The Composite Operator (CJT) Formalism in the $(\lambda\varphi^4 + \eta\varphi^6)_{D=3}$ Model at Finite Temperature

$G.N.J.A\tilde{n}a\tilde{n}os$[1], $N.F.Svaiter$[2]

Centro Brasileiro de Pesquisas Físicas-CBPF

Rua Dr.Xavier Sigaud 150, Rio de Janeiro, RJ 22290-180 Brazil

Abstract

We discuss three-dimensional $\lambda\varphi^4 + \eta\varphi^6$ theory in the context of the $1/N$ expansion at finite temperature. We use the method of the composite operator (CJT) for summing a large sets of Feynman graphs. We analyse the behavior of the thermal square mass and the thermal coupling constant in the low and high temperature limit. The existent of the tricritical point at some temperature is found using this non-pertubative method.

[1]e-mail:gino@lafex.cbpf.br

[2]e-mail:nfuxsvai@lafex.cbpf.br

1   Introduction

The conventional perturbation theory in the coupling or loop expansion can only be used for the study of small quantum corrections to classical results. When discussing truly quantum mechanical effects to any given order in such an expansion, one is not usually able to justify the neglect of yet higher order. In other way, for theories with a large N dimensional internal symmetry group there exist another perturbation scheme, the $1/N$ expansion, which circumvents this criticism. Each term in the $1/N$ expansion contains an infinite subset of terms of the loop expansion. The $1/N$ expansion has the nice property that the leading-order quantum corrections are the same order as the classical quantities. Consequently, the leading order which adequately characterizes the theory in the large N limit preserves much of the nonlinear structure of the full theory. In the next section we derive the effective action to leading order in $1/N$ for three dimensions and consequently the effective potential. In this paper we compute effective thermal mass which depend on the temperature for the field theory with φ^6 interaction on $D = 3$ Euclidean dimensions. Its is known that, in $D > 4$, such theories with φ^4 interaction are in fact free field theory, while in $D < 4$ they have a non-trivial continuum limit as an interacting field theory.

Since both the six-point coupling of $(\eta\varphi^6)_{D=3}$ and the four-point coupling of $(\lambda\varphi^4)_{D=4}$ are dimensionless one expected that have the same continuum limit. However it has been shown that, in the large N limit, the $(\eta\varphi^6)$ theory has a UV fixed point for $D = 3$ and therefore must have a second infra-red fixed point [1]. At least for large N the φ^6 theory is known to be qualitatively different from $(\lambda\varphi^4)_{D=4}$ theory. We study the large N expansion, using the methods of composite operator [2]. The organization of the letter is the following. In section II we briefly discuss the composite operator (CJT) formalism. In section III the thermal gap equation is derived. In section IV the tricritical phenomenon

is presented. Conclusions are given in section V. In this paper we use $c = k_B = \hbar = 1$.

2   The effective Potential(The CJT Formalism)

We are interested here in the most general renormalizable scalar field theories $\lambda \varphi^4 + \eta \varphi^6$

possessing an internal symmetry $O(N)$, in three dimensions. For simplicity we will call

a φ^6 model. We use the method of composite operator developed by Cornwall, Jackiw

and Tomboulis [2, 3] for summing large sets of Feynman graphs by considering only two-

particle irreducible (2PI) graphs express in terms of the exact propagators. This technique

lead to the formulation of the effective action and effective potential which is functional the

vacuum expectation value of both the quantum field $\varphi(x)$ and the time ordered product

$T\{\varphi(x)\varphi(y)\}$ of the fields. Using this method Townsend derived the effective potential of

φ^6 theory to leading order in the $\frac{1}{N}$ expansion for $D \leq 3$ [5] and proof that $\frac{1}{N}$ expansion

is consistent for φ^6 to leading order in $\frac{1}{N}$. The Lagrangian density of the $O(N)$ symmetry

φ^6 theory is :

$$\mathcal{L}(\varphi) = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m_0^2 \varphi^2 - \frac{\lambda_0}{4N!}\varphi^4 - \frac{\eta_0}{6!N^2}\varphi^6, \tag{1}$$

where the quantum field is an N-component vector in the N-dimensional internal symme-

try space. For definiteness, we work at zero-temperature; however, the finite temperature

generalizations can be easily obtained [4]. We are interested in the effective action $\Gamma(\phi)$

which governs the behavior of the expectation values $\varphi_a(x)$ of the quantum field where ϕ

is given by

$$\phi(x) \equiv \frac{\delta W(J)}{\delta J(x)} = < 0|\varphi|0 >, \tag{2}$$

where $W(J)$ is the generating functional for connected Green's functions.

