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## Renormalization of the $N = 1$ Abelian Super-Chern-Simons Theory Coupled to Parity-Preserving Matter

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### Abstract

We analyse the renormalizability of an Abelian  $N=1$  super-Chern-Simons model coupled to parity-preserving matter on the light of the regularization independent algebraic method. The model shows to be stable under radiative corrections and to be gauge anomaly free.

**Key-words:** Renormalization; Chern-Simons; Supersymmetry.

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## 1 Introduction

Our purpose in this letter is to investigate the renormalizability of the  $N=1$  super-Chern-Simons theory [1] coupled in a parity-preserving way with matter supermultiplets [2], by using the method of algebraic renormalization [3]. The latter is based on the BRS-formalism [4] which together with the Quantum Action Principle [5] lead to a regularization independent scheme. The model considered here comes from an  $N=1$  super-QED in the Atiyah-Ward space-time [2] after a dimensional reduction proposed by Nishino in ref. [6] and suitable truncations of some spurious degrees of freedom caused by dimensional reduction of a time coordinate.

## 2 The model and its symmetries

The gauge invariant action for the  $N=1$  super-Chern-Simons theory coupled in a parity-preserving way with matter supermultiplets, in superspace<sup>6</sup>, is given by [1, 2]:

$$\begin{aligned} \Sigma_{inv} = \frac{1}{2} \int dv \{ & \kappa(\Gamma^\alpha W_\alpha) - (\nabla^\alpha \bar{\Phi}_+) (\nabla_\alpha \Phi_+) - (\nabla^\alpha \bar{\Phi}_-) (\nabla_\alpha \Phi_-) + m(\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-) + \\ & - \lambda(\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-)^2 \} . \end{aligned} \quad (1)$$

The coupling of matter coupling here is the same as the one occurring in the super- $\tau_3$  QED model of ref. [2].

The Chern-Simons parameter  $\kappa$  is the inverse of the coupling constant in term of which perturbation expansion is defined. The superspace measure adopted is  $dv \equiv d^3x d^2\theta$ . The gauge superconnection is a spinorial Majorana superfield  $\Gamma_\alpha$ , and matter is represented by the complex scalar superfields  $\Phi_\pm$ , with opposite  $U(1)$ -charges. The covariant spinorial derivatives are defined as follows:

$$\nabla_\alpha \Phi_\pm = (D_\alpha \mp i\Gamma_\alpha) \Phi_\pm \quad \text{and} \quad \nabla_\alpha \bar{\Phi}_\pm = (D_\alpha \pm i\Gamma_\alpha) \bar{\Phi}_\pm \quad , \quad (2)$$

where  $D_\alpha = \partial_\alpha + i\theta^\beta \partial_{\alpha\beta}$ . The superfield-strength  $W_\alpha$  is given by

$$W_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta . \quad (3)$$

The component-field projections read

$$\begin{aligned} \chi_\alpha &= \Gamma_\alpha| \quad , \quad B' = \frac{1}{2} D^\alpha \Gamma_\alpha| \quad , \\ V_{\alpha\beta} &= -\frac{i}{2} D_{(\alpha} \Gamma_{\beta)}| \quad , \quad \lambda_\alpha = \frac{1}{2} D^\beta D_\alpha \Gamma_\beta| \quad . \end{aligned} \quad (4)$$

and

$$\begin{aligned} A_\pm(x) &= \Phi_\pm(x, \theta)| \quad \text{and} \quad \bar{A}_\pm(x) = \bar{\Phi}_\pm(x, \theta)| , \\ \psi_\pm^\alpha(x) &= D^\alpha \Phi_\pm(x, \theta)| \quad \text{and} \quad \bar{\psi}_\pm^\alpha(x) = D^\alpha \bar{\Phi}_\pm(x, \theta)| , \\ F_\pm(x) &= D^2 \Phi_\pm(x, \theta)| \quad \text{and} \quad \bar{F}_\pm(x) = D^2 \bar{\Phi}_\pm(x, \theta)| . \end{aligned} \quad (5)$$

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<sup>6</sup>The notations and conventions adopted throughout the work are those of ref. [7].

