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Critical Fluctuations in Topologically Massive Superconductors

by

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Abstract

We consider a topologically massive Ginzburg-Landau model of superconductivity. In the context of a mean field calculation, we show that there is an increase in the critical temperature driven by the topological term. It is shown that this effect persists even if we take into account the critical fluctuations. The renormalization group analysis gives further insight on this behavior. The fixed point structure is such that the critical exponents tend to their mean field values for very large values of the topological mass. In this sense, the topological term stabilizes the critical fluctuations of the order parameter.

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The Ginzburg-Landau (GL) theory of superconductivity [1] describes very well the phenomenology of conventional superconductors. It is believed that a similar phenomenological theory can be applied to the study of the High temperature superconductors (HTSC). This expectation relies on experimental observation which indicates that the order parameter is just the same in both situations [2]. However, the GL theory neglects the fluctuations of the order parameter which are very important in the HTSC. Consequently, the exponents of the HTSC phase transition differs from the ones given by the GL theory [3]. Theoretically, the fluctuations can be taken into account through the use of renormalization group techniques to study the behavior of the theory in the neighbourhood of the critical point [4]. Another possible path to study the effect of the fluctuations is to compute further corrections to the free energy functional in a systematic way by performing a loop expansion [5].

In this note we study a version of the GL free energy functional where a topological Chern-Simons term [6] is added. Topological models are frequently employed in the construction of quantum models for HTSC which explore the effects of statistical transmutation (anyon superconductivity) [7]. Here we investigate the effect of such a topological contribution in a macroscopic model which generalizes the GL model. We perform mean field calculations very similar to those in the mean field theory proposed by Halperin, Lubensky and Ma (HLM) [8]. Also, we compute the corrections due to the critical fluctuations in the free energy and we obtain the renormaliztion group equations.

Our starting point is the following free energy functional,

$$F[\psi,\vec{A}] = \int d^3x \left[\frac{1}{2}|(\nabla - iq\vec{A})\psi|^2 + \frac{r_0}{2}|\psi|^2 + \frac{u_0}{4!}|\psi|^4 + \frac{1}{2}(\nabla\times\vec{A})^2 + i\frac{\theta}{2}\vec{A}\cdot(\nabla\times\vec{A})\right], \tag{1}$$

where $r_0 = a_0(T-T_0)/T_0$ and $\theta > 0$ is the topological mass. The partition function is given by

$$Z = \int D\vec{A}D\psi^\dagger D\psi \exp(-F[\psi,\vec{A}]). \tag{2}$$

Since F is quadratic in the vector field, the integration over $\vec{A}$ is Gaussian and can be performed exactly. For an uniform ψ this defines the following effective free energy density functional:

$$f_{eff}[\psi] = -\frac{1}{12\pi}[M_+^3(|\psi|^2) + M_-^3(|\psi|^2)] + \frac{r}{2}|\psi|^2 + \frac{u_0}{4!}|\psi|^4, \tag{3}$$

where the calculations were performed in the the Landau gauge and the functions $M_\pm$ are defined by

$$M_\pm^2(|\psi|^2) = q^2|\psi|^2 + \frac{\theta^2}{2} \pm \frac{\theta}{2}\sqrt{\theta^2 + 4q^2|\psi|^2}. \tag{4}$$

The parameter r_0 has been renormalized to $r = a(T - T_c)/T_c$. When $\theta = 0$ Eq.(3) reduces to the effective functional obtained by HLM. The critical temperature T_c does not depend on θ and is the same as in the HLM paper.

The inverse of the susceptibility is obtained for temperatures above the critical temperature by

$$\chi^{-1} = \frac{\partial^2 f_{eff}}{\partial|\psi|^2}_{|\psi|=0}. \tag{5}$$

The critical temperature is defined by the divergence of the susceptibility and we get from Eq.(5) the θ-dependent critical temperature

$$\tilde{T}_c = T_c\left(1 + \frac{q^2\theta}{2\pi a}\right). \tag{6}$$

The susceptibility has a critical behavior with critical exponent $\gamma = 1$ which is the mean field value. Minimizing the effective free energy functional with respect to $|\psi|^2$ and denoting by σ the corresponding value of $|\psi|$ which minimizes f_{eff} we get the expression for the mean field order parameter,

$$\sigma = \sqrt{\frac{6\tilde{a}}{u_0}}\left(1 - \frac{T}{\tilde{T}_c}\right)^{1/2}, \tag{7}$$

where

$$\tilde{a} = a + \frac{q^2\theta}{2\pi}. \tag{8}$$

Therefore the mean field critical exponents are the same as in HLM as it should be. In particular, the above results imply that $\beta = \nu = 1/2$. However, the critical temperature has been increased by a factor of $1+q^2\theta/2\pi^2 a$ with respect to the mean field critical temperature of HLM. The mean field theory of HLM renormalizes negligibly the critical temperature while in the present case this is not necessarily true because we may have arbitrary values of θ which may increase considerably the critical temperature of the superconducting phase transition.

