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THE GAUGE SUPERFIELD IN SIX DIMENSIONS

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ABSTRACT

In D=6 the gauge superfield V has a higher gauge freedom than in D=4. In the Wess-Zumino gauge there are no term with less than two Grassmann variables of each type. We find all its tensor components and compute also the chiral superfield strength which contains the linearized curvature tensor of a "graviton" field, as well as the field intensities of a "gravitino", a "photon", a "photino", and a tri-vector whose field strength is a second rank antisymmetric tensor. The gauge invariant interaction with a chiral field is given and expressed as a function of the components.

Key-words: Supersymmetry; Higher dimensions; Gauge superfield.

1 INTRODUCTION

It has already been pointed out that the order of the differential equations of motion could be related to the dimensionality of space-time[1]. Working with higher order equations presents considerable difficulties, both of mathematical technicalities as well as of physical interpretation. However, for the particular case of plane waves (only one space coordinate and time) a mathetical procedure exists which allows a satisfactory treatment of the problem[3].

In quantum theory, a supersymmetric model seems appropriate mainly because of the improvement in the ultraviolet behaviour of the amplitudes.

For these reasons, we choose as our simplest model, the real U(1) gauge superfield V in six dimensions. The pecularities arising from the fact that $D/2 = 3$, odd number, are of technical origin and do not change substantially - as we will see - the discussion of reference[1] where we choose $D=4n$. The properties of spinors in $D=6$ are taken from Elie Cartan's book[4].

In $D=6$ Weyl spinors have four components and so, the products of Grassmann variables of the same type can only contain up to four factors (as compared with two in $D=4$).

The chiral superfield which has the field strengths associated with V is now a bi-spinor W_{α_1,α_2}[5] whose structure shows that in addition to the usual gauge invariance (i.e. adding a chiral plus anti-chiral fields to V) is also invariant under the addition of a superfield whose covariant derivative (D_α) is chiral. This invariance allows us to adopt the Wess-Zumino gauge in which V has no component with less than two spinor indices of each type. It is in this gauge that the physical meaning of V can be most easily recognized.

The components of the gauge superfield can be named according to their tensor character. We will see that we have a tensor field ("graviton"), a vector-spinor field ("gravitino") a vector field ("photon"), a spinor field ("photino"). It is amusing to see also the appearance of a tri-vector (totally antisymmetric in its three indices) whose field strength is a second rank antisymmetric tensor.

The bi-spinor W_{α_1,α_2} is explicitly evaluated and also the corresponding lagrangian.

Finally, the interaction of the gauge field V with a chiral superfield is computed in the usual way.

2 SPINORS IN SIX DIMENSIONS

For the construction of Dirac matrices $\Gamma_\mu (\mu = 0,\ldots,5)$ we take the six 4x4 hermitian matrices γ_μ with $\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$ (i=1,2,3), $\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, and $\gamma_o = \mathbb{1}$.

We have the 8x8 matrices:

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \tilde{\gamma}_\mu & 0 \end{pmatrix} \ , \ \{\Gamma_\mu,\Gamma_\nu\} = 2\eta_{\mu\nu}(\text{diag. } 1,1\ldots1,-1) \tag{1}$$

where $\tilde{\gamma}_\mu = \gamma_\mu$ for μ=1,...,5 and $\tilde{\gamma}_o = -\gamma_o = -\mathbb{1}$. The "transposition" matrix $\mathbb{C}$ is defined by:

$$\mathbb{C}\Gamma_\mu = -\Gamma_\mu{}^T\mathbb{C} \ , \ \mathbb{C}^2 = \mathbb{1} \tag{2}$$

$$\mathbb{C} = \mathbb{C}^T = \Gamma_0\Gamma_2\Gamma_5 = \begin{pmatrix} 0 & C \\ -C & 0 \end{pmatrix} , \; C = \gamma_2\gamma_5 \tag{3}$$

In D=6, unlike in four dimensions, the conjugate of a Weyl spinor is another Weyl spinor of the same type. The conjugate spinor is defined by[4]

$$\phi^C = C\phi^* \quad ; \quad \bar{\Phi} = \phi^+ C \tag{4}$$

According to[4] (pag.114) the scalar product of two Weyl spinors of different type is defined by

$$\psi \, C\phi = \text{scalar} \; ; \tag{5}$$

Therefore, in order to construct a chiral field with Weyl spinors of the first type we take the Grassman variables as spinors of the second type $\theta_{\dot{\alpha}}$ and $\theta_{\dot{\alpha}}{}^+$.

