CBPF-NF-054/95

# Supersymmetric Generalization of the Tensor Matter Fields

*Vitor Lemes, Alvaro Nogueira* and *J.A.Helayël-Neto*

Centro Brasileiro de Pesquisas Físicas - CBPF
Rua Dr. Xavier Sigaud, 150
22290-180 – Rio de Janeiro, RJ – Brazil

Abstract

The supersymmetric generalization of a recently proposed abelian axial gauge model with antisymmetric tensor matter fields is presented.

**Key-words:** Supersymmetry; Tensor fields.
**PACS numbers:** 11.30.Pb

# 1 Introduction

Recently, a line of investigation has been proposed by Avdeev and Chizhov [1] that consists in treating skew-symmetric rank-2 tensor fields as matter rather than gauge degrees of freedom. The model studied in Ref[1] has been further reassessed from the point of view of renormalization in the framework of BRS quantization [2]. In view of the potential relevance of matter-like tensor fields for phenomenology [1], it is our purpose in this paper to discuss some facts concerning the formulation of an $N = 1$ supersymmetric Abelian gauge model realizing the coupling of gauge fields to matter tensor fields and their partners. One intends here to present a superspace formulation of the model and exploit the possible relevance of extra bosonic supersymmetric partners (complex scalars) for the issue of symmetry breaking.

The present work is outlined as follows: in Section 2, one searches for the supermultiplet that accomodates the matter tensor field and discusses its self-coupling; the coupling to the gauge supermultiplet is pursued in Sections 3 and 4; in Section 5, one couples the well-known O`Raifeartaigh model [5] to the tensor-field supermultiplet and discusses some features concerning spontaneous supersymmetry breaking. Finally, general conclusions are drawn in Section 6.

# 2 Supersymmetrizing the tensor field

Adopting the spinor algebra conventions and the superspace parametrization of Ref.[6], the superfield that accomodates the skew-symmetric rank-2 tensor amongst its components is a spinor multiplet subject to the chirality constraint:

$$\begin{aligned}\Sigma_a = \; & \psi_a + \theta^b \Lambda_{ba} + \theta^2 \mathcal{F}_a - i\theta^c \sigma^{\mu}_{c\dot{c}} \bar{\theta}^{\dot{c}} \partial_\mu \psi_a \\ & - i\theta^c \sigma^{\mu}_{c\dot{c}} \bar{\theta}^{\dot{c}} \partial_\mu \theta^b \Lambda_{ba} - \tfrac{1}{4}\theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu \psi_a,\end{aligned} \tag{2.1}$$

$$\begin{aligned}\overline{\Sigma}_{\dot{a}} = \; & \overline{\psi}_{\dot{a}} + \bar{\theta}_{\dot{b}} \bar{\Lambda}^{\dot{b}}{}_{\dot{a}} + \bar{\theta}^2 \overline{\mathcal{F}}_{\dot{a}} + i\theta^c \sigma^{\mu}_{c\dot{c}} \bar{\theta}^{\dot{c}} \partial_\mu \overline{\psi}_{\dot{a}} \\ & + i\theta^c \sigma^{\mu}_{c\dot{c}} \bar{\theta}^{\dot{c}} \partial_\mu \bar{\theta}_{\dot{b}} \bar{\Lambda}^{\dot{b}}{}_{\dot{a}} - \tfrac{1}{4}\theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu \overline{\psi}_{\dot{a}},\end{aligned} \tag{2.2}$$

$$\overline{D}_{\dot{b}} \Sigma_a = D_b \overline{\Sigma}_{\dot{a}} = 0, \tag{2.3}$$

where $\psi_a$ and $\mathcal{F}_a$ are chiral spinors and $\Lambda_{ba}$, $\bar{\Lambda}_{\dot{b}\dot{a}}$ are decomposed as:

$$\begin{aligned}\Lambda_{ba} &= \varepsilon_{ba} \rho + \sigma^{\mu\nu}_{ba} \lambda_{\mu\nu} \;, \\ \bar{\Lambda}_{\dot{b}\dot{a}} &= -\varepsilon_{\dot{b}\dot{a}} \rho^* - \overline{\sigma}^{\mu\nu}_{\dot{b}\dot{a}} \lambda^*_{\mu\nu} \;.\end{aligned} \tag{2.4}$$

According to the chiral properties of the superfield $\Sigma_a$, the $\lambda_{\mu\nu}$ tensor corresponds to the $(1,0)$-representation of Lorentz group.On the other hand, $\lambda^*_{\mu\nu}$ yields the $(0,1)$-representation. We then write:

$$\begin{aligned} \lambda_{\mu\nu} &= T_{\mu\nu} - i\widetilde{T}_{\mu\nu} \ , \\ \lambda^*_{\mu\nu} &= T_{\mu\nu} + i\widetilde{T}_{\mu\nu} \ , \end{aligned} \tag{2.5}$$

where $\widetilde{T}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}T^{\alpha\beta}$. Notice also that $\widetilde{\lambda}_{\mu\nu} = i\lambda_{\mu\nu}$ and $(\widetilde{\lambda^*_{\mu\nu}}) = -i\lambda^*_{\mu\nu}$.

The canonical dimensions of the component fields read as below:

$$\begin{aligned} d(\psi) &= d(\overline{\psi}) = \tfrac{1}{2} \\ d(\rho) &= d(\lambda_{\mu\nu}) = 1 \\ d(\mathcal{F}) &= d(\overline{\mathcal{F}}) = \tfrac{3}{2} \ . \end{aligned} \tag{2.6}$$

Based on dimensional arguments, we propose the following superspace action for the $\Sigma_a$ superfield:

$$\mathcal{S} = \int d^4x \, d^2\theta d^2\overline{\theta} \ \frac{-1}{32}\{D^a\Sigma_a\overline{D}_{\dot{a}}\overline{\Sigma}^{\dot{a}} + q\Sigma^a\Sigma_a\overline{\Sigma}_{\dot{a}}\overline{\Sigma}^{\dot{a}}\}. \tag{2.7}$$