$\Gamma(\phi)$ can be show to be the sum of one-particle irreducible (1PI) Feynman graphs with

a factor $\phi_a(x)$ on the external line. We may use of the formalism of composite operator

who reduce the problem to summing two particle irreducible (2PI) Feynman graphs by defining a generalized effective action $\Gamma(\phi, G)$ which is a functional not only $\phi_a(x)$, but also of the expectation values $G_{ab}(x, y)$ of the time ordered product of quantum fields $T\{\varphi(x)\varphi(y)\}$.

$$\Gamma(\phi, G) = I(\phi) + \frac{i}{2} Tr \ln G^{-1} + \frac{i}{2} Tr D^{-1}(\phi) G + \Gamma_2(\phi, G) + \ldots \quad . \tag{3}$$

where $I(\phi) = \int dx^D \mathcal{L}(\phi)$, G is a shorthand for the $G_{ab}(x, y)$ and D is defined by

$$i D^{-1} = \frac{\delta^2 I(\phi)}{\delta \phi(x) \delta \phi(y)}, \tag{4}$$

and is shorthand for $D_{ab}(\phi; x, y)$, $\Gamma_2(\phi, G)$ is computed as follows. In the classical action $I(\varphi)$ shift the field φ by ϕ. The new action $I(\varphi + \phi)$ posses term cubic and higher in φ; this define an interaction part $I_{int}(\varphi, \phi)$ where the vertices depend on ϕ. $\Gamma_2(\phi, G)$ is given by sum of all (2PI) vacuum graphs in a theory with vertices determined by $I_{int}(\varphi, \phi)$ and the propagators set equal to $G(x, y)$. The trace and logarithm in eq.(3) are functional.

$$
\begin{aligned}
\mathcal{L}_{int}(\varphi, \phi) =\ & -\frac{1}{2}\left(\frac{\lambda_0 \phi_a}{3N} + \frac{\eta_0 \phi^2 \phi_a}{30N^2}\right)\varphi_a \varphi^2 - \left(\frac{8\eta_0 \phi_a \phi_b \phi_c}{6N^2}\right)\varphi_a \varphi_b \varphi_c - \frac{1}{4!N}\left(\lambda_0 + \frac{\eta_0 \phi^2}{10N}\right)\varphi^4 \\
& - \left(\frac{12\eta_0 \phi_a \phi_b}{6!N^2}\right)\varphi_a \varphi_b \varphi^2 - \frac{1}{5!}\left(\frac{\eta_0 \phi_a}{N^2}\varphi_a \varphi^4\right) - \frac{\eta_0}{6!N^2}\varphi^6 .
\end{aligned}
\tag{5}
$$

The effective action as usually defined is found by solving for $G_{ab}(x, y)$ in the equation

$$\frac{\delta \Gamma(\phi, G)}{\delta G_{ab}(x, y)} = 0 \tag{6}$$

and substituting in the generalized effective action $\Gamma(\phi, G)$.

The vertices in the above equation contains factor $1/N$ or $1/N^2$, but a φ loop gives a factor of N provided the $O(N)$ isospin flows around it alone and not into another part of the graph. We usually call such loops bubbles. Then the leading order in $1/N$ the vacuum graphs are bubbles trees with two or three bubbles at each vertex. The (2PI) graphs are shown in figure.(1). It is straightforward to obtain
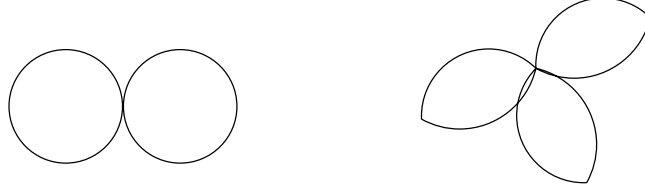
Figure 1: The 2PI vacuum graphs

$$\Gamma_2(\phi, G) = \frac{-1}{4!N} \int d^D x \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) [G_{aa}(x, x)]^2 - \frac{\eta_0}{6!N^2} \int d^D x [G_{aa}(x, x)]^3. \quad (7)$$