We choose a gauge-fixing action of the Landau type:

$$\Sigma_{gf} = \frac{1}{2} s \int dv \widehat{C} D^\alpha \Gamma_\alpha = \int dv \left\{ \frac{1}{2} B D^\alpha \Gamma_\alpha + \widehat{C} D^2 C \right\} , \quad (6)$$

with  $C$ ,  $\widehat{C}$  and  $B$  being respectively, the ghost, the antighost and the Lagrange multiplier superfields.

The action,  $\Sigma_{inv} + \Sigma_{gf}$ , is invariant under the following BRS transformations:

$$\begin{aligned} s\Phi_\pm &= \pm i C \Phi_\pm , & s\bar{\Phi}_\pm &= \mp i C \bar{\Phi}_\pm , \\ s\Gamma_\alpha &= D_\alpha C , & s\widehat{C} &= B , \\ sC &= 0 , & sB &= 0 . \end{aligned} \quad (7)$$

In view of expressing the BRS-invariance of the model in a functional way by a Slavnov-Taylor identity, we add to the action a source term,  $\Sigma_{ext}$ , which contains external sources  $J_\pm$ ,  $\bar{J}_\pm$  coupled to the non-linear BRS variations of the quantum fields. The external coupling then reads

$$\Sigma_{ext} = \int dv \left\{ \bar{J}_+ s\Phi_+ + \bar{J}_- s\Phi_- + s\bar{\Phi}_+ J_+ + s\bar{\Phi}_- J_- \right\} . \quad (8)$$

Then, the Slavnov-Taylor identity obeyed by the complete action

$$\Sigma = \Sigma_{inv} + \Sigma_{gf} + \Sigma_{ext} , \quad (9)$$

is given by

$$\begin{aligned} \mathcal{S}(\Sigma) &= \int dv \left\{ D^\alpha C \frac{\delta \Sigma}{\delta \Gamma^\alpha} + B \frac{\delta \Sigma}{\delta \widehat{C}} + \frac{\delta \Sigma}{\delta \bar{J}_+} \frac{\delta \Sigma}{\delta \Phi_+} + \frac{\delta \Sigma}{\delta \bar{J}_-} \frac{\delta \Sigma}{\delta \Phi_-} + \right. \\ &\quad \left. - \frac{\delta \Sigma}{\delta J_+} \frac{\delta \Sigma}{\delta \bar{\Phi}_+} - \frac{\delta \Sigma}{\delta J_-} \frac{\delta \Sigma}{\delta \bar{\Phi}_-} \right\} = 0 . \end{aligned} \quad (10)$$

The corresponding linearized Slavnov-Taylor operator reads

$$\begin{aligned} \mathcal{S}_\Sigma &= \int dv \left\{ D^\alpha C \frac{\delta \Sigma}{\delta \Gamma^\alpha} + B \frac{\delta \Sigma}{\delta \widehat{C}} + \frac{\delta \Sigma}{\delta \bar{J}_+} \frac{\delta}{\delta \Phi_+} + \frac{\delta \Sigma}{\delta \bar{J}_-} \frac{\delta}{\delta \Phi_-} + \frac{\delta \Sigma}{\delta \Phi_+} \frac{\delta}{\delta \bar{J}_+} + \frac{\delta \Sigma}{\delta \Phi_-} \frac{\delta}{\delta \bar{J}_-} + \right. \\ &\quad \left. - \frac{\delta \Sigma}{\delta J_+} \frac{\delta}{\delta \bar{\Phi}_+} - \frac{\delta \Sigma}{\delta J_-} \frac{\delta}{\delta \bar{\Phi}_-} - \frac{\delta \Sigma}{\delta \bar{\Phi}_+} \frac{\delta}{\delta J_+} - \frac{\delta \Sigma}{\delta \bar{\Phi}_-} \frac{\delta}{\delta J_-} \right\} . \end{aligned} \quad (11)$$