Then, if we take the scalar product between a spinor ψ_α of the first type and the conjugate $\bar{\Phi}_{\dot{\alpha}}$ of the second type we have:

$$\bar{\Phi}C\psi = -\phi^+ CC\psi = \phi_{\dot{\alpha}}{}^+ \delta^{\dot{\alpha}\alpha}\psi_{\dot{\alpha}} = \phi_{\dot{\alpha}}{}^+ \psi^{\dot{\alpha}} \tag{6}$$

where we have introduced a contravariant spinor[6]

$$\psi^{\dot{\alpha}} = \delta^{\alpha\dot{\alpha}}\psi_\alpha \tag{7}$$

Analogously $\phi^\alpha = \delta^{\alpha\dot{\alpha}}\phi_{\dot{\alpha}}$.

We will take as generators of the supersymmetry algebra acting on superfields

$$Q_\alpha = i \frac{\partial}{\partial\theta^\alpha} + \theta^{+}_{\dot\alpha}\gamma^{\mu\dot\alpha}_{\dot\alpha}\partial_\mu$$
$$Q^{+\dot\alpha} = i \frac{\partial}{\partial\theta^{+}_{\dot\alpha}} + \gamma^{\mu\dot\alpha}_{\alpha}\theta^\alpha\partial_\mu \tag{8}$$

with

$$\{Q_\alpha Q^{+\dot\alpha}\} = 2i\,\partial_\alpha{}^{\dot\alpha} \tag{9}$$

and the corresponding covariant derivatives

$$D = \frac{\partial}{\partial\theta} + i\,\theta^{+}\gamma^\mu\partial_\mu$$
$$D^{+} = \frac{\partial}{\partial\theta^{+}} + i\,\gamma^\mu\theta\,\partial_\mu \tag{10}$$

which anticommute with Q and Q^+.

3 THE GAUGE SUPERFIELD

A real superfield V has the general form

$$V = \sum_{s,t=0}^{4} \theta^{+}_{\dot\alpha_1}\ldots\theta^{+}_{\dot\alpha_s} A^{\dot\alpha_1\ldots\dot\alpha_s}_{\alpha_1\ldots\alpha_t}\,\theta^{\alpha_1}\ldots\theta^{\alpha_t} \tag{11}$$

with

$$\left(A^{\dot\alpha_1\ldots\dot\alpha_s}_{\alpha_1\ldots\alpha_t}\right)^* = A^{\dot\alpha_1\ldots\dot\alpha_t}_{\alpha_1\ldots\alpha_s}$$

We shall define the gauge transformation of V by

$$V' = V + i(\bar{\psi}-\psi) \tag{12}$$

where

$$D^{+}_{\dot{\alpha}_1} D^{+}_{\dot{\alpha}_2} \psi = 0 \quad \text{and} \quad D_{\alpha_1} D_{\alpha_2} \bar{\psi} = 0 \tag{13}$$

(double chiral fields)

We shall justify below this definition.

Using (12) we can go over to the Wess-Zumino gauge in which all components with less than two indices of each class are absent

$$V = \sum_{s,t=2}^{4} \theta^{+}_{\dot{\alpha}} \ldots \theta^{+}_{\dot{\alpha}_s} A^{\dot{\alpha}_1 \ldots \dot{\alpha}_s}_{\alpha_1 \ldots \alpha_t} \theta^{\alpha_1} \ldots \theta^{\alpha_t} \tag{14}$$

We shall first analyze the lowest component $A^{\dot{\alpha}_1\dot{\alpha}_2}_{\alpha_1\alpha_2}$ which is antisymmetric in both types of indices. According to Cartan in ref. [4] the product of two spinors of the same type is reduced to the sum of a vector and of a tri-vector. Further the vector is antisymmetric in the spinor indices and the tri-vector is symmetric. So, in our case, we have

$$A^{\dot{\alpha}_1\dot{\alpha}_2}_{\alpha_1\alpha_2} = A_{\mu\nu}(\gamma^{\mu}C)_{\alpha_1\alpha_2}(C\gamma^{\nu})^{\dot{\alpha}_1\dot{\alpha}_2} \tag{15}$$

which shows that the lowest component is equivalent to a second rank tensor whose symmetric and traceless part we may call "graviton". We now take the next "diagonal" component.