Therefore eq.(6) becomes

$$\frac{\delta \Gamma(\phi, G)}{\delta G_{ab}(x, y)} = \frac{1}{2} (G^{-1})_{ab}(x, y) + \frac{i}{2} D^{-1}(\phi) - \frac{1}{12N} \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) [\delta_{ab} G_{cc}(x, x)] \delta^D(x - y)$$

$$- \frac{3\eta_0}{6!N} \delta_{ab} [G_{cc}(x, x)]^2 \delta^D(x - y) = 0. \quad (8)$$

Rewriting this equation, we obtain the gap equation

$$(G^{-1})_{ab}(x, y) = D^{-1}(\phi) + \frac{i}{6N} \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) [\delta_{ab} G_{cc}(x, x)] \delta^D(x - y) +$$

$$\frac{i\eta_0}{5!N^2} \delta_{ab} [G_{cc}(x, x)]^2 \delta^D(x - y). \quad (9)$$

Hence

$$\frac{i}{2} Tr D^{-1} G = \frac{1}{12N} \int d^D x \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) [G_{aa}(x, x)]^2 + \frac{2\eta_0}{6!N^2} \int d^D x [G_{aa}(x, x)]^3 + cte. \quad (10)$$

Using eqs. (9) and (10) in (7) we find the effective action

$$\Gamma(\phi) = I(\phi) + \frac{i}{2} Tr \ln G^{-1} + \frac{1}{4!N} \int d^D x \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) [G_{aa}(x, x)]^2 +$$

$$\frac{2\eta_0}{6!N^2} \int d^D x [G_{aa}(x, x)]^3, \quad (11)$$

where G_{ab} is given implicitly by eq.(9). The last two term on the r.h.s of eq.(11) is the

leading two-loop contribution to the effective action for composite operators in $\lambda \phi^4 + \eta \phi^6$

theory. As usual we may simplify by separating G_{ab} into transverse and longitudinal components, so

$$G_{ab} = (\delta_{ab} - \frac{\phi_a \phi_b}{\phi^2})g + \frac{\phi_a \phi_b}{\phi^2} \tilde{g} \ , \tag{12}$$

in this form we can invert G_{ab},

$$(G)_{ab}^{-1} = (\delta_{ab} - \frac{\phi_a \phi_b}{\phi^2})g^{-1} + \frac{\phi_a \phi_b}{\phi^2} \tilde{g}^{-1} \ . \tag{13}$$

Now we can take the trace respect the indices,

$$G_{aa} = Ng + O(1), \quad (G)_{aa}^{-1} = Ng^{-1} + O(1) \ . \tag{14}$$

From this equation to leading order in $\frac{1}{N}$, G_{ab} is diagonal in a, b. Substituting eq.(14) into eq.(11) and eq.(9) and keep only the leading order one finds the daisy and superdaisy resumed effective potential for the $\lambda \phi^4 + \eta \phi^6$ theory is given by:

$$\begin{aligned}
\Gamma(\phi) &= I(\phi) + \frac{iN}{2} Tr \ln g^{-1} + \frac{N}{4} \int d^D x \left(\lambda_0 + \frac{\eta_0 \phi^2}{10N} \right) g^2(x,x) + \\
&\quad \frac{2N\eta_0}{6!} \int d^D x g^3(x,x) + O(1),
\end{aligned} \tag{15}$$

and the gap equation

$$g^{-1}(x,y) = i \left[\Box + m_0^2 + \frac{\lambda_0}{6}(\frac{\phi^2}{N} + g(x,x)) + \frac{\eta_0}{5!}(\frac{\phi^2}{N} + g(x,x))^2 \right] \delta^D(x - y) + O(\frac{1}{N}) \tag{16}$$

