ISSN 0029 - 3865

CBPF-NF-049/88

THE NUCLEON-AIR NUCLEI INTERACTION PROBABILITY LAW WITH RISING CROSS-SECTION

by

Helio M. PORTELLA* and F.M. de Oliveira CASTRO

Centro Brasileido de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Instituto de Física
Universidade Federal Fluminense
Outeiro de São João Batista, s/n
24210 - Niteroi, RJ . Brasil

ABSTRACT

In this paper, we obtained the negative binomial interaction probability law for the nucleon's interactions with the air-nuclei as a consequence of the respective diffusion equation. The nucleon-air nuclei interaction mean-free path rises with energy of the incident nucleon in the form $\frac{1}{\lambda_N(E)} = \frac{1+a\ell n E/E_o}{\lambda_N^o}$, E_o = 1TeV. In the case of λ_N = constant the distribution law is poissonian.

Key-words: Interaction probability law; Inelastic cross-section; Diffusion equation of nucleous.

If we suppose the average inelasticity (K_N) and the nucleon-air nuclei interaction mean free path (λ_N^o) to be constants, then a nuclen that makes "n" interactions in traversing $x(g/cm^2)$ of the atmosphere will have its energy reduced from $E' = E/(1-K_N)^n$ to E, so that the elementary energy contribution of the primary energy spectrum to the x level differential energy intensity is given by

$$G(E')dE' = G(E/(1-K_N)^n)\frac{dE}{(1-K_N)} \quad (1)$$

Now, assuming "a priori" that the probability of a nucleon making n interactions is given by the Poisson distribution

$$P_n(x) = e^{-x/\lambda_N^o} \frac{(x/\lambda_N)^n}{n!} \quad , \quad (2)$$

G. Brooke et al.[1] obtained for the total flux at the atmosphere depth x, the following expression

$$F_N(x,E) = e^{-x/\lambda_N^o} \sum_{n=0}^{\infty} \frac{(x/\lambda_N^o)^n}{n!} \frac{G(E/(1-K_N)^n)}{(1-K_N)^n} \quad (3)$$

F.M. de Oliveira Castro[2] used the successive approximation method to integrate the following differential equation that describes the diffusion of a nucleon in the atmosphere

$$\frac{\partial F_{N(x,E)}}{\partial_x} = - \frac{F_{N(x,E)}}{\lambda_N} + \frac{F_N(x,E/(1-K_N))}{(1-K_N).\lambda_N} \quad (4)$$

with the initial condition in the general form $F_N(O,E) = G(E)$ and obtained expression (3) without any hypothesis on the interaction's probability law.

In the similar way, and using the same successive approximation method, we integrate exactly the diffusion equation,

$$\frac{\partial F_{N(x,E)}}{\partial x} = \frac{-F_{N(x,E)}}{\lambda_N(E)} + \frac{F_N(x,E/(1-K_N))}{(1-K_N)\lambda_N(\frac{E}{(1-K_w)})} \quad (5)$$

with the initial condition, $F_N(0,E) = G(E)$, where $\lambda_N(E)$ is now the nucleon-air nuclei interaction mean free path expressed by

$$\frac{1}{\lambda_N(E)} = \frac{1 + a\ell n E/E_o}{\lambda_N^o} \quad , \quad E_o = 1\text{TeV}.$$

We obtained the exact solution[3]

$$F_N(x,E) = \sum_{n=0}^{\infty} e^{-x/\lambda_N(E'(1-K_N)^n)} \frac{\Gamma(z+1+n)}{\Gamma(z+1)n!}(1-(1-K_N)^{ax/\lambda_N^o})^n \; x$$

$$x \; \frac{G(E/(1-K_N)^n)}{(1-K_N)^n} \tag{6}$$

where

$\Gamma(z+n+1)$ and $\Gamma(z+1)$ are the usual gamma function,

$$Z = \frac{1 + a\ell n E/E_o}{a\ell n(1/(1-K_N))}$$

and

E' is the initial energy of a nucleon at $x = 0$.

The total nucleon flux $F_N(x,E)$ at depth $x(g/cm^2)$ can also be expressed in the following way

$$F_N(x,E) = \sum_{n=0}^{\infty} P_N(x,E')G(E')x\frac{1}{(1-K_N)^n} \tag{7}$$

where $P_n(x,E')$ is the probability that a nucleon with energy $E' = E/(1-K_N)^n$ at $x = 0$ interacts n times down to the depth x and has the following form

$$P_n(x,E) = e^{-x/\lambda_N(E'(1-K_N)^n)} \frac{\Gamma(z+1+)n}{\Gamma(z+1)n!}(1-(1-K_N)^{ax/\lambda_N^o})^n \tag{8}$$