To check whether such an action is actually the supersymmetric extension of the model that treats $T_{\mu\nu}$ as a matter field [1], we have now to write down eq(2.7) in terms of the component fields $\psi, \rho, \lambda_{\mu\nu}$ and $\mathcal{F}$:

$$\begin{aligned} \mathcal{S} \ &= \textstyle\int d^4x \Big( + \partial^\mu\rho\partial_\mu\rho^* - 16\partial^\mu\lambda_{\mu\nu}\partial_\alpha\lambda^{*\alpha\nu} + i\overline{\mathcal{F}}^{\dot{a}}\overline{\sigma}^\mu_{\dot{a}a}\partial_\mu\mathcal{F}^a - i\overline{\psi}^{\dot{a}}\overline{\sigma}^\mu_{\dot{a}a}\partial_\mu\partial^\nu\partial_\nu\psi^a \\ &-q\tfrac{1}{\sqrt{2}}(\rho^2 - 4\lambda_{\mu\nu}\lambda^{\mu\nu})\tfrac{1}{\sqrt{2}}(\rho^{*2} - 4\lambda^*_{\alpha\beta}\lambda^{*\alpha\beta}) + 4q\lambda^{\mu\nu}\lambda_{\mu\nu}\overline{\mathcal{F}}_{\dot{a}}\overline{\psi}^{\dot{a}} + 4q\lambda^{*\mu\nu}\lambda^*_{\mu\nu}\mathcal{F}^a\psi_a \\ &-2q\mathcal{F}^a\psi_a\overline{\mathcal{F}}_{\dot{a}}\overline{\psi}^{\dot{a}} - q\rho^2\overline{\mathcal{F}}_{\dot{a}}\overline{\psi}^{\dot{a}} - q(\rho^*)^2\mathcal{F}^a\psi_a + \tfrac{q}{2}\psi^a\psi_a\partial^\mu\partial_\mu(\overline{\psi}_{\dot{a}}\overline{\psi}^{\dot{a}}) \\ &-iq\rho\psi^a\sigma^\mu_{a\dot{a}}\partial_\mu(\overline{\psi}^{\dot{a}}\rho^*) + 4q\rho\psi^a\sigma^\mu_{a\dot{a}}\partial_\beta(\lambda^{*\beta}{}_\mu\overline{\psi}^{\dot{a}}) - 4q\lambda_{\mu\beta}\partial^\beta(\rho^*\overline{\psi}_{\dot{a}})\overline{\sigma}^{\mu\dot{a}a}\psi_a \\ &-16iq\lambda_{\mu\alpha}\partial_\beta(\lambda^{*\beta\alpha}\overline{\psi}_{\dot{a}})\overline{\sigma}^{\mu\dot{a}a}\psi_a\Big). \end{aligned} \tag{2.8}$$

Using $\lambda_{\mu\nu} = \frac{1}{4}(T_{\mu\nu} - i\widetilde{T}_{\mu\nu})$, we have that:

$$16\partial^\alpha\lambda_{\alpha\mu}\partial_\beta\lambda^{*\beta\mu} = 2\partial^\alpha T_{\alpha\mu}\partial_\beta T^{\beta\mu} - \frac{1}{2}\partial^\alpha T^{\mu\nu}\partial_\alpha T_{\mu\nu} \tag{2.9}$$

The action above displays the terms proposed by Avdeev et al. in Ref.[1]; besides the anti-symmetric tensor, there appear a complex scalar and a pair of spinors as its supersymmetric partners ($\psi_a$ is a non - physical fermion, whereas $\mathcal{F}_a$ corresponds to a physical Weyl spinor).

# 3 The gauging of the model

In order to perform the gauging of the model described by eq(2.7), one proceeds along the usual lines and introduces a chiral scalar superfield, $\Lambda$, to act as the gauge parameter:

$$\Lambda = (1 - i\theta^a \sigma^\mu_{a\dot{a}} \overline{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4}\theta^2 \overline{\theta}^2 \partial_\mu \partial^\mu)(\phi + \theta^b w_b + \theta^2 \pi) \tag{3.1}$$

$$\overline{\Lambda} = (1 + i\theta^a \sigma^\mu_{a\dot{a}} \overline{\theta}^{\dot{a}} \partial_\mu - \frac{1}{4}\theta^2 \overline{\theta}^2 \partial_\mu \partial^\mu)(\phi^* + \overline{\theta}_{\dot{b}} \overline{w}^{\dot{b}} + \overline{\theta}^2 \overline{\pi}). \tag{3.2}$$

The infinitesimal gauge transformations of the superfields $\Sigma$ and $\overline{\Sigma}$ are:

$$\begin{aligned} \delta\Sigma_a &= ih\Lambda\, \Sigma_a \\ \delta\overline{\Sigma}_{\dot{a}} &= -ih\overline{\Lambda}\, \overline{\Sigma}_{\dot{a}}, \end{aligned} \tag{3.3}$$

and the behaviour of $(D^a\Sigma_a)$ and $(\overline{D_{\dot{a}}}\ \overline{\Sigma^{\dot{a}}})$ under finite tranformations read:

$$\begin{aligned} D^a\, \Sigma'_a &= e^{ih\Lambda}\, (D^a\, \Sigma_a + ihD^a \Lambda\, \Sigma_a) \\ \overline{D_{\dot{a}}}\, \overline{\Sigma'^{\dot{a}}} &= e^{-ih\overline{\Lambda}} (\overline{D_{\dot{a}}}\, \overline{\Sigma^{\dot{a}}} - ih\overline{D_{\dot{a}}}\, \overline{\Lambda}\, \overline{\Sigma^{\dot{a}}}). \end{aligned} \tag{3.4}$$

To gauge-covariantize the superspace derivatives, one introduces a gauge connection superfield:

$$D_a \rightarrow \nabla_a = D_a + ih\Gamma_a, \tag{3.5}$$

in such a way that $\Gamma_a$ transforms like

$$\Gamma'_a = \Gamma_a - D_a\Lambda. \tag{3.6}$$

This yields:

$$(\nabla^a \Sigma_a)' = e^{ih\Lambda} (\nabla^a \Sigma_a). \tag{3.7}$$

To achieve a U(1)-invariant action, one proposes

$$\mathcal{S} = \int d^4x\, d^2\theta d^2\overline{\theta} \left( \nabla^a \Sigma_a\, e^{hV}\, \overline{\nabla_{\dot{a}} \Sigma^{\dot{a}}} \right), \tag{3.8}$$

where V is the real scalar superfield [9] that accomplishes the gauging of supersymmetric QED [10]:

$$V' = V + i\, (\overline{\Lambda} - \Lambda). \tag{3.9}$$

At this point, the gauge sector displays more degrees of freedom that it is actually required to perform the gauging. There are component vector fields in $\Gamma_a$ and $V$. However, we notice that the superfield $\Gamma_a$ is not a true independent gauge potential. Indeed,

$$\Gamma_a = -iD_a V \tag{3.10}$$

reproduces correctly the gauge tranformation of $\Gamma_a$ and, at the same time, eliminates the redundant degrees of freedom that would be otherwise present, if we were to keep

