Non-Parallel Electric and Magnetic Fields in a Gravitational Background, Stationary Gravitational Waves and Gravitons

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The existence of an electromagnetic field with parallel electric and magnetic field components in the presence of a gravitational field is considered. A non-parallel solution is shown to exist. Next, we analyse the possibility of finding stationary gravitational waves in nature. Finally, we construct a $D = 4$ effective quantum gravity model. Tree-level unitarity is verified.

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I Electric and Magnetic Field in a Gravitational Background

Based on a series of papers by Brownstein [1] and Salingaros [2], we consider here the possibility of the existence of an eletromagnetic field whose electric and magnetic field components are parallel in the presence of a gravitational field. The coupling between the electromagnetic sector and the gravitational backgrounds is accomplished by means of the action

$$S = \int \sqrt{-\tilde{g}} \, \left(-\frac{1}{4} \, F_{\mu\nu} F^{\mu\nu} \right) d^4x \ , \qquad (1)$$

where

$$\tilde{g} = det \, (g_{\mu\nu}) \ ,$$

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and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

From the above action, the following field-equations follow:

$$\mathcal{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu} + \Gamma^\beta_{\beta\lambda} F^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} F^{\mu\lambda} = J^\nu \ , \quad (2)$$

$$\mathcal{D}_\mu F_{\nu\beta} + \mathcal{D}_\nu F_{\beta\mu} + \mathcal{D}_\beta F_{\mu\nu} = 0 \ . \quad (3)$$

Choosing the background to be described by the F.R.W metric,

$$dS^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-Ar^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right] \ , \quad (4)$$

the Maxwell equations in the absence of electromagnetic sources are

$$\begin{aligned} \vec{\nabla}\cdot\vec{E} &= g\vec{\nabla}f\cdot\vec{E} \ , \quad (5)\\ \vec{\nabla}\cdot\vec{B} &= 0 \ , \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \ , \\ \vec{\nabla}\times\vec{B} &= \frac{\partial\vec{E}}{\partial t} - g\ \frac{\partial f}{\partial t}\ \vec{E} + g\vec{\nabla}f\times\vec{B} - \Gamma^i_{\mu\beta}F^{\mu\beta} \ , \end{aligned}$$

where

$$g = \frac{\sqrt{1-Ar^2}}{a^3 r^2 sin\theta} \ , \qquad f = \frac{a^3 r^2 sin\theta}{\sqrt{1-Ar^2}} \ , \quad (6)$$

and $A = +1, 0, -1$.

The wave-equations for $\vec{E}$ and $\vec{B}$ are found to be:

$$\begin{aligned} \nabla^2\vec{E} - \frac{\partial^2\vec{E}}{\partial t^2} &= \vec{\nabla}(g\vec{\nabla}f\cdot\vec{E}) - \ \frac{\partial g}{\partial t}\ \frac{\partial f}{\partial t}\vec{E} - g\ \frac{\partial^2 f}{\partial t^2}\vec{E} - g\ \frac{\partial f}{\partial t}\ \frac{\partial\vec{E}}{\partial t} + \quad (7)\\ &\frac{\partial g}{\partial t}\vec{\nabla}f\times\vec{B} + g\ \frac{\partial}{\partial t}\vec{\nabla}f\times\vec{B} + g\vec{\nabla}f\times\frac{\partial\vec{B}}{\partial t} - \frac{\partial}{\partial t}(\Gamma^i_{\mu\nu}F^{\mu\beta}) \ , \\ \nabla^2\vec{B} - \frac{\partial^2\vec{B}}{\partial t^2} &= \vec{\nabla}\times\left(g\ \frac{\partial f}{\partial t}\vec{E} - g\vec{\nabla}f\times\vec{B} + \Gamma^i_{\mu\beta}F^{\mu\beta}\right) \ . \quad (8) \end{aligned}$$

Now, due to the presence of the gravitational background, we have explicitly built up a solution for $\vec{E}$ and $\vec{B}$ that is different from the one obtained by Brownstein [1] and Salingaros [2]. These authors state that it is always possible to find solutions for parallel $\vec{E}$ and $\vec{B}$ in plasma physics or in an astrophysics plasma. However, contrary to their result, we have found non-parallel solutions due to the non-flat background of gravity:

$$\begin{aligned} \vec{E} &= \hat{i}\Big(\sin\theta G(r,t,\theta) - \cos\theta F(r,t)\Big) ka\cos(kz)\cos(\omega t) + \quad (9)\\ &+\vec{j}\Big(\cos\theta F(r,t) - \sin\theta G(r,t,\theta) ka\ \sin(kz)\cos(\omega z)\Big) + \\ &+\vec{k}\Big[\Big(sin\ \theta\ \cos\ \varphi\ F(r,t) + \cos\ \theta\ \cos\ \varphi\ G(r,t,\theta)\Big)ka\ \cos(kz)\cos(\omega t) + \\ &-\Big(\sin\ \theta\ \sin\ \varphi F(r,t) + \cos\theta\ sin\ \varphi\ G_{(r,t,\theta)} ka\ \sin(kz)\cos(\omega t)\Big)\Big] \end{aligned}$$

and

$$\vec{B} = ka\big[\hat{i}\ \sin(kz) + \hat{j}\cos(kz)\big]\cos(\omega t) \quad (10)$$

where the functions $G(r,t,\theta) = \dfrac{a\cot g\theta}{3\dot{a}r}$ and

$$F(r,t) = \frac{2a}{3\dot{a}r} + \frac{Aar}{3\dot{a}(1-Ar^2)} \quad (11)$$

are the metric contribution.