Up to now we have taken into account only the fluctuations which does not change the mean field behavior of the model. To proceed let us investigate the effects of the critical fluctuations. These are obtained by computing the one loop correction arising from the fluctuations of the order parameter. The corrected free energy density is given by

$$f_{eff}^{1-loop} = \frac{r_0}{2}|\psi|^2 + \frac{u_0}{4!}|\psi|^4 + \frac{1}{4\pi^2}\{\int_0^\Lambda dp p^2[\ln(p^2 + M_+^2(|\psi|^2)) + \ln(p^2 + M_-^2(|\psi|^2)) + \ln(p^2 + r_0 + u_0|\psi|^2/2) - 3\ln(p^2)]\}, \tag{9}$$

from which we get the corrected inverse susceptibility for $T > T_c'$, T_c' being the new critical temperature:

$$\chi_{1-loop}^{-1} = \frac{a}{T_c}(T - \tilde{T}_c) + \frac{u_0}{4\pi^2}\int_0^\Lambda dp \frac{p^2}{p^2 + r'}. \tag{10}$$

In the above equation we have replaced r_0 by $r' = \alpha(T - T_c') = \chi_{1-loop}^{-1}$ since the error involved is beyond one loop order. The critical temperature is

$$T_c'(\theta) = \tilde{T}_c(\theta) - \frac{u_0 T_c \Lambda}{4\pi^2 a}. \tag{11}$$

Thus, the critical fluctuations have the effect of lowering the critical temperature. However, $T_c'(0) < T_c'(\theta)$ and therefore the topological term increases the degree of order with respect to the standard GL model. This effect is better elucidated if one uses Eq.(6) to rewrite (11) as

$$T_c'(\theta) = T_c + \frac{\Lambda T_c}{2\pi a}\left(q^2\overline{\theta} - \frac{u_0}{2}\right), \tag{12}$$

where we have chosen to measure θ in units of Λ, that is, $\theta = \Lambda\overline{\theta}$. From Eq. (12) we see that the effect of the critical fluctuations can be supressed by the topological mass. In fact, if $u_0 = 2q^2\overline{\theta}$ we have $T_c' = T_c$. Thus, the topological term acts as a factor which compensates the disorder introduced by the critical fluctuations. By attributing typical BCS values to both q^2 and u_0 in Eq.(12) we get that $\overline{\theta} = u_0/2q^2 \sim 10^{-2}$ in order to suppress the effects of the fluctuations of the order parameter on the critical temperature. Since for a typical BCS situation $u_0/2q^2 \sim 10^{-2}$ we may choose $\overline{\theta}$ such that $\overline{\theta} >> u_0/2q^2$ and $T_c' \approx \tilde{T}_c$. Therefore, in a topologically massive GL model the critical temperature can be considerably enhanced even when the critical fluctuations are taken into account. The fluctuations arising from the vector potential $\vec{A}$ dominates over the fluctuations of the order parameter.

The critical behavior is better analysed through renormalization group (RG) techniques. The case with $\theta = 0$ was already analysed by many authors [4]. The RG study in the ultraviolet limit was carried over in the case of Chern-Simons scalar QED without a self-coupling of the scalar field and show a trivial behavior of the Chern-Simons coupling, at least in the context of perturbation theory [9]. We are interested in the infrared behavior and, therefore, the ultraviolet cutoff is kept fixed. We shall work with Wilson's version of the RG in its perturbative form [10]. Although the presence of the cutoff spoils gauge invariance, it can be shown that as soon as the cutoff is removed gauge invariance is recovered [11]. In fact, many studies using Wilson's RG are being performed in gauge theories [12]. Up to one loop order it is possible to perform an ϵ-expansion because the antisymmetric part of the vector field propagator does not contribute at this order. Alternatively, if we want higher orders we can use dimensional reduction techniques to obtain the desired power of ϵ.