The operation of  $\mathcal{S}_\Sigma$  over the fields and the external sources is given by

$$\begin{aligned} \mathcal{S}_\Sigma \phi &= s\phi , & \phi &= \Phi_\pm, \bar{\Phi}_\pm, \Gamma_\alpha, C, \widehat{C} \text{ and } B , \\ \mathcal{S}_\Sigma J_+ &= -\frac{\delta \Sigma}{\delta \bar{\Phi}_+} , & \mathcal{S}_\Sigma J_- &= -\frac{\delta \Sigma}{\delta \bar{\Phi}_-} , \\ \mathcal{S}_\Sigma \bar{J}_+ &= \frac{\delta \Sigma}{\delta \Phi_+} , & \mathcal{S}_\Sigma \bar{J}_- &= \frac{\delta \Sigma}{\delta \Phi_-} . \end{aligned} \quad (12)$$

The gauge condition, the ghost equation and the antighost equation [3] for (9) read

$$\frac{\delta\Sigma}{\delta B} = \frac{1}{2}D^\alpha\Gamma_\alpha \quad , \quad (13)$$

$$\frac{\delta\Sigma}{\delta\widehat{C}} = D^2C \quad , \quad (14)$$

$$-i\frac{\delta\Sigma}{\delta C} = \Delta_{\text{class}} \quad , \quad (15)$$

$$\Delta_{\text{class}} = iD^2\widehat{C} - \bar{J}_+\Phi_+ + \bar{J}_-\Phi_- - \bar{\Phi}_+J_+ + \bar{\Phi}_-J_- \quad .$$

Note that the right-hand sides being linear in the quantum fields, will not be submitted to renormalization.

Moreover, from the antighost equation (15) and from the Slavnov-Taylor identity (10) we get the  $U(1)$  rigid invariance

$$\mathcal{W}_{\text{rigid}}\Sigma = 0 \quad , \quad (16)$$

where  $\mathcal{W}_{\text{rigid}}$  is the Ward operator of  $U(1)$  rigid symmetry defined by

$$\mathcal{W}_{\text{rigid}} = \int dv \left\{ \Phi_+ \frac{\delta}{\delta\Phi_+} - \Phi_- \frac{\delta}{\delta\Phi_-} - \bar{\Phi}_+ \frac{\delta}{\delta\bar{\Phi}_+} + \bar{\Phi}_- \frac{\delta}{\delta\bar{\Phi}_-} \right\} \quad . \quad (17)$$

The Slavnov-Taylor identity (10), the constraints (13–15) and the rigid invariance (16) form an algebra that takes the form:

$$\mathcal{S}_{\mathcal{F}}\mathcal{S}(\mathcal{F}) = 0 \quad , \quad \forall \mathcal{F} \quad , \quad (18)$$

$$\mathcal{S}_{\mathcal{F}}\mathcal{S}_{\mathcal{F}} = 0 \quad \text{if} \quad \mathcal{S}(\mathcal{F}) = 0 \quad , \quad (19)$$

$$\frac{\delta\mathcal{S}(\mathcal{F})}{\delta B} - \mathcal{S}_{\mathcal{F}}\left(\frac{\delta\mathcal{F}}{\delta B} - \frac{1}{2}D^\alpha\Gamma_\alpha\right) = \left(\frac{\delta\mathcal{F}}{\delta\widehat{C}} - D^2C\right) \quad , \quad (20)$$

$$\frac{\delta\mathcal{S}(\mathcal{F})}{\delta\widehat{C}} + \mathcal{S}_{\mathcal{F}}\frac{\delta\mathcal{F}}{\delta\widehat{C}} = 0 \quad , \quad (21)$$

$$-i\int dv \frac{\delta}{\delta C}\mathcal{S}(\mathcal{F}) + \mathcal{S}_{\mathcal{F}}\int dv \left(-i\frac{\delta}{\delta C}\mathcal{F} - \Delta_{\text{class}}\right) = \mathcal{W}_{\text{rigid}}\mathcal{F} \quad , \quad (22)$$

$$\mathcal{W}_{\text{rigid}}\mathcal{S}(\mathcal{F}) - \mathcal{S}_{\mathcal{F}}\mathcal{W}_{\text{rigid}}\mathcal{F} = 0 \quad , \quad (23)$$

where  $\mathcal{F}$  is an arbitrary functional of ghost number zero.

