_{exp.}$
^{12}C (2.1 GeV/nucleon)	9.005	6 ± 9	15.6	9 ± 12	45 56	75 ± 14
^{20}Ne (2.1 GeV/nucleon)	23.3	32 ± 11	41.06	43 ± 12	121.7	153 ± 18
^{40}Ar (1.8 GeV/nucleon)	63.4	——	113 46	132 ± 17	345 9	348 ± 34
^{56}Fe (1.7 GeV/nucleon)	120.82	88 ± 14	218.1	217 ± 20	680.3	601 ± 54

TABLE II

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