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Magnetic Levitation with diamagnetic boundaries

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Abstract

The present paper is proposed to expose and explain experiments involving stable magnetic levitation. Despite the well established Earnshaw´s theorem, that avoids the levitation of magnetic and electrical charges in the presence of static electromagnetic and gravitational fields, one shows that there is at least two situations where the existence of diamagnetic materials provides a stable levitation configuration. In this sense, some experimental devices - composed by magnets, supports and diamagnetic boundaries - were set up and succeed in yielding stable levitation of magnetic bodies.

Key-words: Diamagnetic Levitation, Earnshaw, Magnetic Levitation.

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1 Introduction

The observation of levitating bodies has awoken great interest over the latest years, both by the fascination inherent to this phenomenon and the innumerous possibilities of technological applications in systems where it is extremely desirable to eliminate any type of mechanic contact, friction or dissipation. The history of this discovery goes back to 1939 when W. Braunbeck [1] succeeded in levitating small pieces of graphite in a region permeated by the field of a permanent magnet. Some years after (1947) V. Arkadiev [2] used a lead disk immersed into liquid helium (a superconductor) to levitate diminutive magnets. Since the publication of this pioneer result the experiments with superconductor levitation received great attention, increased more yet in the 80´s years with the discovery of the high-T_c superconductivity [3]. These new materials have become the magnetic levitation process especially stable due to the pinning mechanism [4] usual to the type-II superconductors. In 1996 the observation of a new kind of magnetic levitation caught the attention of scientific community: the *Levitron*^{TM}[5],[6], [7], that consists basically in a magnetic dipole in shape of a top, spinning and floating above a fixed base with permanent magnetization. The interesting fact is that the mathematical formulation used to explain the Levitron can be also employed with success for approaching the levitation of diamagnetic samples.

The levitation of purely magnetic origin is a motion of surprise between the physicists due to the well known Earnshaw´s theorem [8] (1842) that establishes the impossibility of finding a stable equilibrium point inside a configuration of static fields (of electric, magnetic or gravitational nature) [9]. By another way, one can say that the Earnshaw´s theorem does not allow the existence of a minimum for the potential magnetic energy, as it is necessary for observation of a stable equilibrium point. The presence of diamagnetic materials inside magnetic fields, however, modifies the shape of the potential energy, creating the possibility of a minimum point without violation of the Earnshaw´s theorem [9],[10], [11].

Diamagnetic materials have negative magnetic susceptibility, repulse and are repulsed by a strong magnetic field. When a external magnetic field acts on the electrons of a diamagnet sample, they tend to adjust their orbital motion in such a way to create small permanent currents that oppose to the external field. The graphite and the bismuth are the two materials that present the major (absolute) magnetic susceptibility, being therefore the most suitable for accomplishment of levitation experiments (for requiring magnetic fields of minor intensity) [11].

The first evidence that diamagnetic materials could act as a particular case beyond the generality of the Earnshaw´s theorem was given by Lord Kelvin in 1847 [12], eight years after the publication of this theorem. At the epoch he demonstrated qualitatively that these materials could be put in stable equilibrium when immersed into magnetic fields. The theoretical and experimental demonstration of this possibility was given in 1939 by W. Braunbeck [1], in whose experiments was verified the achievable character of the process. After this publication, there were several attempts of obtaining levitation with usage of diamagnetic materials. In 1956 Boerdijk used a horizontal graphite sheet (inside a magnetic field) to stabilize the levitation of a magnet above it [13]; in 1981 Ponizovskii presented some devices constructed to yield the levitation of diamagnets inside magnetic fields, and he also utilized pyrolytic graphite at stabilizing configurations for accomplishing levitation of small magnets [14]. It is important observe that both Boerdijk and Ponizovskii exposed the effect of the diamagnetic stabilization, but without developing a satisfactory theory to explain it. At the beginning of 90s, this issue gained again notoriety after that E. Beaugnon and R. Tournier [15] achieved the levitation of drops of water and ethanol, pieces of wood, plastics and other organic materials. This work showed that the major part of organic materials are levitable, since all have almost the same diamagnetic specific susceptibility [16]. In 1996 Weilert *et al.* [17] levitated drops of liquid helium in order to study noncoalescence effects.