The RG equations are obtained by integrating out the fast modes and performing an appropriate rescaling of the momentum variables and fields. The flow equations are better expressed in terms of dimensionless parameters defined through $r = \Lambda^2\overline{r}$, $u = S_d^{-1}\Lambda^{4-d}\overline{u}$, $f = q^2 = S_d^{-1}\Lambda^{4-d}\overline{f}$ and $\theta = \Lambda\overline{\theta}$ where $S_d = 2^{1-d}\pi^{-d/2}/\Gamma(d/2)$. The result, extrapolated to $4 - d = 1$, is given by

$$\frac{d\overline{r}}{dt} = (2-\eta)\overline{r} + \frac{2\overline{u}}{3(1+\overline{r})} + \frac{3\overline{f}}{(1+\overline{\theta}^2)} \tag{13}$$

$$\frac{d\overline{u}}{dt} = (1-2\eta)\overline{u} - \frac{5\overline{u}^2}{3(1+\overline{r})^2} - \frac{18\overline{f}^2}{(1+\overline{\theta}^2)^2} \tag{14}$$

$$\frac{d\overline{f}}{dt} = (1-\eta_A)\overline{f} \tag{15}$$

where the anomalous dimensions for the scalar and gauge fields are given respectively by

$$\eta = -3\frac{\overline{f}}{(1+\overline{r})(1+\overline{\theta}^2)} \tag{16}$$

and

$$\eta_A = \frac{\overline{f}}{3(1+\overline{r})}. \tag{17}$$

There is no flow for the parameter $\overline{\theta}$ as expected perturbatively. From the above flow equations it is readily seen that all critical exponents will depend on $\overline{\theta}$. Therefore, the parameter $\overline{\theta}$ drive the system into different universality classes.

The fixed point structure is well known for $\overline{\theta} = 0$. It is found that the superconducting fixed point $(\overline{r}^*, \overline{u}^*, \overline{f}^*)$ has a complex value for $\overline{u}^*$ if the number of components of the order parameter is less than 365.9 [13]. This behavior is usually interpreted as indicating a first order phase transition driven by fluctuations.

However, this behavior is changed for $\overline{\theta} > 0$. In this situation, real superconducting fixed points are found. Typically, we find two physical superconducting fixed points, one with two attractive infrared directions and one infrared repulsive while the other one has two infrared repulsive directions and one attractive. For instance, for $\overline{\theta} = 5$ and $\overline{\theta} = 10$ we find the following fixed points with two infrared attractive directions $(-0.28, 2.16, 0.44)$ and $(-0.21, 0.43, 2.37)$, respectively. This type of behavior confirms the already mentioned fact that the topological mass damps the critical fluctuations of the order parameter. Indeed, for very large values of $\overline{\theta}$, the critical exponents tend to their mean field values. For example, for $\overline{\theta} = 100$ we find for the exponent of the order parameter correlation function, the value $\eta = -0.0006$. We conclude that the mean field exponents becomes exact for very large values of the topological mass.

To summarize, we studied a topologically massive version of the GL model. The mean field theory gives a critical temperature larger than the usual one by a factor proportional to the topological mass. This gives an enhancement of the critical temperature driven by topological effects. Even when the fluctuations of the order parameter are taken into account this enhancement persists and we conclude that the topological term has the effect of increase the order of the system. In this sense the topological term may be thought as an inductor of higher critical temperatures in unconventional superconductors. The fixed point structure exhibits superconducting fixed points with real values for $\overline{u}^*$ if $\overline{\theta} > 0$. The critical exponents have a $\overline{\theta}$ dependence such that they tend to the mean field values for $\overline{\theta}$ very large. This means that the Chern-Simons term has the effect of stabilize the critical fluctuations.

It is an interesting question if this topological macroscopic theory could have some relevance to the HTSC case. In order to discuss this case it is necessary to improve considerably the proposed model. The HTSC are highly anysotropic and we are dealing here with an isotropic situation. Also, the coherence length is very small as compared to the coherence length of conventional superconductors and the effects of the critical fluctuations are very important. This means that a more careful study of the RG equations is necessary with an explicit evaluation of the critical exponents. However, it is quite remarkable that the critical region for these materials are large as compared to conventional superconductors due to the smallness of the correlation lenght. Therefore, Ginzburg-Landau like models are suitable for a macroscopic description of the superconducting phase transition in HTSC.

We hope that this note could estimulate further investigation on the subject, both from the theoretical and experimental point of view.

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