Now it is convenient to continue all momenta to Euclidean values ($p_0 = ip_4$) and take the following Ansatz for $g(x,y)$,

$$g(x,y) = \int \frac{d^D p}{(2\pi)^D} \frac{\exp^{i(x-y)p}}{p^2 + M^2(\phi)} \tag{17}$$

and substituting this in eq.(16) we get the expression for the gap equation:

$$M^2(\phi) = m_0^2 + \frac{\lambda_0}{6} \left(\frac{\phi^2}{N} + F(\phi) \right) + \frac{\eta_0}{5!} \left(\frac{\phi^2}{N} + F(\phi) \right)^2 \ , \tag{18}$$

where $F(\phi)$ is given by

$$F(\phi) = \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 + M^2(\phi)},$$ (19)

and the effective potential in Euclidean-space can be express,

$$Veff(\phi) = V_0(\phi) + \frac{N}{2} \int \frac{d^D p}{(2\pi)^D} \ln\left[p^2 + M^2(\phi)\right] - \frac{N}{4!}(\lambda_0 + \frac{\eta_0\phi^2}{10N})F(\phi)^2 - \frac{2N\eta F(\phi)^3}{6!}.$$ (20)

where $V_0(\phi)$ is the classical potential.

3 The Gap-Equation for the model $(\lambda\varphi^4 + \eta\varphi^6)_{D=3}$ at Finite Temperature

Let us suppose that our system is in equilibrium with a thermal bath. At the one-loop approximation the thermal mass and coupling constant for the $\lambda\varphi^4$ model in a D-dimensional Euclidean space have been obtained in a previous work [6] and for the the theory $(\lambda\varphi^4)_D$ is obtained using the composite operator method [7]. It is clear that for $D = 3$ the $(\lambda\phi^4)_D$ model is not the most general perturbative renormalizable model and we are able to introduce a ϕ^6 term preserving the renormalizability of theory. For $D = 3$ the $\lambda\varphi^4 + \eta\varphi^6$ model is the most general O(N) symmetric model which preserves the perturbative renormalizability. Most general models with cubic symmetry can be also studied. In the conclusion we will briefly discuss this model.

To study the temperature effects in quantum field theory the most used methods is the imaginary time Green function approach [4], which amounts to replace the continuous four momenta k_4 by discrete ω_n an integration by a summation $(\beta = \frac{1}{T})$:

$$k_4 \rightarrow \omega_n = \frac{2\pi n}{\beta}, \quad n = 0, \pm 1, \pm 2, ...$$

$$\int \frac{d^D k}{(2\pi)^D} \rightarrow \sum_n \frac{1}{\beta} \int \frac{d^{D-1} k}{(2\pi)^{D-1}}.$$ (21)

Then the gap equation at finite temperature for the theory $\lambda\varphi^4 + \eta\varphi^6$ is given by (see eq.(18)),

$$M_\beta^2(\phi) = m_0^2 + \frac{\lambda_0}{6}\left(\frac{\phi^2}{N} + F_\beta(\phi)\right) + \frac{\eta_0}{5!}\left(\frac{\phi^2}{N} + F_\beta(\phi)\right)^2, \tag{22}$$

where

$$F_\beta(\phi) = \frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^{D-1}p}{(2\pi)^{D-1}}\frac{1}{\omega_n^2 + p^2 + M_\beta^2(\phi)}. \tag{23}$$

In order to regularized this expression we use a mixing between dimensional regularization and analytic regularization. For this purpose we define the expression $I_\beta(d,s)$ as :

$$I_\beta(D,s,m) = \frac{1}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\frac{1}{(\omega_n^2 + k^2 + m^2)^s}. \tag{24}$$