$\Gamma_a$ and $V$ as gauge superfields. Therefore, the locally U(1)- invariant action takes over the form:

$$\begin{aligned}\mathcal{S} &= \int d^4x\, d^2\theta \;\; + \tfrac{-1}{128}\left(\overline{D}^2(e^{-V}D^a e^V)\overline{D}^2(e^{-V}D_a e^V)\right)\\ &+ \int d^4x\, d^2\theta d^2\overline{\theta}\;\; \tfrac{-1}{32}\left(\nabla^a\Sigma_a\, e^{hV}\; \overline{\nabla_{\dot a}\Sigma^{\dot a}} + q\Sigma^a\Sigma_a e^{2hV}\overline{\Sigma}_{\dot a}\overline{\Sigma}^{\dot a}\right),\end{aligned} \tag{3.11}$$

where

$$\begin{aligned}\nabla^a\Sigma_a &= D^a\Sigma_a + hD^aV\,\Sigma_a\\ \overline{\nabla_{\dot a}\Sigma^{\dot a}} &= \overline{D_{\dot a}\Sigma^{\dot a}} + h\overline{D_{\dot a}}V\,\overline{\Sigma^{\dot a}}.\end{aligned} \tag{3.12}$$

The $\theta$-expansion for the superfield $V$ brings about the following component fields:

$$\begin{aligned}V &= C + \theta^a b_a + \overline{\theta}_{\dot a}\overline{b}^{\dot a} + \theta^a\overline{\theta}^{\dot a}\sigma^{\mu}_{a\dot a}A_\mu\\ &+ \theta^2\lambda + \overline{\theta}^2\overline{\lambda} + \theta^2\overline{\theta}_{\dot a}\overline{\gamma}^{\dot a} + \overline{\theta}^2\theta^a\gamma_a + \theta^2\overline{\theta}^2\Delta,\end{aligned} \tag{3.13}$$

where C, $\lambda$, $\overline{\lambda}$ and $\Delta$ are scalars $b_a$ and $\gamma_a$ are spinors and $A_\mu$ is the U(1)-gauge field. The gauge transformation of these fields read as below:

$$\begin{aligned}\delta C &= i(\phi^* - \phi),\;\; \delta\lambda = -i\pi,\;\; \delta\overline{\lambda} = i\overline{\pi},\\ \delta b_a &= -iw_a,\;\; \delta\overline{b}_{\dot a} = i\overline{w}_{\dot a}\\ \delta\Delta &= \tfrac{i}{4}\partial^\mu\partial_\mu(\phi - \phi^*),\;\; \delta A^\mu = -\partial^\mu(\phi + \phi^*),\\ \delta\gamma_a &= \tfrac{1}{2}\sigma^{\mu}_{a\dot a}\partial_\mu\overline{w}^{\dot a},\;\; \delta\overline{\gamma}_{\dot a} = -\tfrac{1}{2}\overline{\sigma}_{\dot a a}\partial_\mu w^a.\end{aligned} \tag{3.14}$$

As already known, for the sake of component-field calculations, one usually works in the so-called Wess-Zumino gauge, where C, $b_a$ and $\lambda$ are gauged away. The expansion of the exponential of the gauge superfield simplifies, in this gauge, according to:

$$e^{hV} = 1 + h\theta^a\sigma^{\mu}_{a\dot a}\overline{\theta}^{\dot a}A_\mu + h\theta^2\overline{\theta}_{\dot a}\overline{\gamma}^{\dot a} + h\overline{\theta}^2\theta^a\gamma_a + h\theta^2\overline{\theta}^2\Delta + \frac{1}{4}h^2\theta^2\overline{\theta}^2A^\mu A_\mu. \tag{3.15}$$

Using this gauge, the transformations of the matter fields are:

$$\psi_a = ih\phi\psi_a,\;\; \delta\rho = ih\phi\rho,\;\; \delta\lambda_{\mu\nu} = ih\phi\lambda_{\mu\nu},\;\; \delta\mathcal{F}_a = ih(\phi\mathcal{F}_a); \tag{3.16}$$

we get thereby the following transformations for the components $T_{\mu\nu}$ and $\widetilde{T}_{\mu\nu}$:

$$\delta T_{\mu\nu} = h\phi\widetilde{T}_{\mu\nu}, \quad \delta\widetilde{T}_{\mu\nu} = -h\phi T_{\mu\nu} \tag{3.17}$$

These are precisely the Abelian gauge transformations for the tensor field as firstly proposed in Ref.[1].

# 4 Component-field action in the Wess-Zumino gauge.

Having adopted the component fields as defined in the previous sections, lengthy algebraic computations yield the following action in the Wess-Zumino gauge:

