CBPF-NF-051/01

Affine Malcev Algebra and $N = 8$ KdV*

F. Toppan

CBPF - CCP, Rua Dr. Xavier Sigaud 150, cep 22290-180, Rio de Janeiro (RJ) Brazil
E-mail: toppan@cbpf.br

In this talk I report the results of two recent papers concerning the realization of the $N = 8$ supersymmetry from the division algebra of the octonions. At first I discuss a Sugawara realization for the "Non-associative $N = 8$ SCA" in terms of a superaffinization of the algebra of octonions. Next, I discuss the fact that the $N = 8$ SCA provides a generalized Poisson brackets structure for an $N = 8$ super-KdV.

Key-word: Supersymmetry.

*Talk given at the 9th International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY'01) Dubna, June 2001. To appear in the proceedings.

1 Introduction

In this talk I report some results obtained in two recent papers. In the first one [1], the superaffinization of the octonionic algebra has been introduced extending a procedure originally introduced in [2] in the purely bosonic case. In [1] it has been further proven that such superaffinization is related (via a suitable Sugawara limiting procedure) to the so-called "Non-associative $N = 8$ Superconformal algebra" first introduced in [3]. This algebra is constructed with the help of the octonionic structure constants and does not fulfill the Jacobi identities (due to the non-associativity of the octonions). This is the reason why it presents a central extension, overcoming a no-go theorem which states that no central extensions are allowed for *ordinary* superconformal algebras with $N > 4$. The presence of the central extension makes it tempting to consider the $N = 8$ SCA as a generalized Poisson brackets structure which induces a generalized KdV dynamics with large N supersymmetries. This is the content of [4], in which the first example of an $N = 8$ KdV has been constructed. Previous examples of supersymmetric extensions of KdV were given in [5] for the $N = 2$ case and in [6] in the $N = 4$ case.

The existence of both the $N = 8$ KdV and the $N = 8$ Sugawara realization relating the affine superfields with the superconformal ones, induces an $N = 8$ dynamics on the affine superfields which generalizes both the NLS and the mKdV systems of equations.

2 The $N = 8$ superaffine Malcev algebra

It consists of a superalgebra satisfying the Malcev property. It is constructed in terms of manifest $N = 1$ superfields $\Psi_a(X)$ and the totally antisymmetric octonionic structure constants C_{abc} (Greek indices run from 1 to 7, while Latin indices from 0 to 7), whose only non-vanishing components are given by

$$C_{123} = C_{147} = C_{165} = C_{246} = C_{257} = C_{354} = C_{367} = 1. \quad (1)$$

The algebra is explicitly given by

$$\{\Psi_a(X), \Psi_b(Y)\} = 2C_{abc}\Psi_c(Y)\delta(X,Y) + k \cdot \Pi(\tau_a \cdot \tau_b) D_Y \delta(X,Y), \quad (2)$$

where k is an affine central charge and $\Pi(\tau_a \cdot \tau_b)$ denotes the projection over the identity $\mathbf{1}$ in the composition law of the octonions τ_a's.

The existence under a suitable limit of a Sugawara reconstruction formula for the fields entering the $N = 8$ SCA from the affine fields $\Psi_a(X)$, guarantees in particular that (2) is $N = 8$ supersymmetric. While the Sugawara formula will not be reported here, the "Non-associative $N = 8$ SCA" is explicitly given by

$$\begin{aligned}
\{T(x), T(y)\} &= -\frac{1}{2}{\partial_y}^3\delta(x-y) + 2T(y)\partial_y\delta(x-y) + T'(y)\delta(x-y),\\
\{T(x), Q(y)\} &= \frac{3}{2}Q(y)\partial_y\delta(x-y) + Q'(y)\delta(x-y),\\
\{T(x), Q_\alpha(y)\} &= \frac{3}{2}Q_\alpha(y)\partial_y\delta(x-y) + {Q_\alpha}'(y)\delta(x-y),
\end{aligned}$$