Using analytic extension of the inhomogeneous Epstein zeta function it is possible to obtain the analytic extension of $I_\beta(D,s,m_\beta)$;

$$I_\beta(D,s,m) = \frac{m^{D-2s}}{(2\pi^{1/2})^D\Gamma(s)}\left[\Gamma\left(s - \frac{D}{2}\right) + 4\sum_{n=1}^{\infty}\left(\frac{2}{mn\beta}\right)^{D/2-s}K_{D/2-s}(mn\beta)\right] \tag{25}$$

where $K_\mu(z)$ is the modified Bessel function of third kind. Fortunately for $D = 3$ the analytic extension of the function $I_\beta(D,s,m_\beta)$ is finite and can be express in a closed form [8] (note in $D = 3$ we have no pole, at least in this approximation), and particular

$$F_\beta(\phi) = I_\beta(3,1,M_\beta(\phi)) = -\frac{M_\beta(\phi)}{4\pi}\left(1 + \frac{2\ln(1 - e^{-M_\beta(\phi)\beta})}{M_\beta(\phi)\beta}\right). \tag{26}$$

If we have no spontaneous symmetric breaking, we define the thermal effective mass as

$$\left.\frac{\partial^2 Veff(\phi)}{\partial\phi_a^2}\right|_{\phi=0} = m_\beta^2 \tag{27}$$

and is not difficult to show that,

$$m_\beta^2 = M_\beta^2(0) = m^2 + \frac{\lambda}{6}F_\beta(0) + \frac{\eta}{5!}F_\beta(0)^2 \tag{28}$$

and

$$F_\beta(0) = -\frac{m_\beta}{4\pi}\left(1 + \frac{2\ln(1 - e^{-m_\beta\beta})}{m_\beta\beta}\right). \tag{29}$$

where the m^2, λ and η are the finite parameter of the theory at tree-level. So eq.(28) and eq.(29) determined the behavior of the effective mass with the temperature and from this relations it is possible to show that the mass increase with the temperature.

And the expression for the effective coupling constant can be derived from the relation given by:

$$\frac{\partial^4 V eff(\phi)}{\partial \phi_a^4}\bigg|_{\phi=0} = 3\frac{\partial^2 M_\beta^2(\phi)}{\partial^2 \phi_a}\bigg|_{\phi=0} \qquad (30)$$

where $M_\beta^2(\phi)$ is given by eq.(18), so after some algebra we find that,

$$\lambda_\beta = \frac{\lambda + \frac{\eta}{10}F_\beta(0)}{1 - \left[\frac{\lambda_0}{6} + \frac{2\eta_0}{5!}F_\beta(0)\right]\frac{\partial F_\beta(0)}{\partial m_\beta^2}}; \qquad (31)$$

This is the thermal effective coupling constant at leading order. In order to found the behavior of the thermal effective coupling constant with the temperature, we used the solution for the eq.(28) and replace in eq.(31). From this we conclude that the effective thermal coupling constant increase with the temperature. In the next section we discuss the existence of the tricritical phenomenon in this model at finite temperature.

4   The tricritical phenomenon

In the last section we obtained the thermal correction to the square mass m_β^2 an the coupling constant λ_β in absence of spontaneous symmetry breaking. The tricritical phenomenon occurs when $\lambda_\beta = m_\beta^2 = 0$. If it happens we conclude that spontaneous symmetry breaking must be occur. At tree level this happen when the classical potential $V_0(\phi)$, develops an absolute minimum for $\phi^2 \neq 0$. If we consider quantum effects, then from eq.(20) we have the relation(we discus at $T = 0$)

$$\frac{\partial V}{\partial \phi_a} = \left[m^2 + \frac{\lambda}{6}\left(\frac{\phi^2}{N} + F(\phi)\right) + \frac{\eta}{5!}\left(\frac{\phi^2}{N} + F(\phi)\right)^2\right]\phi_a \qquad (32)$$

If we have spontaneous symmetry breaking the next relation must be satisfied in terms of the renormalized parameter,

$$\left[m^2 + \frac{\lambda}{6} \left(\frac{\phi^2}{N} + F(\phi) \right) + \frac{\eta}{5!} \left(\frac{\phi^2}{N} + F(\phi) \right)^2 \right] = 0 \ . \tag{33}$$