$$
\begin{aligned}
\mathcal{S} \;=& \int d^4x \Big( -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + 2\Delta^2 + i\overline{\gamma}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\gamma^a + \partial^\mu\rho\partial_\mu\rho^* - 16\partial^\mu\lambda_{\mu\nu}\partial_\alpha\lambda^{*\alpha\nu} \\
&+ i\overline{\mathcal{F}}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\mathcal{F}^a - i\overline{\psi}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\partial^\nu\partial_\nu\psi^a - i\tfrac{h}{2}\partial^\mu\rho A_\mu\rho^* + i\tfrac{h}{2}\rho A^\mu\partial_\mu\rho^* \\
&+ 2h\rho\Delta\rho^* + \tfrac{h^2}{4}\rho A^\mu A_\mu\rho^* - h\gamma^a\mathcal{F}_a\rho^* - h\rho\overline{\gamma}_{\dot a}\overline{\mathcal{F}}^{\dot a} - i\tfrac{h}{2}\rho\gamma^a\sigma^{\mu}_{a\dot a}\partial_\mu\overline{\psi}^{\dot a} \\
&- i\tfrac{h}{2}\rho^*\overline{\gamma}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\psi^a + i\tfrac{h}{2}\partial_\mu\rho^*\psi^a\sigma^{\mu}_{a\dot a}\overline{\gamma}^{\dot a} - i\tfrac{h}{2}\partial_\mu\rho\gamma^a\sigma^{\mu}_{a\dot a}\overline{\psi}^{\dot a} - \tfrac{h}{4}\psi^a\sigma^{\mu}_{a\dot a}A_\mu\partial^\nu\partial_\nu\overline{\psi}^{\dot a} \\
&+ \tfrac{h}{4}\overline{\psi}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}A_\mu\partial^\nu\partial_\nu\psi^a + ih\Delta\overline{\psi}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\psi^a + ih\Delta\psi^a\sigma^{\mu}_{a\dot a}\partial_\mu\overline{\psi}^{\dot a} - \tfrac{h}{4}\partial^\nu A_\nu\overline{\psi}^{\dot a}\overline{\sigma}^{\mu}_{\dot a a}\partial_\mu\psi^a \\
&+ \tfrac{h}{4}\partial_\nu A^\nu\psi^a\sigma^{\mu}_{a\dot a}\partial_\mu\overline{\psi}^{\dot a} + \tfrac{h}{2}\mathcal{F}^a\sigma^{\mu}_{a\dot a}A_\mu\overline{\mathcal{F}}^{\dot a} - \tfrac{h}{8}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\mu\nu}\partial_\alpha\overline{\psi}^{\dot a} \\
&+ \tfrac{h}{8}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\alpha\sigma^\nu)_{a\dot a}F_{\nu\alpha}\overline{\psi}^{\dot a} + h\rho^*F_{\mu\nu}\lambda^{\mu\nu} + h\rho F_{\mu\nu}\lambda^{*\mu\nu} \\
&+ 2ih(4\partial_\mu\lambda^{*\mu\nu}\lambda_{\nu\alpha}A^\alpha - 4\partial^\mu\lambda_{\mu\nu}A_\alpha\lambda^{*\nu\alpha}) \\
&- i\tfrac{h^2}{2}\gamma^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}\lambda_{\mu\nu}A_\alpha\overline{\psi}^{\dot a} + \tfrac{h}{2}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\overline{\gamma}^{\dot a}\partial_\nu\lambda^*_{\mu\alpha} - \tfrac{h}{2}\partial_\nu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\lambda^*_{\mu\alpha}\overline{\gamma}^{\dot a} \\
&+ i\tfrac{h^2}{2}\overline{\gamma}^{\dot a}(\overline{\sigma}^\mu\sigma^\alpha\overline{\sigma}^\beta)_{\dot a a}A_\mu\psi^a\lambda^*_{\alpha\beta} - \tfrac{h}{2}\gamma^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\partial_\mu\lambda_{\alpha\beta}\overline{\psi}^{\dot a} + \tfrac{h}{2}\gamma^a(\sigma^a\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\lambda_{\alpha\beta}\partial_\mu\overline{\psi}^{\dot a} \\
&+ \tfrac{h}{2}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a} + \tfrac{h}{4}\partial_\mu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a} \\
&+ \tfrac{h}{4}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\alpha\sigma^\nu)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a} + h^2\Delta\psi^a\sigma^{\mu}_{a\dot a}A_\mu\overline{\psi}^{\dot a} + \tfrac{i}{16}h^2\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\mu\nu}A_\alpha\overline{\psi}^{\dot a} \\
&+ i\tfrac{h^2}{16}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\alpha\mu}A_\nu\overline{\psi}^{\dot a} - i\tfrac{h^2}{2}\gamma^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\lambda_{\alpha\beta}A_\mu\overline{\psi}^{\dot a} \\
&- i\tfrac{h^2}{4}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}A_\nu A_\alpha\overline{\psi}^{\dot a} - i\tfrac{h^2}{8}\partial_\mu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}A_\nu A_\alpha\overline{\psi}^{\dot a} \\
&+ i\tfrac{h^2}{4}\psi^a A_\nu A_\mu(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\partial_\alpha\overline{\psi}^{\dot a} + i\tfrac{h^2}{8}\psi^a A_\nu A_\mu(\sigma^\nu\overline{\sigma}^\alpha\sigma^\mu)_{a\dot a}\partial_\alpha\overline{\psi}^{\dot a} \\
&+ \tfrac{h^3}{8}\psi^a(\sigma^\nu\overline{\sigma}^\alpha\sigma^\mu)_{a\dot a}A_\nu A_\alpha A_\mu\overline{\psi}^{\dot a} - h^2\gamma^a\psi_a\overline{\gamma}_{\dot a}\overline{\psi}^{\dot a} + 4h^2A^\nu A_\mu\lambda_{\nu\beta}\lambda^{*\beta\mu} \\
&- q\tfrac{1}{\sqrt{2}}(\rho^2 - 4\lambda_{\mu\nu}\lambda^{\mu\nu})\tfrac{1}{\sqrt{2}}(\rho^{*2} - 4\lambda^*_{\alpha\beta}\lambda^{*\alpha\beta}) + 4q\lambda^{\mu\nu}\lambda_{\mu\nu}\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} + 4q\lambda^{*\mu\nu}\lambda^*_{\mu\nu}\mathcal{F}^a\psi_a \\
&- 2q\mathcal{F}^a\psi_a\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} - q\rho^2\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} - q(\rho^*)^2\mathcal{F}^a\psi_a + \tfrac{q}{2}\psi^a\psi_a\partial^\mu\partial_\mu(\overline{\psi}_{\dot a}\overline{\psi}^{\dot a}) \\
&- iq\rho\psi^a\sigma^{\mu}_{a\dot a}\partial_\mu(\overline{\psi}^{\dot a}\rho^*) + 4q\rho\psi^a\sigma^{\mu}_{a\dot a}\partial^\beta(\lambda^*_{\beta\mu}\overline{\psi}^{\dot a}) - 4q\lambda_{\mu\beta}\partial^\beta(\rho^*\overline{\psi}_{\dot a})\overline{\sigma}^{\mu\dot a a}\psi_a \\
&- 16iq\lambda_{\mu\alpha}\partial_\beta(\lambda^{*\beta\alpha}\overline{\psi}_{\dot a})\overline{\sigma}^{\mu\dot a a}\psi_a - qh\Delta\psi^a\psi_a\overline{\psi}_{\dot a}\overline{\psi}^{\dot a} - iq\tfrac{h}{2}A_\mu\psi^a\psi_a\partial^\mu(\overline{\psi}_{\dot a}\overline{\psi}^{\dot a}) \\
&+ iq\tfrac{h}{2}A_\mu\partial^\mu(\psi^a\psi_a)\overline{\psi}_{\dot a}\overline{\psi}^{\dot a} - q\tfrac{h^2}{2}A^\mu A_\mu\psi^a\psi_a\overline{\psi}_{\dot a}\overline{\psi}^{\dot a} - qh\rho\gamma^a\psi_a\overline{\psi}_{\dot a}\overline{\psi}^{\dot a} - qh\rho^*\overline{\gamma}_{\dot a}\overline{\psi}^{\dot a}\psi^a\psi_a \\
&- qh\gamma^a\sigma^{\mu\nu}_{ab}\psi^b\overline{\psi}_{\dot a}\overline{\psi}^{\dot a}\lambda_{\mu\nu} + qh\overline{\gamma^{\dot a}}\overline{\sigma}^{\mu\nu}_{\dot a\dot b}\lambda^*_{\mu\nu}\overline{\psi}^{\dot b}\psi^a\psi_a - qh\rho\psi^a\sigma^{\mu}_{a\dot a}A_\mu\overline{\psi}^{\dot a}\rho^* \\
&- 2iqh\rho\psi^a A_\mu(\sigma^\mu\overline{\sigma}^\alpha\sigma^\beta)_{a\dot a}\lambda^*_{\alpha\beta}\overline{\psi}^{\dot a} + 2iqh\rho^*\psi^a(\sigma^\beta\overline{\sigma}^\alpha\sigma^\mu)_{a\dot a}A_\mu\lambda_{\alpha\beta}\overline{\psi}^{\dot a} \\
&+ 2qh\psi^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\gamma\overline{\sigma}^\mu\sigma^\nu)_{a\dot a}\lambda_{\alpha\beta}A_\gamma\lambda^*_{\mu\nu}\overline{\psi}^{\dot a}\Big).
\end{aligned}
\tag{4.1}
$$