In this case, is easily demonstrate that

$$\sum_{n=0}^{\infty} P_n(x,E') = 1,0 \tag{9}$$

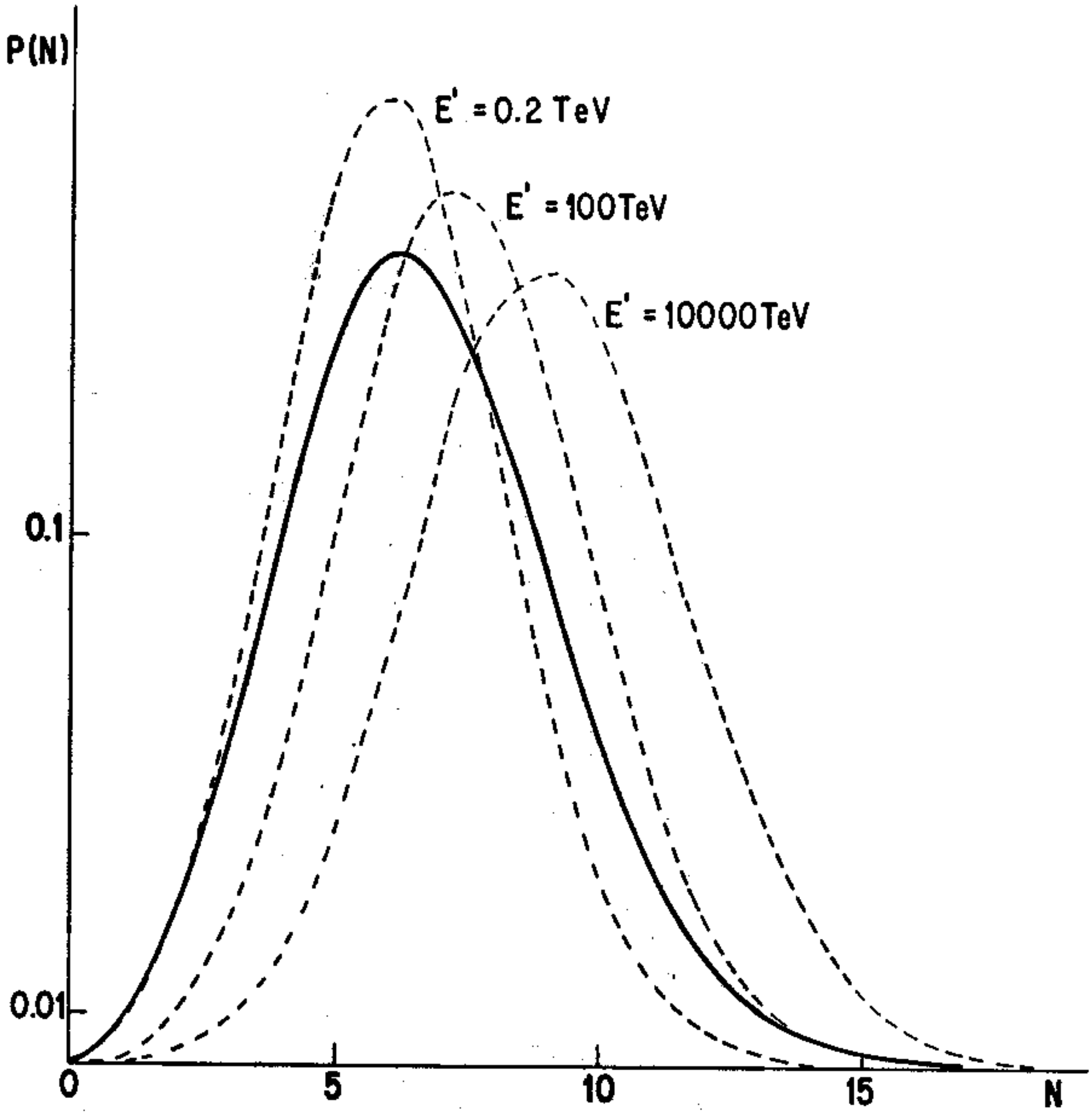
With the use of the values $K_N = 0.5, x = 540g/cm^2$ $\lambda_N = 80g/cm^2$, and $a = 0.06$, we show the dependence of $P_n(x,E')$ on E' and compare our solution with the Poisson distribution (case of λ_N^o constant). see fig. 1.

ACKNOWLEDGEMENT

The authors would express their thanks to members of Brazil-Japan Collaboration on Chacalthaya Emulsion Chamber Experiment specially to N. Arata, R.H.C. Maldonado, C.E. Navia Ojeda for valuable discussions and encouragement during the realization of this paper.

FIGURE CAPTION

Fig. 1 - Nucleon-air nuclei interaction probability law. Solid curves (——) Poisson law for $\lambda_N^0 = 80\ g/cm^2$ = constant broken curves (-·-·-·-) interaction probability law for $\lambda_N(E) = \lambda_N^0/(1+a\ell nE/E_0)$, E_0 = 1TeV, a = 0.06.


P(N)
E' = 0.2 TeV
E' = 100TeV
E' = 10000TeV
0.1
0.01
0
5
10
15
N

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