But form eq.(19) $F(\phi)$ can be express as (using dimensional regularization)

$$F(\phi) = -\frac{1}{4\pi} \left[m^2 + \frac{\lambda}{6} \left(\frac{\phi^2}{N} + F(\phi) \right) + \frac{\eta}{5!} \left(\frac{\phi^2}{N} + F(\phi) \right)^2 \right]^{\frac{1}{2}} \ . \tag{34}$$

From the last two relations we conclude that $F(\phi) = 0$ at the minimum. This implies that eq.(33) becomes

$$m^2 + \frac{\lambda}{6} \left(\frac{\phi^2}{N} \right) + \frac{\eta}{5!} \left(\frac{\phi^2}{N} \right)^2 = 0 \ . \tag{35}$$

This is the same equations as for the tree approximation, but in terms of the renormalized parameters. And the effective square mass vanishes at the minimum i.e $M^2(\phi) = 0$, this is consequence of the Goldstone's theorem. From eq.(35) provides tree possibilities for symmetry breaking :

$$\begin{aligned} &1. \quad \lambda \geq 0 \ , \quad m^2 < 0 \\ &2. \quad \lambda < 0 \ , \quad m^2 < 0 \\ &3. \quad \lambda < 0 \ , \quad m^2 \geq 0. \end{aligned} \tag{36}$$

If we consider the region 3 at the tree level and for $|\lambda|$

$$\frac{5\lambda^2}{8\eta} \ > \ m^2 \ . \tag{37}$$

sufficiently large we have two real positive zeros for $V_0(\phi)$. In this case for $m^2 > 0$ at $\frac{5\lambda^2}{8\eta} = m^2$ we have a first order transition as shown in figure.(2). Then the point $m^2 = 0$, $\lambda = 0$ is the tricritical point, which the first and second order transitions meet here. Of course the relation in eq.(37) was found by requiring that the minimum of the classical potential in region 3 be absolute. If we consider quantum corrections this relation must be modified. At finite temperature the effective parameter m^2 and λ change with the
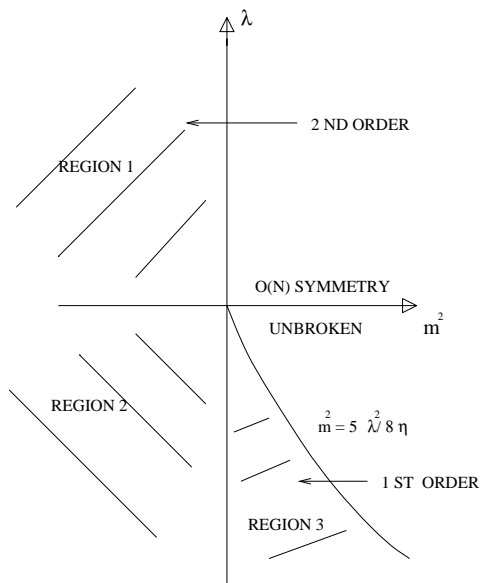
Figure 2: The Regions in λ , m^2 of symmetry breaking for fixed η

temperature as we discussed in the preceding section. At sufficiently high temperature we guess form eq.(31) that there is no symmetry breaking. Then in order to use the relations of the last section we first started when the symmetry is present and from this we prove that it can be exist an intermediate temperature β_c at which $m^2_{\beta_c} = 0$, $\lambda_{\beta_c} = 0$.

First we note that the eq.(26) is valid only for $m^2_\beta > 0$ and this expression is not useful to find the critical temperature. So we regularized this expression in the limit $m^2_\beta = 0$, $(m^2_\beta > 0)$ and for $D = 3 - \epsilon$ we get,

$$\lim_{\substack{m_\beta \to 0 \ (m_\beta > 0)}} I_\beta(3 - \epsilon, s = 1, m_\beta) = \frac{\Gamma(\frac{1-\epsilon}{2}) \, \zeta(1-\epsilon)}{2(\pi)^{3/2 - \epsilon/2} \beta_c^{1-\epsilon}} \tag{38}$$

the finite part $Ir(\beta_c)$ given by (we use the expansion of $\zeta(z)$ an $\Gamma(z)$ function in order to find the regular part),

$$Ir(\beta_c) = \frac{1}{2\pi\beta_c} \ln\left(\frac{k}{\pi \beta_c^2 \mu_c^2}\right); \tag{39}$$