We should stress here a remarkable difference with respect to the case of the chiral and anti-chiral scalar superfields (Wess-Zumino model [3]), namely, the minimal coupling of $\Sigma$ and $\overline{\Sigma}$ to the gauge sector necessarily affects the $\Sigma$ - superfield self-interaction terms as one reads off from eq (3.11). The gauging of the $U(1)$ - symmetry enriches the self-interactions of the tensor field not only through its fermionic supersymmetric partners, but also through the introduction of the gauge boson and the gaugino among the matter self-interaction terms. This is so because the model presented here is based on a single spinor superfield. Had we introduced a couple of spinor superfields, $\Sigma_a$ and $\mathcal{T}_a$, with opposite $U(1)$ charges:

$$\begin{aligned} \Sigma'_a &= e^{ih\Lambda}\Sigma_a, \\ \mathcal{T}'_a &= e^{-ih\Lambda}\mathcal{T}_a, \end{aligned} \tag{4.2}$$

a self-interacting term of the form $(\Sigma^a\mathcal{T}_a\overline{\Sigma}_{\dot{a}}\overline{\mathcal{T}}^{\dot{a}})$ would automatically be invariant whenever the symmetry is gauged, and there would be no need for introducing the vector superfield to ensure local invariance. Such a mixed self-interacting term could, in principle, be thought of as a possible source for a mass term for the spinor superfields, whenever the physical scalar component $\rho$ develops a non-trivial vacuum expectation value. Nevertheless, by analysing the $\rho$ - field interactions in the scalar potential, one concludes that there is no room for spontaneous symmetry breaking as induced by the component field ( and, similarly, for its counterpart inside $\mathcal{T}$ ). On the other hand, we could think to introduce a gauge-invariant mass term of the form

$$\mathcal{S}_{mass} = \int d^4x\,(d^2\theta\;\, i\frac{m}{16}\Sigma^a\mathcal{T}_a - d^2\overline{\theta}\;\, i\frac{m}{16}\overline{\Sigma}_{\dot{a}}\overline{\mathcal{T}}^{\dot{a}}); \tag{4.3}$$

however, a mixed mass term like the one above introduces two massive excitations of the type $k^4 = m^4$ that are simultaneously present in the spectrum. So, regardless the sign of $m^2$, a tachyon shall always be present; hence such a mass term is disregarded.

## 5 Supersymmetry Breaking

Due to the spinorial character of the superfield $\Sigma_a$, it cannot be used to accomplish a spontaneous supersymmetry breaking. Indeed, Lorentz invariance is lost whenever $\Sigma_a$ acquires a non-trivial vacuum expectation value. The idea in the present section is to couple, in a gauge invariant manner, the well-known O`Raifeartaigh model [5] to the spinor superfield $\Sigma_a$, so as to understand the issue of mass generation for $\Sigma_a$ via spontaneous supersymmetry breakingdown. The model we adopt to discuss such a matter is defined by the action below:

$$
\begin{aligned}
\mathcal{S} &= \int d^4x\, d^2\theta d^2\overline{\theta}\left(\overline{\phi}\phi + \overline{\phi_+}e^{hV}\phi_+ + \overline{\phi_-}e^{-hV}\phi_-\right) \\
&+ \int d^4x\, d^2\theta\left(\frac{1}{2}m\phi^2 + \mu\phi_+\phi_- + f\phi + g\phi\phi_+\phi_- + G\Sigma^a\Sigma_a\phi_-\phi_-\right) \\
&+ \int d^4x\, d^2\overline{\theta}\left(\frac{1}{2}m\overline{\phi}^2 + \mu\overline{\phi_+\phi_-} + f\overline{\phi} + g\overline{\phi\phi_+\phi_-} + G\overline{\Sigma}_{\dot{a}}\overline{\Sigma}^{\dot{a}}\overline{\phi_-\phi_-}\right) \qquad (5.1)\\
&+ \int d^4x\, d^2\theta\ \tfrac{-1}{128}\left(\overline{D}^2(e^{-V}D^ae^V)\overline{D}^2(e^{-V}D_ae^V)\right) \\
&+ \int d^4x\, d^2\theta d^2\overline{\theta}\ \tfrac{-1}{32}\left(\nabla^a\Sigma_a\, e^{hV}\ \overline{\nabla_{\dot{a}}\Sigma^{\dot{a}}} + q\Sigma^a\Sigma_a e^{2hV}\overline{\Sigma}_{\dot{a}}\overline{\Sigma}^{\dot{a}}\right),
\end{aligned}
$$

where the chiral scalar superfields $\phi$, $\phi_+$ and $\phi_-$ are parametrized as follows:

$$
\begin{aligned}
\phi &= (1 - i\theta^a\sigma^\mu_{a\dot{a}}\overline{\theta}^{\dot{a}}\partial_\mu - \tfrac{1}{4}\theta^2\overline{\theta}^2\partial_\mu\partial^\mu)(A + \theta^b\xi_b + \theta^2 b) \\
\phi_+ &= (1 - i\theta^a\sigma^\mu_{a\dot{a}}\overline{\theta}^{\dot{a}}\partial_\mu - \tfrac{1}{4}\theta^2\overline{\theta}^2\partial_\mu\partial^\mu)(A_+ + \theta^b\xi_{+b} + \theta^2 b_+) \qquad (5.2)\\
\phi_- &= (1 - i\theta^a\sigma^\mu_{a\dot{a}}\overline{\theta}^{\dot{a}}\partial_\mu - \tfrac{1}{4}\theta^2\overline{\theta}^2\partial_\mu\partial^\mu)(A_- + \theta^b\xi_{-b} + \theta^2 b_-).
\end{aligned}
$$

m and $\mu$ are mass parameters, f has dimension of $mass^2$, whereas g and G are dimensionless coupling constants. $\Sigma_a$ and $\phi_-$ have opposite U(1) - charges. This action in terms of components reads:

$$
\begin{aligned}
\mathcal{S} &= \int d^4x\, 4\Big(4\{\partial^\mu A^*\partial_\mu A + \partial^\mu A^*_-\partial_\mu A_- + \partial^\mu A^*_+\partial_\mu A_+\} + 4\{b^*b + b^*_-b_- + b^*_+b_+\} \\
&+2i\{\overline{\xi}^{\dot{a}}\overline{\sigma}^\mu_{\dot{a}a}\partial_\mu\xi^a + \overline{\xi}^{\dot{a}}_-\overline{\sigma}^\mu_{\dot{a}a}\partial_\mu\xi^a_- + \overline{\xi}^{\dot{a}}_+\overline{\sigma}^\mu_{\dot{a}a}\partial_\mu\xi^a_+\}\Big) \\
&+ \int d^4x\,\Big(16hA^*_+\Delta A_+ + 8ihA^*_+\partial^\mu A_\mu A_+ + 4h^2A^*_+A^\mu A_\mu A_+ - 8hA^*_+\gamma^a\xi_{+a} \\
&-8h\overline{\xi}_{+\dot{a}}\overline{\gamma}^{\dot{a}}A_+ + 4h\xi^{+a}\sigma^\mu_{a\dot{a}}A_\mu\overline{\xi}^{\dot{a}}_+ + 16ih\partial^\mu A^*_+A_\mu A_+\Big) \\
&+ \int d^4x\,\Big(-16hA^*_-\Delta A_- - 8ihA^*_-\partial^\mu A_\mu A_- + 4h^2A^*_-A^\mu A_\mu A_- + 8hA^*_-\gamma^a\xi_{-a} \\
&+8h\overline{\xi}_{-\dot{a}}\overline{\gamma}^{\dot{a}}A_- - 4h\xi^{-a}\sigma^\mu_{a\dot{a}}A_\mu\overline{\xi}^{\dot{a}}_- - 16ih\partial^\mu A^*_-A_\mu A_-\Big) \\
&+ \int d^4x\,\Big(m(-4bA + \xi^a\xi_a) + m(-4b^*A^* + \overline{\xi}_{\dot{a}}\overline{\xi}^{\dot{a}}) - 4fb - 4fb^*\Big) \\
&+ \int d^4x\, 2\Big(\mu(-2b_+A_- - 2b_-A_+ + \xi^a_+\xi_{a-}) + \mu(-2b^*_+A^*_- - 2b^*_-A^*_+ + \overline{\xi}_{\dot{a}+}\overline{\xi}^{\dot{a}}_-)\Big)
\end{aligned}
$$