Renewed the interest by the theme, some other works have arisen: A. Geim *et al.* [18] and M. Berry & A. Geim [10] obtained the levitation of various diamagnetic samples - including an alive frog - inside the a solenoid subjected to intense magnetic field of 16T. In this last article it is accomplished the evaluation of stability zones (regions where can occur the stable levitation inside the solenoid) based on the formalism developed to the Levitron. Finally, more recently, Geim *et al.* [9] and Simon & Geim [19] presented experiment and theory related to the levitation of small magnets inside experimental setups with diamagnetic stabilizing boundaries (disks and cylindric sheets). The use of these boundaries allows the constitution of a region of horizontal and vertical stability.

2 Earnshaw´s Theorem and stability conditions

In 1842 Samuel Earnshaw published a simple result of large validity, concerning all the interactions among particles mediated by forces that depend upon the square inverse of the distance [8], [19]: the Earnshaw´s theorem, as it is known. It asserts that no particle interacting through a $1/r^2$ force can reach the position of stable equilibrium. Going to a more fundamental level, one can notice that it is a mere consequence of mathematical structure of the $1/r$-type potentials, whose laplacian is null. In this way the total potential $U(r)$ - associated to a system of interactions of this nature - must always satisfy the Laplace equation:

$$\nabla^2 U(\overrightarrow{r}) = 0, \quad \text{where: } U(\overrightarrow{r}) = \sum C_j/r_j \ . \tag{1}$$

Being null the laplacian of the potential energy, there is no point of minimum for the energy of this system, occurring only saddle points at the tridimensional space [20]. The gravitational, electrostatic and magnetostatic interactions are all depending upon the inverse square distance, therefore they do not provide potential wells (region where there exist conditions for occurring stable equilibrium of a test particle).

For happening magnetic levitation the equilibrium must be stable, that is to say, the force $\overrightarrow{F}(r)$ must be conservative at the surroundings of the minimum point [22]. A necessary condition is:

$$\oint \overrightarrow{F}(\overrightarrow{r}).d\overrightarrow{s} = 0, \tag{2}$$

where the integral is over any small closed surface containing the equilibrium point. From the divergence theorem one has:

$$\overrightarrow{\nabla}.\overrightarrow{F}(\overrightarrow{r}) < 0. \tag{3}$$

Be U the potential energy of a body of volume V and magnetic susceptibility χ, subject to the actions of a gravitational and magnetic field:

$$U = mgz - \frac{\chi V}{2\mu_o} B^2(\mathbf{r}). \tag{4}$$

One can then easily obtain the actuating force over this body:

$$\overrightarrow{F}(\overrightarrow{r}) = -\overrightarrow{\nabla} U(\mathbf{r}) \Rightarrow \overrightarrow{\nabla}.\overrightarrow{F}(\mathbf{r}) = -\nabla^2 U(\mathbf{r}), \tag{5}$$

$$\text{If: } \overrightarrow{\nabla}.\overrightarrow{F}(\mathbf{r}) < 0 \Rightarrow \nabla^2 U(\mathbf{r}) > 0. \tag{6}$$

Due to the fact the field $\overrightarrow{B}(\overrightarrow{r})$ to have null divergence and be irrotational, its components satisfy the Laplace equation individually, what implies:

$$\nabla^2 B^2(\overrightarrow{r}) = 2\left[(\nabla B_x)^2 + (\nabla B_y)^2 + (\nabla B_z)^2\right] \Rightarrow \tag{7}$$

$$\Rightarrow \nabla^2 B^2(\mathbf{r}) > 0. \tag{8}$$

According to the equations (5), (6), (8) one notices that only the diamagnetic materials can be levitated, being impossible the levitation of paramagnets, unless the same be immersed into a region of major magnetic susceptibility. The equation (8) is the key of the question: the laplacian of $\mathbf{B}^2$ is not null, despite the individual components of $\overrightarrow{B}$ all satisfy $\nabla^2 B_i = 0$. This fact opens the possibility for the existence of a minimum point of energy (that depends linearly on $\mathbf{B}^2$).