$$A^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3}_{\alpha_1\alpha_2\alpha_3} = \varepsilon^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4} \varepsilon_{\alpha_1\alpha_2\alpha_3\alpha_4} A^{\alpha_4}_{\dot{\alpha}_4} \tag{16}$$

But now $A^{\alpha}_{\dot{\alpha}}$ is neither symmetric nor antisymmetric, so we must have both terms in the already mentioned reduction:

$$A^{\alpha}_{\dot{\alpha}} = A_{\mu}\tilde{\gamma}^{\mu\alpha}_{\dot{\alpha}} + i\,A_{\nu_1\nu_2\nu_3}\left(\tilde{\gamma}^{\nu_1}\gamma^{\nu_2}\tilde{\gamma}^{\nu_3}\right)^{\alpha}_{\dot{\alpha}} \tag{17}$$

where $A_{\nu_1\nu_2\nu_3}$ is a completely antisymmetric real self-dual tensor. The real veçtor A_{μ} can be called "photon field".

The highest component is equivalent to a scalar:

$$A^{\dot{\alpha}_1\ldots\dot{\alpha}_4}_{\alpha_1\ldots\alpha_4} = \varepsilon^{\dot{\alpha}_1\ldots\dot{\alpha}_4}\varepsilon_{\alpha_1\ldots\alpha_4}D \quad (D = D^*,\ \text{auxiliary field}) \tag{18}$$

Let us go over to the off diagonal components:

$$A^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3}_{\alpha_1\alpha_2} = \varepsilon^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}\,A^{\mu}_{\dot{\alpha}_4}(\gamma_{\mu}C)_{\alpha_1\alpha_2} \tag{19}$$

$A^{\mu}_{\dot{\alpha}}$ is a vector-spinor which we may call "gravitino". Now

$$A^{\dot{\alpha}_1\ldots\dot{\alpha}_4}_{\alpha_1\alpha_2} = \varepsilon^{\dot{\alpha}_1\ldots\dot{\alpha}_4}\,\lambda_{\mu}(\gamma^{\mu}C)_{\alpha_1\alpha_2} \tag{20}$$

where λ_{μ} is a complex vector field. Finally

$$A^{\dot{\alpha}_1\ldots\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3} = \varepsilon^{\dot{\alpha}_1\ldots\dot{\alpha}_4}\,\varepsilon_{\alpha_1\ldots\alpha_4}A^{\alpha_4} \tag{21}$$

The spinor field A^{α} may be called "photino".

4 THE FIELD STRENGTH SUPERFIELD

We shall take as the chiral superfield strength, (see ref.[5]) the following bi-spinor:

$$W_{\alpha_1\alpha_2} = - W_{\alpha_2\alpha_1} = \overset{4}{D}{}^{+} D_{\alpha_1} D_{\alpha_2} V \tag{22}$$

where

$$\overset{4}{D}{}^{+} = \frac{1}{4!}\, \varepsilon_{\dot\alpha_1\ldots\dot\alpha_4} D^{+\dot\alpha_1}\ldots D^{+\dot\alpha_4} \tag{23}$$

$W_{\alpha_1\alpha_2}$ is obviously chiral. Further, it is invariant under the gauge transformation (12), (13). The proof is direct for $\bar\psi$. The invariance of (22) under the addition to V of ψ (being (13)) can be easily seen using the anticommutation relations between D and D^+.

Direct evaluation of (22), for $\theta^+=0$, gives:

$$\begin{aligned}
W^{o}_{\alpha_1\alpha_2} = \varepsilon_{\dot\alpha_1\ldots\dot\alpha_4}\Big[& 2A^{\dot\alpha_1\ldots\dot\alpha_4}_{\alpha_2\alpha_1} + \theta^{\beta_1}\Big\{-2i\partial^{\dot\alpha_4}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_1]\beta_1} + 2i\partial^{\dot\alpha_4}_{\beta_1} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_2\alpha_1} + \\
& + 6A^{\dot\alpha_1\ldots\dot\alpha_4}_{\alpha_2\alpha_1\beta_1}\Big\} + \theta^{\beta_1}\theta^{\beta_2}\Big\{-\partial^{\dot\alpha_3}_{\alpha_1}\partial^{\dot\alpha_4}_{\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\beta_1\beta_2} + 2\partial^{\dot\alpha_4}_{\beta_1}\partial^{\dot\alpha_3}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\alpha_1]\beta_2} + \\
& + \partial^{\dot\alpha_1}_{\beta_1}\partial^{\dot\alpha_2}_{\beta_2} A^{\dot\alpha_3\dot\alpha_4}_{\alpha_2\alpha_1} - 3i\partial^{\dot\alpha_4}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_1]\beta_1\beta_2} + 6i\partial^{\dot\alpha_4}_{\beta_1} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_2\alpha_1\beta_2} - 12A^{\dot\alpha_1\ldots\dot\alpha_4}_{\alpha_1\alpha_2\beta_1\beta_2}\Big\} \\
& + \theta^{\beta_1}\theta^{\beta_2}\theta^{\beta_3}\Big\{-\partial^{\dot\alpha_3}_{\alpha_1}\partial^{\dot\alpha_4}_{\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\beta_1\beta_2\beta_3} + 3\partial^{\dot\alpha_4}_{\beta_1}\partial^{\dot\alpha_3}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\alpha_1]\beta_2\beta_3} + 3\partial^{\dot\alpha_3}_{\beta_1}\partial^{\dot\alpha_4}_{\beta_2} A^{\dot\alpha_1\dot\alpha_2}_{\alpha_2\alpha_1\beta_3} - \\
& - 4i\partial^{\dot\alpha_4}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_1]\beta_1\beta_2\beta_3} + 12i\partial^{\dot\alpha_4}_{\beta_1} A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_2\alpha_1\beta_2\beta_3}\Big\} + \theta^{\beta_1}\cdots\theta^{\beta_4} \\
& \Big\{-\partial^{\dot\alpha_3}_{\alpha_1}\partial^{\dot\alpha_4}_{\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\beta_1\ldots\beta_4} + 4\partial^{\dot\alpha_4}_{\beta_1}\partial^{\dot\alpha_3}_{[\alpha_2} A^{\dot\alpha_1\dot\alpha_2}_{\alpha_1]\beta_2\beta_3\beta_4} + 6\partial^{\dot\alpha_3}_{\beta_1}\partial^{\dot\alpha_4}_{\beta_2} A^{\dot\alpha_1\dot\alpha_2}_{\alpha_2\alpha_1\beta_3\beta_4}\Big\}\Big]
\end{aligned} \tag{24}$$

(Compare with the four-dimensional case, see [7]).

As the lowest component, (24) has the complex vector field λ_μ (see eq.(20)). The highest component contains second derivatives of λ^*_μ. The next to lowest component contains what we called the gravitino (eq.(19)) and photino (eq.(21)) fields, whose complex conjugates appear in the next to highest component. The "middle" component has the tensor field (eq.(15)), the real vector and trivector fields (eq.(17)), and the auxiliary field (eq.(18)).

5 THE LAGRANGIAN

We obtain the Lagrangian by first taking the scalar product $\varepsilon^{\alpha_1 \ldots \alpha_4} W^o_{\alpha_1\alpha_2} W^o_{\alpha_3\alpha_4}$, and then looking for the highest θ-component (which is supersymmetric).

$$\mathcal{L} \equiv \varepsilon^{\alpha_1 \ldots \alpha_4} W^o_{\alpha_1\alpha_2} W^o_{\alpha_3\alpha_4}\Big|_{\theta^4} + \text{h.c.} \tag{25}$$

We shall not write down $\mathcal{L}$ explicitly, but we will point out some results which we think interesting.

The product of the lowest component of $W^o_{\alpha_1\alpha_2}$ times the highest component of $W^o_{\alpha_3\alpha_4}$ gives the Lagrangian for the complex vector field:

$$\mathcal{L}_1 = A_{\alpha_2\alpha_1}^{\dot\alpha_1\ldots\dot\alpha_4}\Big(-\partial_{\alpha_3}^{\dot\beta_3}\partial_{\alpha_4}^{\dot\beta_4}A_{\beta_1\ldots\beta_4}^{\dot\beta_1\dot\beta_2} + 8\partial_{\beta_1}^{\dot\beta_4}\partial_{\alpha_4}^{\dot\beta_3}A_{\alpha_3\beta_2\beta_3\beta_4}^{\dot\beta_1\dot\beta_2} + $$
$$+ 6\partial_{\beta_1}^{\dot\beta_3}\partial_{\beta_2}^{\dot\beta_4}A_{\alpha_4\alpha_3\beta_3\beta_4}^{\dot\beta_1\dot\beta_2}\Big)\varepsilon^{\alpha_1\ldots\alpha_4}\varepsilon^{\beta_1\ldots\beta_4}\varepsilon_{\dot\alpha_1\ldots\dot\alpha_4}\varepsilon_{\dot\beta_1\ldots\dot\beta_4}$$