$$+\int d^4x\, 2\Bigg(g(-2bA_+A_- - 2Ab_+A_- - 2AA_+b_- + A\xi^a_+\xi_{a-} + A_-\xi^a\xi_{a+} + A_+\xi^a\xi_{a-})$$
$$+g(-2b^*A^*_+A^*_- - 2A^*b^*_+A^*_- - 2A^*A^*_+b^*_- + A^*\overline{\xi}_{\dot a+}\overline{\xi}^{\dot a-} + A^*_-\overline{\xi}_{\dot a}\overline{\xi}^{\dot a}_+ + A^*_+\overline{\xi}_{\dot a}\overline{\xi}^{\dot a}_-)\Bigg)$$
$$+\int d^4x\, 4G\Bigg((-2\mathcal{F}^a\psi_a + \rho^2 + 4\lambda_{\mu\nu}\lambda^{\mu\nu})(A_-)^2 + (-2\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} + (\rho^*)^2 + 4\lambda^*_{\mu\nu}\lambda^{*\mu\nu})(A^*_-)^2$$
$$+2\psi^b\psi_b(-4b_-A_- + \xi^a_-\xi_{a-}) + 2\overline{\psi}_{\dot b}\overline{\psi}^{\dot b}(-4b^*_-A^*_- + \overline{\xi}_{\dot a-}\overline{\xi}^{\dot a-})$$
$$-2(\rho\psi^b - \sigma^{\mu\nu\, ba}\lambda_{\mu\nu}\psi_a)(\xi_{b-}A_-) + 2(\rho^*\overline{\psi}_{\dot b} - \lambda^*_{\mu\nu}\overline{\sigma}^{\mu\nu}_{\dot b\dot a}\overline{\psi}^{\dot a})(\overline{\xi}^{\dot b}_-A^*_-)\Bigg)$$
$$+\int d^4x\,\Bigg(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + 2\Delta^2 + i\overline{\gamma}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\gamma^a + \partial^\mu\rho\partial_\mu\rho^* - 16\partial^\mu\lambda_{\mu\nu}\partial_\alpha\lambda^{*\alpha\nu}$$
$$+i\overline{\mathcal{F}}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\mathcal{F}^a - i\overline{\psi}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\partial^\nu\partial_\nu\psi^a - i\tfrac{h}{2}\partial^\mu\rho A_\mu\rho^* + i\tfrac{h}{2}\rho A^\mu\partial_\mu\rho^*$$
$$+2h\rho\Delta\rho^* + \tfrac{h^2}{4}\rho A^\mu A_\mu\rho^* - h\gamma^a\mathcal{F}_a\rho^* - h\rho\overline{\gamma}_{\dot a}\overline{\mathcal{F}}^{\dot a} - i\tfrac{h}{2}\rho\gamma^a\sigma^\mu_{a\dot a}\partial_\mu\overline{\psi}^{\dot a}$$
$$-i\tfrac{h}{2}\rho^*\overline{\gamma}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\psi^a + i\tfrac{h}{2}\partial_\mu\rho^*\psi^a\sigma^\mu_{a\dot a}\overline{\gamma}^{\dot a} - i\tfrac{h}{2}\partial_\mu\rho\gamma^a\sigma^\mu_{a\dot a}\overline{\psi}^{\dot a} - \tfrac{h}{4}\psi^a\sigma^\mu_{a\dot a}A_\mu\partial^\nu\partial_\nu\overline{\psi}^{\dot a}$$
$$+\tfrac{h}{4}\overline{\psi}^{\dot a}\overline{\sigma}^\mu_{\dot a a}A_\mu\partial^\nu\partial_\nu\psi^a + ih\Delta\overline{\psi}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\psi^a + ih\Delta\psi^a\sigma^\mu_{a\dot a}\partial_\mu\overline{\psi}^{\dot a} - \tfrac{h}{4}\partial^\nu A_\nu\overline{\psi}^{\dot a}\overline{\sigma}^\mu_{\dot a a}\partial_\mu\psi^a$$
$$+\tfrac{h}{4}\partial_\nu A^\nu\psi^a\sigma^\mu_{a\dot a}\partial_\mu\overline{\psi}^{\dot a} + \tfrac{h}{2}\mathcal{F}^a\sigma^\mu_{a\dot a}A_\mu\overline{\mathcal{F}}^{\dot a} - \tfrac{h}{8}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\mu\nu}\partial_\alpha\overline{\psi}^{\dot a}$$
$$+\tfrac{h}{8}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\alpha\sigma^\nu)_{a\dot a}F_{\nu\alpha}\overline{\psi}^{\dot a} + h\rho^*F_{\mu\nu}\lambda^{\mu\nu} + h\rho F_{\mu\nu}\lambda^{*\mu\nu}$$
$$+2ih(4\partial_\mu\lambda^{*\mu\nu}\lambda_{\nu\alpha}A^\alpha - 4\partial^\mu\lambda_{\mu\nu}A_\alpha\lambda^{*\nu\alpha})$$
$$-i\tfrac{h^2}{2}\gamma^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}\lambda_{\mu\nu}A_\alpha\overline{\psi}^{\dot a} + \tfrac{h}{2}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\overline{\gamma}^{\dot a}\partial_\nu\lambda^*_{\mu\alpha} - \tfrac{h}{2}\partial_\nu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\lambda^*_{\mu\alpha}\overline{\gamma}^{\dot a}$$
$$+i\tfrac{h^2}{2}\overline{\gamma}^{\dot a}(\overline{\sigma}^\mu\sigma^\alpha\overline{\sigma}^\beta)_{\dot a a}A_\mu\psi^a\lambda^*_{\alpha\beta} - \tfrac{h}{2}\gamma^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\partial_\mu\lambda_{\alpha\beta}\overline{\psi}^{\dot a} + \tfrac{h}{2}\gamma^a(\sigma^a\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\lambda_{\alpha\beta}\partial_\mu\overline{\psi}^{\dot a}$$
$$+\tfrac{h}{2}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a} + \tfrac{h}{4}\partial_\mu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a}$$
$$+\tfrac{h}{4}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\alpha\sigma^\nu)_{a\dot a}A_\nu\partial_\alpha\overline{\psi}^{\dot a} + h^2\Delta\psi^a\sigma^\mu_{a\dot a}A_\mu\overline{\psi}^{\dot a} + \tfrac{i}{16}h^2\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\mu\nu}A_\alpha\overline{\psi}^{\dot a}$$
$$+i\tfrac{h^2}{16}\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}F_{\alpha\mu}A_\nu\overline{\psi}^{\dot a} - i\tfrac{h^2}{2}\gamma^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\mu)_{a\dot a}\lambda_{\alpha\beta}A_\mu\overline{\psi}^{\dot a}$$
$$-i\tfrac{h^2}{4}\partial_\mu\psi^a(\sigma^\mu\overline{\sigma}^\nu\sigma^\alpha)_{a\dot a}A_\nu A_\alpha\overline{\psi}^{\dot a} - i\tfrac{h^2}{8}\partial_\mu\psi^a(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}A_\nu A_\alpha\overline{\psi}^{\dot a}$$
$$+i\tfrac{h^2}{4}\psi^aA_\nu A_\mu(\sigma^\nu\overline{\sigma}^\mu\sigma^\alpha)_{a\dot a}\partial_\alpha\overline{\psi}^{\dot a} + i\tfrac{h^2}{8}\psi^aA_\nu A_\mu(\sigma^\nu\overline{\sigma}^\alpha\sigma^\mu)_{a\dot a}\partial_\alpha\overline{\psi}^{\dot a}$$
$$+\tfrac{h^3}{8}\psi^a(\sigma^\nu\overline{\sigma}^\alpha\sigma^\mu)_{a\dot a}A_\nu A_\alpha A_\mu\overline{\psi}^{\dot a} - h^2\gamma^a\psi_a\overline{\gamma}_{\dot a}\overline{\psi}^{\dot a} + 4h^2A^\nu A_\mu\lambda_{\nu\beta}\lambda^{*\beta\mu}$$
$$-q\tfrac{1}{\sqrt{2}}(\rho^2 - 4\lambda_{\mu\nu}\lambda^{\mu\nu})\tfrac{1}{\sqrt{2}}(\rho^{*2} - 4\lambda^*_{\alpha\beta}\lambda^{*\alpha\beta}) + 4q\lambda^{\mu\nu}\lambda_{\mu\nu}\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} + 4q\lambda^{*\mu\nu}\lambda^*_{\mu\nu}\mathcal{F}^a\psi_a$$
$$-2q\mathcal{F}^a\psi_a\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} - q\rho^2\overline{\mathcal{F}}_{\dot a}\overline{\psi}^{\dot a} - q(\rho^*)^2\mathcal{F}^a\psi_a + \tfrac{q}{2}\psi^a\psi_a\partial^\mu\partial_\mu(\overline{\psi}_{\dot a}\overline{\psi}^{\dot a})$$
$$-iq\rho\psi^a\sigma^\mu_{a\dot a}\partial_\mu(\overline{\psi}^{\dot a}\rho^*) + 4q\rho\psi^a\sigma^\mu_{a\dot a}\partial^\beta(\lambda^*_{\beta\mu}\overline{\psi}^{\dot a}) - 4q\lambda_{\mu\beta}\partial^\beta(\rho^*\overline{\psi}_{\dot a})\overline{\sigma}^{\mu\dot a a}\psi_a$$
$$-16iq\lambda_{\mu\alpha}\partial_\beta(\lambda^{*\beta\alpha}\overline{\psi}_{\dot a})\overline{\sigma}^{\mu\dot a a}\psi_a - qh\Delta\psi^a\psi_a\overline{\psi}_{\dot a}\overline{\psi}^{\dot a} - iq\tfrac{h}{2}A_\mu\psi^a\psi_a\partial^\mu(\overline{\psi}_{\dot a}\overline{\psi}^{\dot a})$$

$$+iq\frac{h}{2}A_\mu\partial^\mu(\psi^a\psi_a)\overline{\psi}_{\dot{a}}\overline{\psi}^{\dot{a}} - q\frac{h^2}{2}A^\mu A_\mu\psi^a\psi_a\overline{\psi}_{\dot{a}}\overline{\psi}^{\dot{a}} - qh\rho\gamma^a\psi_a\overline{\psi}_{\dot{a}}\overline{\psi}^{\dot{a}} - qh\rho^*\overline{\gamma}_{\dot{a}}\overline{\psi}^{\dot{a}}\psi^a\psi_a$$
$$-qh\gamma^a\sigma^{\mu\nu}_{ab}\psi^b\overline{\psi}_{\dot{a}}\overline{\psi}^{\dot{a}}\lambda_{\mu\nu} + qh\overline{\gamma^{\dot{a}}}\overline{\sigma}^{\mu\nu}_{\dot{a}\dot{b}}\lambda^*_{\mu\nu}\overline{\psi}^{\dot{b}}\psi^a\psi_a - qh\rho\psi^a\sigma^\mu_{a\dot{a}}A_\mu\overline{\psi}^{\dot{a}}\rho^*$$
$$-2iqh\rho\psi^a A_\mu(\sigma^\mu\overline{\sigma}^\alpha\sigma^\beta)_{a\dot{a}}\lambda^*_{\alpha\beta}\overline{\psi}^{\dot{a}} + 2iqh\rho^*\psi^a(\sigma^\beta\overline{\sigma}^\alpha\sigma^\mu)_{a\dot{a}}A_\mu\lambda_{\alpha\beta}\overline{\psi}^{\dot{a}}$$
$$+2qh\psi^a(\sigma^\alpha\overline{\sigma}^\beta\sigma^\gamma\overline{\sigma}^\mu\sigma^\nu)_{a\dot{a}}\lambda_{\alpha\beta}A_\gamma\lambda^*_{\mu\nu}\overline{\psi}^{\dot{a}}\Bigg). \tag{5.3}$$

In the scalar sector of this component-field action there is room for spontaneous supersymmetry breaking, the broken phase being characterized by a non-trivial vacuum expectation value [11]. One then sees that, due to the appearance of a coupling term of the type $\rho^2 A_-^2$, whenever supersymmetry and the U(1)-symmetry are broken by $\langle A_-\rangle \neq 0$, the $\rho$-field and the tensor field acquire a physical mass, being therefore split from the fermionic degrees of freedom present in $\Sigma_a$. The mass splitting that occurs for $\Sigma_a$ takes place at the expenses of a supersymmetry breaking triggered by the superfield $\phi_-$.