II Stationary Gravitational Waves and Gravitons

Now, we analyse the possibility of finding stationary gravitational waves. From a phenomenological viewpoint, a distribution of black holes could play the role of knots for the non-propagating gravitational waves. We postulate the equation that may lead to this sort of waves to be of the form

$$R_{\mu\nu} = \kappa \Lambda h_{\mu\nu} \ , \quad (1)$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu} \ , \quad (2)$$

where Λ is the cosmological constant. These equations yield:

$$\partial_\beta \partial_\nu h^\beta_\mu + \partial_\beta \partial_\mu h^\beta_\nu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h^\beta_\beta = \Lambda h_{\mu\nu} \ . \quad (3)$$

Now, solutions of the form

$$h_{\mu\nu} = C_{\mu\nu}(z) f(t) \ , \quad (4)$$

$$h_{\mu\nu} = \begin{pmatrix} A_{00} & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & A_{00} \end{pmatrix} e^{i\tilde{k}z} cos\ \omega t \ , \quad (5)$$

can be found, where A_{00}, A_{11} and A_{12} are free parameters, whereas $\tilde{k} = \sqrt{\Lambda - \omega^2}$ is the wave number. Having in mind that Λ is a small number, the frequency ω must be extremely small. This forces us to search for a mechanism to detect such low-frequency stationary waves.

The equations of motion are derived from the Lagrangian density

$$\mathcal{L}_H = \frac{1}{2} H^{\mu\nu} \Box H_{\mu\nu} - \frac{1}{4} H \Box H - \frac{1}{2}\ H^{\mu\nu} \partial_\mu \partial_\alpha H^\alpha_\nu - \frac{1}{2}\ H^{\mu\nu} \partial_\nu \partial_\alpha H^\alpha_\mu - \frac{1}{2}\ \Lambda H^{\mu\nu} H_{\mu\nu} + \frac{1}{4}\ \Lambda H^2 \ , \quad (6)$$

where

$$H^\alpha_\nu = h^\alpha_\nu - \frac{1}{2}\ \delta^\alpha_\nu h$$

and the bilinear form operator of lagrangian (2.17) is given by

$$\Theta_{\mu\nu,\kappa\lambda} = (\Box - \Lambda) P^{(2)} - \Lambda P^{(1)}_m + \frac{5}{2} (\Box - \Lambda) P^{(0)}_s - \frac{(\Lambda + 3\Box)}{2}\ P^{(0)}_w$$
$$+ \frac{\sqrt{3}}{2} (\Lambda - \Box)\ P^{(0)}_{sw} + \frac{\sqrt{3}}{2}\ (\Lambda - \Box)\ P^{(0)}_{ws} \ , \quad (7)$$

and $P^{(i)}$, $i = 0, 1, 2$, are spin projection operators in the space of rank-2 symmetric tensors. The graviton propagator is given by:

$$\langle T(h_{\mu\nu}(x); h_{\kappa\lambda}(y))\rangle = i \Theta^{-1}_{\mu\nu,\kappa\lambda} \delta^4(x - y) \quad (8)$$

where

$$\Theta^{-1} = [X P^{(2)} + Y P^{(1)}_m + Z P^{(0)}_s + W P^{(0)}_w + R P^{(0)}_{sw} + S P^{(0)}_{ws}]_{\mu\nu,\kappa\lambda} \quad (9)$$

with

$$X = -\ \frac{1}{\Lambda - \Box} \ , \qquad Y = -\ \frac{1}{\Lambda} \ , \qquad Z = -\ \frac{\Lambda + 3\Box}{\Lambda^2 + 8\Lambda\Box - 9\Box^2} \ , \quad (10)$$
$$W = -\ \frac{5}{\Lambda - 9\Box} \ , \qquad R = -\ \frac{\sqrt{3}}{\Lambda + 9\Box} \qquad \text{and} \qquad S = -\ \frac{\sqrt{3}}{\Lambda + 9\Box} \ .$$

From this propagator, a current-current amplitude is obtained and the tree-level unitarity [3] is discussed. Three massive excitations are found: They are a spin-2 quantum with mass equal to $k^2 = \Lambda$ and two massive spin-0 quanta with masses equal to $k^2 = \Lambda$ and $k^2 = -\frac{1}{9}\Lambda$. The spin-2 is a physical one: the imaginary part of the residue of the amplitude at the pole $k^2 = \Lambda$ is positive, so that it does not lead to a ghost. It remains to be shown that the tachyonic pole, $k^2 = -\frac{1}{9}\Lambda$, is non-dynamical or decouples through some constraint on the sources.

We conclude, then, that in a gravitational background it is always possible to find non-parallel electric and magnetic fields. It is the gravitational field that breaks the parallel configuration of $\vec{E}$ and $\vec{B}$ [1,2]. Furthermore, a stationary gravitational wave equation is postulated and a particular solution is found. We argue that such a solution is likely to be found in Black Hole distributions. Finally, we set up an effective quantum gravity model where the necessary condition for the tree-level unitarity for the spin-2 sector is respected. The model is infrared finite though non-renormalizable in the ultraviolet limit.

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