And now, using eq.(20):

$$\mathcal{L}_1 = 2\lambda^\mu \partial_\mu \partial^\nu \lambda^*_\nu - \lambda^\mu \Box \lambda^*_\mu \tag{26}$$

Consider now the Lagrangian corresponding to the tensor field (Cf.eq.(15)) which we take, for simplicity, to be symmetric and traceless. From (24) and (25) this Lagrangian is:

$$\mathcal{L}_2 = \left\{-\partial^{\dot\alpha_3}_{\alpha_1}\partial^{\dot\alpha_4}_{\alpha_2}A^{\dot\alpha_1\dot\alpha_2}_{\beta_1\beta_2} + 4\partial^{\dot\alpha_4}_{\beta_1}\partial^{\dot\alpha_3}_{\alpha_2}A^{\dot\alpha_1\dot\alpha_2}_{\alpha_1\beta_2} + \partial^{\dot\alpha_1}_{\beta_1}\partial^{\dot\alpha_2}_{\beta_2}A^{\dot\alpha_3\dot\alpha_4}_{\alpha_2\alpha_1}\right\} \cdot$$
$$\left\{-\partial^{\dot\beta_3}_{\alpha_3}\partial^{\dot\beta_4}_{\alpha_4}A^{\dot\beta_1\dot\beta_2}_{\beta_3\beta_4} + 4\partial^{\dot\beta_4}_{\beta_3}\partial^{\dot\beta_3}_{\alpha_4}A^{\dot\beta_1\dot\beta_2}_{\alpha_3\beta_4} + \partial^{\dot\beta_1}_{\beta_3}\partial^{\dot\beta_2}_{\beta_4}A^{\dot\beta_3\dot\beta_4}_{\alpha_4\alpha_3}\right\} \varepsilon^{\alpha_1\ldots\alpha_4}\varepsilon^{\beta_1\ldots\beta_4}\varepsilon_{\dot\alpha_1\ldots\dot\alpha_4}\varepsilon_{\dot\beta_1\ldots\dot\beta_4} \tag{27}$$

After use of (15) and some "gammology", we obtain from eq.(27):

$$\mathcal{L}_2 \approx \Box A_{\mu\nu}\Box A^{\mu\nu} - 2\partial_\lambda\partial^\mu A_{\mu\nu}\partial^\lambda\partial_\rho A^{\rho\nu} + \partial^\mu\partial^\nu A_{\mu\nu}\partial^\rho\partial^\sigma A_{\rho\sigma} \tag{28}$$

We now define:

$$F_{\mu\nu\sigma\lambda} = \partial_\lambda\partial_\nu A_{\mu\sigma} + \partial_\sigma\partial_\mu A_{\nu\lambda} - \partial_\lambda\partial_\mu A_{\nu\sigma} - \partial_\sigma\partial_\nu A_{\mu\lambda}\ , \tag{29}$$

which is antisymmetric in $\mu\nu$ and in $\sigma\lambda$ (symmetric under the interchange of $\mu\nu$ with $\sigma\lambda$). Then, up to a divergence, we have:

$$\mathcal{L}_2 = \frac{1}{4}F^{\mu\nu\sigma\lambda}F_{\mu\nu\sigma\lambda} \tag{30}$$

Note the similarity between (29) and the linearized curvature tensor of gravitation (see, for example reference [8]). Note also that the related equations of motion are of the fourth order).

For the Lagrangian corresponding to $A^{\dot\alpha_1\dot\alpha_2\dot\alpha_3}_{\alpha_1\alpha_2\alpha_3}$, eqs. (24) and

(25) give:

$$\mathcal{L}_3 = (\partial_{\alpha_1}^{\dot\alpha_4} A_{\alpha_2\beta_1\beta_2}^{\dot\alpha_1\dot\alpha_2\dot\alpha_3} + \partial_{\beta_1}^{\dot\alpha_4} A_{\alpha_2\alpha_1\beta_2}^{\dot\alpha_1\dot\alpha_2\dot\alpha_3})(\partial_{\alpha_3}^{\dot\beta_4} A_{\alpha_4\beta_3\beta_4}^{\dot\beta_1\dot\beta_2\dot\beta_3} + \partial_{\beta_3}^{\dot\beta_4} A_{\alpha_4\alpha_3\beta_4}^{\dot\beta_1\dot\beta_2\dot\beta_3})\, \varepsilon^{\alpha_1\ldots\alpha_4}\, \varepsilon^{\beta_1\ldots\beta_4}\, \varepsilon_{\dot\alpha_1\ldots\dot\alpha_4}\, \varepsilon_{\dot\beta_1\ldots\dot\beta_4} \tag{31}$$