# 6 Conclusions

The supersymmetrization of the matter tensor field first investigated in Ref.[1] has been worked out here in terms of a spinor chiral superfield, $\Sigma_a$, whose kinetic and self-interacting terms have been found in $N = 1$ - superspace. The gauging of the model reveals some peculiarities, such as the need of gauge fields appearing in the matter self-interactions.

Extra bosonic degrees of freedom that accompany the fermionic partners cannot be the source for spontaneous symmetry or supersymmetry breaking, as it could in principle be thought. The reason is that Lorentz invariance prevents $\Sigma_a$ from developing non-trivial vacuum expectation value. Mass for the tensor field and its partners may be generated by spontaneous supersymmetry breaking.

# 7 Conventions

$$\sigma^\mu = (1, \sigma), \quad \overline{\sigma}^\mu = (1, -\sigma), \quad \overline{\sigma}^\mu_{\dot{b}a} = \sigma^\mu_{a\dot{b}}. \tag{7.1}$$

Where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0, & 1 \\ 1, & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0, & -i \\ i, & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix} \tag{7.2}$$

In addition the matrices $\sigma^{\mu\nu}$ and $\overline{\sigma}^{\mu\nu}$ are given by

$$\begin{aligned} \sigma^{\mu}_{a\dot{a}}\overline{\sigma}^{\nu\dot{a}b} &= \eta^{\mu\nu}\delta_a^{\ b} - i(\sigma^{\mu\nu})_a^{\ b} \\ \overline{\sigma}^{\mu\dot{a}a}\sigma^{\nu}_{a\dot{b}} &= \eta^{\mu\nu}\delta^{\dot{a}}_{\ \dot{b}} - i(\overline{\sigma}^{\mu\nu})^{\dot{a}}_{\ \dot{b}} \end{aligned} \tag{7.3}$$

and the trace is

$$\sigma^{\mu}_{a\dot{a}}\overline{\sigma}^{\nu\dot{a}b}\sigma^{\alpha}_{b\dot{b}}\overline{\sigma}^{\beta\dot{b}a} = 2\Big(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} + i\varepsilon^{\mu\nu\alpha\beta}\Big), \tag{7.4}$$

where $\varepsilon^{0123} = 1$.

The summation convention is:

$$\theta\eta = \theta^a\eta_a, \qquad \overline{\theta}\overline{\eta} = \overline{\theta}_{\dot{a}}\overline{\eta}^{\dot{a}}. \tag{7.5}$$

Where lowering and raising of indices are effected through

$$\theta^a = \varepsilon^{ab}\theta_b, \qquad \theta_a = \varepsilon_{ab}\theta^b, \tag{7.6}$$

with $\varepsilon_{ab} = -\varepsilon_{ba}$, (the same for dotted indices). Differentation with respect to the anticommuting parameters $\theta_a$, $\overline{\theta}_{\dot{a}}$ is defined by

$$\frac{\partial}{\partial\theta^a}\theta^b = \delta_a^{\ b} \qquad \frac{\partial}{\partial\overline{\theta}^{\dot{a}}}\overline{\theta}^{\dot{b}} = \delta_{\dot{a}}^{\ \dot{b}}. \tag{7.7}$$

Covariant derivatives with respect to the supersymmetry transformations are:

$$\begin{aligned} D_a &= \tfrac{\partial}{\partial\theta^a} - i\sigma^{\mu}_{a\dot{a}}\overline{\theta}^{\dot{a}}\partial_\mu \\ \overline{D}_{\dot{a}} &= -\tfrac{\partial}{\partial\overline{\theta}^{\dot{a}}} + i\theta^a\sigma^{\mu}_{a\dot{a}}\partial_\mu, \end{aligned} \tag{7.8}$$

and they obey the anticommutation relations

$$\{D_a, \overline{D}_{\dot{a}}\} = 2i\sigma^{\mu}_{a\dot{a}}\partial_\mu \qquad \{D_a, D_b\} = 0 = \{\overline{D}_a, \overline{D}_{\dot{a}}\}. \tag{7.9}$$

## Acknowledgements

We wish to thank Marco Antonio de Andrade for helpful discussions. The *Conselho Nacional de Desenvolvimento Cientifico e Tecnologico*, $CNPq$-Brasil is gratefully acknowledged for the financial support.

# References


[1] L. V. Avdeev and M. V. Chizhov, *Phys. Lett.* B321 (1994) 212;
L. V. Avdeev and M. V. Chizhov, *A queer reduction of degrees of freedom*, preprint $JINR$ Dubna, $hep-th/$9407067;

[2] Vitor Lemes, Ricardo Renan, S. P. Sorella, *Phys. Lett.* B344 (1995) 158;

[3] J. Wess and B. Zumino, *Phys. Lett.* B49 (1974) 52;

[4] P.van Nieuwenhuizen, *Nucl. Phys.* B60 (1973) 478;

[5] L. O`Raifeartaigh, *Nucl. Phys.* B96 (1975) 331;

[6] O. Piguet and K. Sibold, Renormalized supersymmetry, *Birkhäuser Press (Boston, 1986).*

[7] O. Piguet, "Renormalisation en théorie quantique des champs" and "Renormalisation des théories de jauge", lectures of the *"Troisième cycle de la physique en Suisse Romande" (1982-1983)*;

[8] C. Itzykson and J.-B. Zuber, "Quantum field theory", *McGraw-Hill 1985*;

[9] J. Wess, B. Zumino, *Nucl. Phys.* B78 (1974) 1;
S. Ferrara, B. Zumino *Nucl. Phys.* B79 (1974) 413;.

[10] S. J. Gates, Jr. , M. T. Grisaru, M. Roček and W. Siegel, "Superspace or one thousand and one lessons in supersymmetry" *Benjamin, Massachusetts, 1983*;

[11] F.Feruglio, J.A.Helayël-Neto and F.Legovini *Nucl. Phys.* B249 (1985) 533;.