The expression (5) establishes a necessary condition for the existence of a energy minimum. However, this condition is not enough, being still necessary to require that the concavity of the energy surface be positive (sufficient condition), constituting a well potential around the minimum point. Therefore, the sufficient condition reads:

$$\partial^2 B^2/\partial x^2 > 0;\ \partial^2 B^2/\partial y^2 > 0;\ \partial^2 B^2/\partial z^2 > 0. \tag{9}$$

In a system with rotational symmetry, the magnetic field has only the radial and axial components (B_r, B_z). It is possible to write these components as a Taylor expansion (up second order) around the equilibrium point. Making use of the equations $\nabla.\overrightarrow{B} = 0$ and $\nabla \times \overrightarrow{B} = 0$, one derives the following expressions:

$$B_z(r,z) = B_o + B_z^{'}.z + (1/2)B_z^{''}.z^2 - (1/4)B_z^{''}.r^2, \tag{10}$$

$$B_r(r,z) = -(B_z^{'}/2).r - (B_z^{''}/2)r.z, \tag{11}$$

where: $B_z^{'} = \partial B_z/\partial z|_{z=z_o}$, $B_z^{''} = \partial^2 B_z/\partial^2 z|_{z=z_o}$, and z_o is the equilibrium point.

Departing from the expressions (10, 11), the square magnitude of the magnetic field (up to second order) is:

$$B^2(r,z) = B_o^2 + (2B_oB_z^{'})z + (B_oB_z^{''} + B_z^{'2})z^2 + + \frac{1}{4}(B_z^{'\,2} - 2B_oB_z^{''})r^2. \tag{12}$$

In terms of this last equation, one can rewrite the stability conditions (9) in cylindrical coordinates:

$$\partial^2 B^2/\partial z^2 > 0 \Rightarrow (B_oB_z^{''} + B_z^{'2}) > 0, \tag{13}$$

(Vertical stability)

$$\partial^2 B^2/\partial r^2 > 0 \Rightarrow (B_z^{'\,2} - 2B_oB_z^{''}) > 0. \tag{14}$$

(Horizontal stability)

Therefore, for diamagnetic samples it is always possible to find out small regions (around the point $B_z^{''} = 0$) where the stability equations (13) and (14) are satisfied and the levitation becomes possible.

At the case of magnetic samples (a magnetic dipole, for example), the things are not so simple. Its interaction energy with a magnetic field depends linearly on the magnitude of the field $B(r,z)$, so that its total potential energy is given by:

$$U(r) = mgz - \overrightarrow{M}.\overrightarrow{B}. \tag{15}$$

From (12) one can take the magnitude of $B(r,z)$:

$$B = B_o + B_z^{'}z + \frac{1}{2}B_z^{''}z^2 + \frac{1}{4}(\frac{B_z^{'2}}{2B_o} - B_z^{''})r^2, \tag{16}$$

which substituted into (15) leads to the following expression for the total potential energy of the dipole:

$$U(r) = -M\{B_o + [B_z^{'} - \frac{mg}{M}].z + \frac{1}{2}B_z^{''}z^2 + \frac{1}{4}(\frac{B_z^{'2}}{2B_o} - B_z^{''})r^2\}. \quad (17)$$

In the last equation the term linear in z must be null to assure the vertical equilibrium between the weight and the lifting magnetic force. The new stability conditions now reads:

$$-\frac{M}{2}B_z^{''} > 0, \quad (18)$$

$$\frac{M}{4}(B_z^{''} - \frac{B_z^{'2}}{2B_o}) > 0. \quad (19)$$

At the proximity of the inflection point $\left(B_z^{''} = 0\right)$, around which it happens the levitation of the last case, the second derivative can be negative or positive; if it is negative, the vertical stability is assured, but not the horizontal; if it is positive, there is no vertical stability. In this case, the equilibrium will be achieved only after the introduction of stabilizing diamagnetic boundaries.