Using eqs. (16) and (17) we arrive at:

$$\mathcal{L}_3 \approx \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + 36\partial^{\nu_1} A_{\nu_1\nu_2\nu_3}\partial^\mu A_\mu^{\ \nu_2\nu_3} - 6\partial_\mu A_{\nu_1\nu_2\nu_3}\partial^\mu A^{\nu_1\nu_2\nu_3} \tag{32}$$

We define a rotational operator on a tri-vector as:

$$G^{\nu_5\nu_6} = \frac{1}{3!}\,\partial_{\nu_1} A_{\nu_2\nu_3\nu_4}\,\varepsilon^{\nu_1\ldots\nu_6} \equiv \partial_{\nu_1}\tilde{A}^{\nu_1\nu_5\nu_6} \tag{33}$$

where $\tilde{A}$ is the dual of A.

As the tri-vector is self-dual, it can be shown that both terms involving $A_{\nu_1\nu_2\nu_3}$ in (32) are proportional to $G^{\mu\nu}G_{\mu\nu}$. Then:

$$\mathcal{L}_3 = F_{\mu\nu}F^{\mu\nu} + K\, G^{\mu\nu}G_{\mu\nu} \tag{34}$$

(κ some numerical constant).

6 INTERACTION

The gauge invariant interaction between the superfield V and a chiral field Φ is given in the usual way by:

$$\mathcal{L}_{\pm} = \bar{\Phi} e^{V} \phi = \phi_o^{+} e^{-i\Lambda} e^{V} e^{-i\Lambda} \phi_o = \phi_o^{+} e^{-i\Lambda} (1+V+\frac{V^2}{2}) e^{-i\Lambda} \phi_o \qquad (35)$$

where

$$\Lambda = \theta^{+}_{\dot{\alpha}} \gamma^{\mu\,\dot{\alpha}}_{\alpha} \theta^{\alpha} \partial_{\mu} \qquad (36)$$

The first term in the parenthesis (of (35)) corresponds to the free Lagrangian for the chiral field. We shall discuss only the simplest term of the interaction, namely

$$\mathcal{L}' = \Phi_o^{+} V \Phi_o \Big|_{\overset{4}{\theta}\overset{4}{\theta}{}^{+}} \qquad (37)$$

where

$$\Phi_o = \sum_{s=0}^{4} \psi_{\alpha_1 \ldots \alpha_s} \theta^{\alpha_1} \cdots \theta^{\alpha_s} \qquad (38)$$

Introducing (38) and (14) in (37) and taking the coefficients of $\overset{4}{\theta}\overset{4}{\theta}{}^{+}$ we get:

$$\mathcal{L}' = \sum_{s,t=2}^{4} \bar{\psi}^{\dot{\alpha}_{s+1} \cdots \dot{\alpha}_4} A^{\dot{\alpha}_1 \ldots \dot{\alpha}_s}_{\alpha_1 \ldots \alpha_t} \psi_{\alpha_{t+1} \ldots \alpha_4} \varepsilon^{\alpha_1 \ldots \alpha_4} \varepsilon_{\dot{\alpha}_1 \ldots \dot{\alpha}_4} \qquad (39)$$

Or, "in extenso":

$$\begin{aligned}\mathcal{L}' = \Big\{ & \bar{\psi}^{\dot{\alpha}_1\dot{\alpha}_2} A^{\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2} \psi_{\alpha_3\alpha_4} + \bar{\psi}^{\dot{\alpha}_1\dot{\alpha}_2} A^{\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3} \psi_{\alpha_4} + \bar{\psi}^{\dot{\alpha}_1\dot{\alpha}_2} A^{\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\ldots\alpha_4} \psi + \\ & + \bar{\psi}^{\dot{\alpha}_1} A^{\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2} \psi_{\alpha_3\alpha_4} + \bar{\psi}^{\dot{\alpha}_1} A^{\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3} \psi_{\alpha_4} + \bar{\psi}^{\dot{\alpha}_1} A^{\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} \psi + \\ & + \bar{\psi} A^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2} \psi_{\alpha_3\alpha_4} + \bar{\psi} A^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3} \psi_{\alpha_4} + \bar{\psi} A^{\dot{\alpha}_1\dot{\alpha}_2\dot{\alpha}_3\dot{\alpha}_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} \psi \Big\} \varepsilon^{\alpha_1 \ldots \alpha_4} \varepsilon_{\dot{\alpha}_1 \ldots \dot{\alpha}_4}\end{aligned}$$

Now using (15) to (21).