$$
\begin{aligned}
\{T(x),J_\alpha(y)\} &= J_\alpha(y)\partial_y\delta(x-y)+J_\alpha{}'(y)\delta(x-y),\\
\{Q(x),Q(y)\} &= -\frac{1}{2}\partial_y{}^2\delta(x-y)++\frac{1}{2}T(y)\delta(x-y),\\
\{Q(x),Q_\alpha(y)\} &= -J_\alpha(y)\partial_y\delta(x-y)-\frac{1}{2}J_\alpha{}'(y)\delta(x-y),\\
\{Q(x),J_\alpha(y)\} &= -\frac{1}{2}Q_\alpha(y)\delta(x-y),\\
\{Q_\alpha(x),Q_\beta(y)\} &= -\frac{1}{2}\delta_{\alpha\beta}\partial_y{}^2\delta(x-y)+C_{\alpha\beta\gamma}J_\gamma(y)\partial_y\delta(x-y)+\\
&\quad +\frac{1}{2}(\delta_{\alpha\beta}T(y)+C_{\alpha\beta\gamma}J_\gamma{}'(y))\delta(x-y),\\
\{Q_\alpha(x),J_\beta(y)\} &= \frac{1}{2}(\delta_{\alpha\beta}Q(y)-C_{\alpha\beta\gamma}Q_\gamma(y))\delta(x-y),\\
\{J_\alpha(x),J_\beta(y)\} &= \frac{1}{2}\delta_{\alpha\beta}\partial_y\delta(x-y)-C_{\alpha\beta\gamma}J_\gamma(y)\delta(x-y).
\end{aligned}
\tag{3}
$$

3 The $N = 8$ KdV

The $N = 8$ SCA can be regarded as a generalized Poisson brackets structure. By looking for the most general hamiltonian with the right dimension and $N = 8$-invariant, we arrive at a unique solution (up to the normalizing factor), namely

$$
H = \int dx(-T^2 - Q'Q - Q'_\alpha Q_\alpha + J''_\alpha J_\alpha), \tag{4}
$$

The equations of motion derived from (4) with the $N = 8$ SCA Poisson brackets correspond to the $N = 8$ generalization of KdV. They are explicitly given by

$$
\begin{aligned}
\dot{T} &= -T''' - 12T'T - 6Q''_a Q_a + 4J''_\alpha J_\alpha,\\
\dot{Q} &= -Q''' - 6T'Q - 6TQ' - 4Q''_\alpha J_\alpha + 2Q_\alpha J''_\alpha - 2Q'_\alpha J'_\alpha,\\
\dot{Q}_\alpha &= -Q_\alpha{}''' - 2QJ''_\alpha - 6TQ'_\alpha - 6T'Q_\alpha + 2Q'J'_\alpha + 4Q''J_\alpha -\\
&\quad 2C_{\alpha\beta\gamma}(Q_\beta J''_\gamma - Q'_\beta J'_\gamma - 2Q''_\beta J_\gamma),\\
\dot{J}_\alpha &= -J_\alpha{}''' - 4T'J_\alpha - 4TJ'_\alpha + 2QQ'_\alpha + 2Q'Q_\alpha - C_{\alpha\beta\gamma}(4J_\beta J''_\gamma + 2Q_\beta Q'_\gamma).
\end{aligned}
\tag{5}
$$

Acknowledgments

It is a pleasure for me to acknowledge my collaborators H.L. Carrion and M. Rojas from CBPF.

References

[1] H.L. Carrion, M. Rojas and F. Toppan, "An $N = 8$ Superaffine Malcev Algebra and Its $N = 8$ Sugawara", Preprint CBPF-NF-011/01.

[2] E.P. Osipov, *Phys. Lett.* **B 214**, 371 (1988); *Lett. Math. Phys.* **18**, 35 (1989).

[3] F. Englert *et al.* *J. Math. Phys.* **29**, 281 (1988).

[4] H.L. Carrion, M. Rojas and F. Toppan, "Division Algebras and Extended $N = 2, 4, 8$ Super-KdVs", Preprint CBPF-NF-012/01.

[5] P. Mathieu, *Phys. Lett.* **B 203**, 287 (1988); C.A. Laberge and P. Mathieu, *Phys. Lett.* **B 215**, 718 (1988); P. Labelle and P. Mathieu, *J. Math. Phys.* **32** 923 (1991).

[6] F. Delduc and E. Ivanov, *Phys. Lett.* **B 309**, 312(1993); F. Delduc, E. Ivanov and S. Krivonos, *J. Math. Phys.* **37** 1356 (1996).