Finally, the action (9) is invariant under the discrete symmetries displayed below:

Parity:

$$\begin{aligned} \Phi_\pm &\longleftrightarrow \bar{\Phi}_\pm \quad , \quad J_\pm \longleftrightarrow -\bar{J}_\pm \quad , \\ \Gamma_\alpha &\longrightarrow -\Gamma_\alpha \quad , \quad B \longrightarrow -B \quad , \quad C \longrightarrow -C \quad \widehat{C} \longrightarrow -\widehat{C} \quad . \end{aligned} \quad (24)$$

$G$ -parity:

$$\begin{aligned} \Phi_\pm &\longrightarrow -\Phi_\pm \quad , \quad \bar{\Phi}_\pm \longrightarrow -\bar{\Phi}_\pm \quad , \quad J_\pm \longrightarrow -J_\pm \quad , \quad \bar{J}_\pm \longrightarrow -\bar{J}_\pm \quad , \\ \Gamma_\alpha &\longrightarrow \Gamma_\alpha \quad , \quad B \longrightarrow B \quad , \quad C \longrightarrow C \quad , \quad \widehat{C} \longrightarrow \widehat{C} \quad . \end{aligned} \quad (25)$$

The ultraviolet and infrared dimensions,  $d$  and  $r$  respectively, as well as the ghost numbers,  $\Phi\Pi$ , and the Grassmann parity,  $GP$ , of all superfields and superspace objects ( $\theta_\alpha$  and  $D_\alpha$ ) are collected in Table 1.

	$\Gamma_\alpha$	$\Phi_\pm$	$C$	$\widehat{C}$	$B$	$J_\pm$	$\theta_\alpha$	$D_\alpha$
$d$	1/2	1/2	0	1	1	3/2	-1/2	1/2
$r$	1/2	3/2	0	1	1	3/2	-1/2	1/2
$\Phi\Pi$	0	0	1	-1	0	-1	0	0
$GP$	1	0	1	1	1	1	1	1

Table 1: UV and IR dimensions,  $d$  and  $r$ , ghost numbers,  $\Phi\Pi$ , and Grassmann parity,  $GP$ .

### 3 Renormalization

The renormalizability of the model runs according to the usual procedure of studying its stability – which amounts to check that, in the quantum theory, the possible counterterms can be reabsorbed by a redefinition of the initial parameters of the model – and the determination of the possible anomalies.

#### 3.1 Stability

In order to study the stability of the model under radiative corrections, we introduce an infinitesimal perturbation in the classical action  $\Sigma$  by means of an integrated local functional  $\Sigma^c$  – which, in the perturbative quantum theory, would be considered as an infinitesimal quantum correction, i.e. a counterterm –

$$\Sigma \rightarrow \Sigma + \epsilon \Sigma^c \quad , \quad (26)$$

where  $\epsilon$  is an infinitesimal parameter. The functional  $\Sigma^c$  has the same quantum numbers as the action in the tree approximation.

The perturbed action must satisfy, to the order  $\epsilon$ , the same equations as  $\Sigma$ , i.e., the Slavnov-Taylor identity, the gauge condition, the ghost and antighost equations together with rigid invariance. This implies the conditions

$$\mathcal{S}_\Sigma \Sigma^c = 0 \quad , \quad (27)$$

$$\frac{\delta \Sigma^c}{\delta B} = 0 \quad , \quad \frac{\delta \Sigma^c}{\delta \widehat{C}} = 0 \quad , \quad \frac{\delta \Sigma^c}{\delta C} = 0 \quad , \quad (28)$$

$$\mathcal{W}_{\text{rigid}} \Sigma^c = 0 \quad . \quad (29)$$

Notice that  $\Sigma^c$  has also to be invariant under the discrete symmetries (24–25).

The condition (27), due to nilpotency of the linearized Slavnov-Taylor operator,  $\mathcal{S}_\Sigma$ , constitutes a cohomology problem in the sector of ghost number zero. The most general invariant counterterms,  $\Sigma^c$ , solution of the constraints (27–29), are given by the invariant pieces of the classical action (1):<sup>7</sup>

$$\Sigma_1^c = \int dv \Gamma^\alpha W_\alpha \quad , \quad \Sigma_2^c = \int dv \{ (\nabla^\alpha \bar{\Phi}_+) (\nabla_\alpha \Phi_+) + (\nabla^\alpha \bar{\Phi}_-) (\nabla_\alpha \Phi_-) \} \quad ,$$