We can rewrite (39) in the form

$$\mathcal{L}_1' \cong 4\bar{\psi}_\mu A^{\mu\nu}\psi_\nu + 2\psi_\mu A_\mu^{*\alpha}\psi_\alpha + \bar{\psi}_\mu \lambda_\mu^* \psi + 2\bar{\psi}^{\dot{\alpha}} A_{\dot{\alpha}}^{\mu}\psi_\mu +$$

$$+ \bar{\psi}^{\dot{\alpha}}\bar{A}_{\dot{\alpha}}\psi + \bar{\psi}A^{\alpha}\psi_\alpha + \bar{\psi}\lambda^{\mu}\psi_\mu + A_\nu \psi^{\dot{\alpha}}\bar{\gamma}_{\dot{\alpha}}^{\nu\alpha} + iA_{\nu_1\nu_2\nu_3}\bar{\psi}^{\dot{\alpha}}\left(\bar{\gamma}_{\nu_1}\gamma_{\nu_2}\bar{\gamma}_{\nu_3}\right)_{\dot{\alpha}}^{\alpha}\psi_\alpha$$

$$+ \bar{\psi}D\psi \tag{40}$$

where ψ_μ is the vector associated with $\psi_{\alpha_1\alpha_2}$

$$\psi_{\alpha_1\alpha_2} = (\gamma^{\mu}C)_{\alpha_1\alpha_2}\psi_\mu$$

We shall write the eq. of motion corresponding to the symmetric and traceless tensor field whose free Lagrangian is given by (28) or (30). The interaction comes from the first term of (40) and the additional ones coming from the rest of (35).

The equation is

$$\partial^{\sigma}\partial^{\lambda}F_{\mu\sigma,\nu\lambda} = j_{\mu\nu}^{v} + j_{\mu\nu}^{sp} + j_{\mu\nu}^{sc} \tag{41}$$

where $F_{\mu\sigma,\nu\lambda}$ is given by (29) and the currents are given by the following expressions

$$j_{\mu\nu}^{v} = \bar{\psi}_\mu\psi_\nu \tag{42}$$

$$j_{\mu\nu}^{sp} = i\partial_\mu\bar{\psi}^{\dot{\alpha}}\gamma_{\nu\dot{\alpha}}^{\alpha}\psi_\alpha + hc \tag{43}$$

$$j_{\mu\nu}^{sc} = A_{\mu\nu}\bar{\psi}\psi + \partial_\mu\bar{\psi}\partial_\nu\psi + \partial_\mu\partial_\nu\bar{\psi}\psi + hc \tag{44}$$

where the symmetric and traceless parts in μ,ν are to be taken.

As a second example, we obtain for the tri-vector field from (32), (33), (34) and (40) the following equation

$$\partial_{[\nu_1} G_{\nu_2 \nu_3]} = \psi^{\dot{\alpha}} \left(\tilde{\gamma}_{[\nu_1} \gamma_{\nu_2} \gamma_{\nu_3]}\right)^{\alpha}_{\dot{\alpha}} \psi_{\alpha} \tag{45}$$

where [ ] means antisymmetrization in the three indices

7 DISCUSSIONS

The gauge superfield in D=6 has some interesting features. It contains all the massless fields we are acquainted with in four dimensions, namely: a graviton, a gravitino, a photon, and a photino; together with a tri-vector which, like the photon, has a second rank antisymmetric tensor as field intensity.

Of course, having chosen an abelian U(1) group of gauge transformations we can not expect neither non-linear field strength, nor a correct gravitational interaction with other fields. (We get a linearized curvature tensor for the graviton intensity).

We also see that the "main" field component (lowest gauge invariant component) obeys a fourth order equation of motion while the gravitino field obeys a third order differential equation.

The next steps will be to construct the canonical tensor associated to the superfield and to properly quantize the model. These studies are under way.

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