where $\mu_c \approx m$ is the mass parameter and k is a numeric constant. The infrared-induced uncertainties can be subsumed into single parameter μ_c [9]. So the tricritical phenomenon

occurs when

$$m_\beta^2 = 0 = m^2 + \frac{\lambda}{6}Ir(\beta_c) + \frac{\eta}{5!}Ir^2(\beta_c),$$

$$\lambda_\beta = 0 = \lambda + \frac{\eta}{10}Ir(\beta_c). \tag{40}$$

And from this relations we conclude that if the tricritical phenomena occurs we must be keep the next two following relation:

$$\lambda^2 = \frac{6\eta m^2}{5}$$

$$T_c^2 = \frac{\mu_c^2 \pi}{k} \exp\left(\frac{-\lambda\pi}{5\eta T_c}\right). \tag{41}$$

Where T_c is the critical temperature at which the tricritical phenomenon occurs. Of course the parameters (m^2, λ) must be in the region 3 in order to occur the tricritical phenomena an obey the relation eq.(41).

5   Conclusions

We have done in this paper an analysis of the vector model $\lambda\varphi^4 + \eta\phi^6$ in $D = 3$ Euclidean dimensions at finite temperature. The form of the thermal corrections to the mass and coupling constant have been found using resummation methods in the leading order $\frac{1}{N}$ approximation (Hartree-Fock approximation). We conclude that these parameters increase with the temperature. This ressult is consistent with previous work [7] in the sense that we are able to recover in the limit $\eta = 0$.

We discussed the existent of the tricritical phenomenon at finite temperature and we have been found an expression for the critical temperature at which the thermal effective mass and coupling constant vanishes, and this shown that the tricritical phenomena occurs at an intermediate temperature T_c.

A natural extension of this work is to study the cubic anisotropic model with the

following Lagrange density

$$\mathcal{L}_{int} = \lambda_1 \sum_{i=1}^{N} (\phi_i \phi_i)^2 + \eta_1 \sum_{i=1}^{N} (\phi_i \phi_i)^3 + \lambda_2 \sum_{i=1}^{N} (\phi_i)^4 + \eta_2 \sum_{i=1}^{N} (\phi_i)^6. \tag{42}$$

In $D = 4$ with the $\lambda \phi^4$ model appear different fixed points (a Gaussian, Heisenberg, Ising and cubic) [10] and between them the Heisenberg fixed point is the only stable when $M < 4$. For $M > 4$ the cubic fixed point becomes stable. This model is under investigation by the authors.

6   Acknowlegements

References

[1] William A.Barden, Moshe Moshe and Myron Bander, Phys.Rev.Lett. **52**, 1118 (1984)

[2] J.M.Cornwall, R.Jackiw and E.Tomboulis, Phys.Rev.D **10**, 2428 (1974).

[3] R.Jackiw, *Diverses topics in Theoretical and Mathematical Physics*, World Scientific Publishing Co.Pte.Ltd (1995).

[4] L.Dolan and R.Jackiw, Phys.Rev.D **9**, 3320 (1974), J.Kapusta, D.B.Reiss and S.Rudaz, Nucl.Phys. **B263**, 207 (1986),

[5] P.K.Townsend, Phys.Rev.D **12**, 2269 (1975) , Nucl.Phys. **B118**, 199 (1977).

[6] A.P.C.Malbouisson and N.F.Svaiter, Physica A **233**, 573 (1996).

[7] G.N.J Añaños, A.P.C.Malbouisson and N.F.Svaiter, Nucl.Phys. **B547**, 221 (1999)

[8] G.N.J.Ananos and N.F.Svaiter, Physica A **241**, 627 (1997).

[9] M.B. Einhorn and D.R.T. Jones, Nucl.Phys. **B398**, 611 (1993), G. Bimonte, D. Iñiguez, A. Tarancón and C.L. Ullod, Nucl.Phys.B **490**, 701 (1997).

[10] D.Amit in Field Theory, the Renormalization Group and Critical Phenomena, (McGraw-Hill International Book Company, 1978)