3 Diamagnetic Stabilizing Boundaries

Truly speaking, there are two configurations yielding magnetic levitation with diamagnets: (i) the incidence of very large magnetic fields ($10^4 - 10^5 G$) on diamagnetic objects; (ii) the incidence of conventional intensity magnetic fields on small magnetic dipoles (in association to diamagnetic stabilizing sheets). In the latter case, the required magnetic field may be generated by small permanent magnets and its intensity can be of just a few Gauss. In the first configuration the stabilization of the levitation comes merely from the diamagnetic nature of the test particle ($\chi < 0$) which, immersed into a magnetic field, presents potential energy depending on the square of the $B(r, z)$ magnitude. The use of high intensity fields is then necessary to produce such a magnetization ($M = \frac{\chi V}{\mu_o}B$) on the diamagnetic test body whose interaction with the external field is enough to compensate the weight force. In the second configuration the test particle has already an intrinsical magnetization and its potential energy depends linearly on the product $\overrightarrow{M}.\overrightarrow{B}$. Hence there is no necessity of high magnitude fields for compensating the gravitational action, but the levitation in this case just becomes assured in the presence of diamagnetic boundaries (whose interaction with the magnetic field and the dipole makes arise a kind of well potential around the minimum point).

The influence of the diamagnetic sheets upon the test magnet can be estimated through a technique similar to the method of images [19], [21], since a dipole near a diamagnetic sheet induces currents, whose effects may be reproduced by the presence of an image charge with magnetization reduced to:

$$\frac{\mu - 1}{\mu + 1}\overrightarrow{M}. \quad (20)$$

For $\chi \ll 1$ the magnetization of the charge image is reduced to half of the original $\left(\frac{1}{2}\overrightarrow{M}\right)$ and in trading of a perfect diamagnetic sheet (superconductor sheet, $\chi = -1$), one has "total reflection", resulting in an image charge with same original magnetization.

Supposing a dipole field approximation, one can determine the magnetic field $\overrightarrow{B}_I$ of the image charge, whose contribution to the energy of the test dipole $\overrightarrow{M}$ is:

$$U = -\frac{1}{2}\overrightarrow{M}.\overrightarrow{B}. \quad (21)$$

Let L be the separation between the two parallel diamagnetic sheets. Carrying out an expansion of the dipole fields around the levitation point, one obtains the following contribution to the potential energy of the test magnetic dipole:

$$U(z) = \frac{6\mu_o M^2|\chi|}{\pi L^5} z^2. \tag{22}$$

The inclusion of this term into the equations (17), (18), (19) leads to an interval for the second derivative so that there can occur levitation:

$$B_z'^2/2B_o < B_z'' < 2k/M. \tag{23}$$

The latter equation encloses an interval for values of B_z'' for which the stable levitation is achievable. The experiments described in the following section explore and confirm this theoretical possibility.

4 The experiment

Two systems have been elaborated to achieve magnetic levitation. The first one is composed by two parallel graphite sheets, two magnetic dipoles and a permanent magnet. Each graphite disk employed was 50 mm large in diameter and 5 mm in thickness. The two test bodies were a magnetic slab (with 17mm in length and 1,5 mm in thickness) and a small magnetic disk (6mm in diameter and 2mm in height). In the second experimental device the graphite sheets were substituted by bismuth ones (35mm in diameter and 3 mm in thickness). At both experiments a magnetic field (generated by a permanent magnet located above the sheets) was applied on the region between the them. The distance L between the sheets was 5 mm in Fig.1 and 8 mm in Fig. 2.

Figure 1: Experimental setup composed by bismuth sheets.

5 Final Remarks

In this work, we have presented the theory for the levitation of small generic diamagnetic samples (in the presence of very high magnetic fields) and for the levitation of magnetic dipoles (in the presence of usual magnitude magnetic fields associated to stabilizing diamagnetic sheets). The experiments carried out succeeded in obtaining stable levitation of the described magnetic dipoles, as shown in Figs. 1, 2 and 3. These outcomes are in perfect agreement with the exposed theory. A nice and interesting follow-up

Figure 2: Experimental setup composed by graphite sheets.

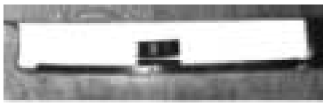

Figure 3: Close view into figure1.

on this work would be the attempting of producing translation and rotation in regime of permanent magnetic levitation.

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