<sup>7</sup>Note that the combination of counterterms given by

$$\frac{1}{2} \mathcal{S}_\Sigma \int dv (\bar{J}_+ \Phi_+ + \bar{J}_- \Phi_- - \bar{\Phi}_+ J_+ - \bar{\Phi}_- J_-) = -\Sigma_2^c + m \Sigma_3^c - 2\lambda \Sigma_4^c \quad ,$$

$$\Sigma_3^c = \int dv (\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-) , \quad \Sigma_4^c = \int dv (\bar{\Phi}_+ \Phi_+ - \bar{\Phi}_- \Phi_-)^2 . \quad (30)$$

The corresponding coefficients are fixed by suitable normalization conditions, which may be chosen as below:

$$\begin{aligned} C_{\beta\alpha} D^2(p) \frac{\partial}{\partial p^2} \Gamma_{\Gamma\Gamma}^{\alpha\beta} \Big|_{p=p(\mu)}^{\theta=0} &= \frac{1}{2} k , & \frac{1}{2} D^2(p) \frac{\partial}{\partial p^2} \Gamma_{\bar{\Phi}_\pm \Phi_\pm} \Big|_{p=p(\mu)}^{\theta=0} &= -\frac{1}{2} , \\ \Gamma_{\bar{\Phi}_\pm \Phi_\pm} \Big|_{p^2=m^2}^{\theta=0} &= 0 , & \Gamma_{\bar{\Phi}_\pm \Phi_\pm \bar{\Phi}_\pm \Phi_\pm} \Big|_{p=p(\mu)}^{\theta=0} &= -\lambda , \end{aligned} \quad (31)$$

where  $\mu$  is an energy scale,  $p(\mu)$  some reference set of 4-momenta at this scale and  $D_\alpha(p) = \partial_\alpha + \theta^\beta p_{\alpha\beta}$ .

### 3.2 Anomaly

Here, our goal, in order to complete the proof of the renormalizability of the model, is to show that it is possible to extend the symmetry preserving perturbative expansion to all orders, i.e., we must prove that is possible to define a quantum vertex functional  $\Gamma$ :

$$\Gamma = \Sigma + \mathcal{O}(\hbar) , \quad (32)$$

such that

$$\mathcal{S}(\Gamma) = 0 , \quad (33)$$

and

$$\frac{\delta\Gamma}{\delta B} = \frac{1}{2} D^\alpha \Gamma_a , \quad \frac{\delta\Gamma}{\delta \widehat{C}} = D^2 C , \quad -i \frac{\delta\Gamma}{\delta C} = \Delta_{\text{class}} , \quad \mathcal{W}_{\text{rigid}} \Gamma = 0 . \quad (34)$$

According to the Quantum Action Principle [5], the Slavnov-Taylor identity (10) will be broken at the quantum level, as follows

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma = \Delta + \mathcal{O}(\hbar\Delta) . \quad (35)$$

where  $\Delta$ , at lowest order in  $\hbar$ , is an integrated local functional with ghost number 1, UV dimension  $\leq 2$  and IR dimension  $\geq 2$ .

By using the algebra (18–23) written for the functional  $\Gamma$  we get the following set of constraints for the breaking  $\Delta$ :

$$\mathcal{S}_\Sigma \Delta = 0 , \quad (36)$$

and

$$\frac{\delta\Delta}{\delta B} = 0 , \quad \frac{\delta\Delta}{\delta \widehat{C}} = 0 , \quad \int dv \frac{\delta}{\delta C} \Delta = 0 , \quad \mathcal{W}_{\text{rigid}} \Delta = 0 . \quad (37)$$

The condition (36) represents the Wess-Zumino consistency condition. It constitutes a cohomology problem analogous to the one determining the counterterm  $\Sigma^c$ , now in the sector of ghost number one. Its solution can always be written as a sum of a trivial

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is “trivial”, being a  $\mathcal{S}_\Sigma$ -variation. The corresponding coefficient – which may be fixed by the second normalization condition (31) – corresponds to the (nonphysical) renormalization of the matter field amplitude. The other three counterterms correspond to the renormalization of the physical parameters  $\kappa$ ,  $m$  and  $\lambda$ .

cocycle  $\mathcal{S}_\Sigma \widehat{\Delta}^{(0)}$ , where  $\widehat{\Delta}^{(0)}$  has ghost number 0, and of nontrivial elements belonging to the cohomology of  $\mathcal{S}_\Sigma$  (11) in the sector of ghost number one:

$$\Delta^{(1)} = \mathcal{A}^{(1)} + \mathcal{S}_\Sigma \widehat{\Delta}^{(0)} . \quad (38)$$

The trivial cocycle  $\mathcal{S}_\Sigma \widehat{\Delta}^{(0)}$  can be absorbed into the vertex functional  $\Gamma$  as a noninvariant integrated local counterterm  $-\widehat{\Delta}^{(0)}$ . On the other hand, a nonzero  $\mathcal{A}^{(1)}$  would represent a possible anomaly.

Considering the third of the conditions (37), which  $\Delta^{(1)}$  has to satisfy, it can be concluded that the latter has the form

$$\Delta^{(1)} = \int dv K^{\alpha(0)} D_\alpha C . \quad (39)$$

By analyzing the Slavnov-Taylor operator  $\mathcal{S}_\Sigma$  (11) and the Wess-Zumino consistency condition (36), one sees that the breaking  $\Delta^{(1)}$  has UV and IR dimensions bounded by  $d \leq 2$  and  $r \geq 2$ . Therefore, the dimensions of  $K^{\alpha(0)}$  must be bounded by  $d \leq \frac{3}{2}$  and  $r \geq \frac{3}{2}$ . It has ghost number 0.

Now, after solving all the conditions  $K^{\alpha(0)}$  has to fulfill, we see that it may be expanded in a local basis as:

$$K^{\alpha(0)} = \sum_{i=1}^3 a_i K_i^{\alpha(0)} , \quad (40)$$

where

$$K_1^{\alpha(0)} = \Gamma^\alpha \Gamma^2 , \quad K_2^{\alpha(0)} = \bar{\Phi}_+ \Gamma^\alpha \Phi_+ , \quad K_3^{\alpha(0)} = \bar{\Phi}_- \Gamma^\alpha \Phi_- . \quad (41)$$

We may thus write

$$K^{\alpha(0)} D_\alpha C = \sum_{i=1}^3 a_i K_i^{\alpha(0)} D_\alpha C = \sum_{i=1}^3 a_i \mathcal{S}_\Sigma \widehat{K}_i^{(0)} = \mathcal{S}_\Sigma \widehat{K}^{(0)} , \quad (42)$$

where

$$\widehat{K}_1^{(0)} = -\frac{1}{2}(\Gamma^2)^2 , \quad \widehat{K}_2^{(0)} = -\bar{\Phi}_+ \Gamma^2 \Phi_+ , \quad \widehat{K}_3^{(0)} = -\bar{\Phi}_- \Gamma^2 \Phi_- . \quad (43)$$

Therefore, the breaking  $\Delta^{(1)}$  is BRS trivial and given by

$$\Delta^{(1)} = \int dv K^{\alpha(0)} D_\alpha C = \mathcal{S}_\Sigma \int dv \widehat{K}^{(0)} = \mathcal{S}_\Sigma \widehat{\Delta}^{(0)} . \quad (44)$$

This means that  $\mathcal{A}^{(1)} = 0$  in (38), which implies the implementability of the Slavnov-Taylor identity to every order through the absorption of the noninvariant counterterm  $-\widehat{\Delta}^{(0)}$ .

Of course, invariant counterterms may still be arbitrarily added at each order. However the result of the discussion on the stability of the classical theory shows that these counterterms correspond to a renormalization of the parameters of the theory. Their coefficients are fixed by the normalization conditions (31).

In conclusion, the use of the algebraic method of renormalization has allowed us to show that  $N=1$  super-Chern-Simons model coupled to parity-preserving matter is perturbatively renormalizable to all orders. First, the study of the stability has led the

conclusion that counterterms can be reabsorbed by a redefinition of the initial parameters of the model. Next, gauge anomalies have been proven to be absent. We stress that this is a purely algebraic analysis, valid to all orders and does not involve any regularization scheme, nor any Feynman graph calculation.

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