CBPF-NF-047/99

Spontaneous emission between an unusual pair of plates

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Abstract

We compute the modification in the spontaneous emission rate for a two-level atom when it is located between two parallel plates of different nature: a perfectly conducting plate ($\epsilon \to \infty$) and an infinitely permeable one ($\mu \to \infty$). We also discuss the case of two infinitely permeable plates. We compare our results with those found in the literature for the case of two perfectly conducting plates.

PACS numbers: 12.20.-m; 32.80.-t

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Using thermodynamic arguments and assuming that thermal equilibrium between matter and radiation is allways achieved, Einstein [1] was able to demonstrate that, besides stimulated emisson, excited atoms must also decay spontaneously. Even an "isolated" excited atom in vacuum must inevitably decay to the ground state. In other words, an excited stationary state of an atom is not actually a stationary state and we can say that spontaneous emission is in fact not a property of an isolated atom, but of an atom-vacuum system [2]. In the context of QED, we can say that the ultimate reason for spontaneous emission of excited atoms is the interaction of the atom with the quantized electromagnetic field of the vacuum state. As a consequence, any modification in the vacuum electromagnetic field, caused for instance by cavities, can modify in principle the radiative properties of atomic systems. We can say that the presence of material walls in the vicinity of atomic systems renormalizes their transition frequencies as well as the widths of their spectral lines. The branch of physics that is concerned with the influence of the environment of an atomic system in its radiative properties is called generically Cavity QED and the above examples are only two among many others (for a review see for instance ref(s) [3,4]). Here we shall be concerned with one of the above effects, namely, the influence of boundary conditions (BC) imposed on the radiation field in the spontaneous emission rate of a two-level atom. It is worth mentioning that Cavity QED was born precisely by the observation of Purcell [5] half a century ago that spontaneous emission process associated with nuclear magnetic moment transitions at radio frequencies could be enhanced if the system were coupled to a ressonant external electric circuit placed in the vicinity of the system. However, we can say that the first detailed papers on this subject were those written by Casimir and Polder [6] in which, among other things, forces between polarizable atoms and metallic walls were treated, and by Casimir in his seminal work that brought about the Casimir effect [7]. Since then, Cavity QED has attracted the attention of many physicists, both theoretical and experimentalists. Particularly, the effects of the proximity of plane walls to atomic systems have been investigated: for instance, Morawitz [8] discussed both classically and quantum-mechanically the influence of a plane mirror in the spontaneous emission rate of a two-level atom. A few years later, Drexhage [9] observed experimentally the oscillatory behaviour of the lifetime on the distance to the mirror. The QED of charged particles between two parallel mirrors was discussed extensively by Barton [10,11], who was the first to compute explicitly the influence of two parallel perfect conducting plates in the spontaneous emission rate for a spherically averaged atomic transition [10]. Barton's result was rederived by Philpott [12] with a similar method and by Milonni and Knight [13] in the context of the image method. An interesting feature of the modified spontaneous emission rate between two conducting mirrors is the fact that for the case of a transition dipole moment parallel to the plates there must be a strong suppression for $2L/\lambda_0 < 1$, where L is the distance between the plates and λ_0, the transition wavelength (see for instance ref. [14]). This inhibited spontaneous emission has been observed experimentally by Hulet, Hilfer and Kleppner [16]. Many other interesting experiments have been done and we suggest for the interested reader the reviews by Haroche and Kleppner [2] and Hinds [17] and references therein.

In this letter we compute the spontaneous emission rate for a two-level atom when it is located between two parallel plates of different nature ($\epsilon \to \infty$ and $\mu \to \infty$) and between two infinitely permeable plates ($\mu \to \infty$), and then, we compare our results with those found in the literature [10,12,13] for the case of two perfectly conducting plates. Though analogous, the results are different, since when we change the boundary conditions on the photon field, the vacuum field modes also change. As expected, a strong suppression also occurs for both cases treated here. However, curious as it may seem, this suppression occurs when the transition dipole moment is perpendicular to the plates, in contrast to the suppression when the dipole moment is parallel to the plates that occurs for the two perfectly conducting plates.

Our starting point is the general expression for the spontaneous emission rate of a transition $2 \to 1$ of a two-level atom, which is given by:

$$A_{21}(\boldsymbol{r}) = \frac{4\pi^2\omega_0{}^2}{\hbar}\sum_{\alpha}\frac{1}{\omega_\alpha}|\boldsymbol{A}_\alpha(\boldsymbol{r})\cdot\boldsymbol{d}_{12}|^2\delta(\omega_\alpha - \omega_0), \tag{1}$$

where ω_0 corresponds to the transition frequency, $\boldsymbol{d}_{12}$ is the transition dipole moment and each mode $\boldsymbol{A}_\alpha(\boldsymbol{r})$ of the vacuum field is characterized by a wave vector $\boldsymbol{k}$ and a polarization λ_0 (see for instance ref. [14]).

The first setup we will consider consists of two infinite parallel surfaces (the plates) one of which will be considered to be a perfect conductor ($\epsilon \to \infty$) while the other is supposed to be perfectly permeable ($\mu \to \infty$). Also, we will choose Cartesian axes in such a way that the axis $\mathcal{OZ}$ is perpendicular to both surafces. The perfectly conducting surface will be placed at $z = 0$ and the permeable one, at $z = L$. The electromagnetic fields must satisfy the following boundary conditions: **(a)** the tangential components E_x and E_y of the electric field as well as the normal component B_z of the magnetic field must vanish on the metallic plate at $z = 0$. **(b)** The tangential components B_x and B_y of the magnetic field must vanish on the permeable plate at $z = L$. It is convenient to work with the vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$ in the Coulomb gauge in which $\boldsymbol{\nabla}\cdot\boldsymbol{A}(\boldsymbol{r}, t) = 0$, $\boldsymbol{E}(\boldsymbol{r}, t) = -\partial\boldsymbol{A}(\boldsymbol{r}, t)/\partial t$ and $\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r}, \boldsymbol{t})$. With this choice of gauge, the above boundary conditions can be written as conditions imposed on the vector potential components:

$$A_x(x, y, 0) = A_y(x, y, 0) = \frac{\partial A_z}{\partial z}(x, y, 0) = 0 \tag{2}$$

$$\frac{\partial A_x}{\partial z}(x, y, L) = \frac{\partial A_y}{\partial z}(x, y, L) = A_z(x, y, L) = 0 \tag{3}$$

The mode functions for this case are [15]:

$$\boldsymbol{A}_{\boldsymbol{k}1}(\boldsymbol{r}) = \left(\frac{2}{V}\right)^{1/2}(\boldsymbol{k}_{\parallel}\times\hat{\boldsymbol{z}})\sin\left[(n+\frac{1}{2})\frac{\pi z}{L}\right]e^{i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}} \tag{4}$$

and

$$\boldsymbol{A}_{\boldsymbol{k}2}(\boldsymbol{r}) = \left(\tfrac{1}{k}\right)\left(\tfrac{2}{V}\right)^{1/2}e^{i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}}\times$$

$$\times\left\{k_{\parallel}\hat{\boldsymbol{z}}\cos\left[(n+\tfrac{1}{2})\tfrac{\pi z}{L}\right] - i\tfrac{\pi}{L}(n+\tfrac{1}{2})\boldsymbol{k}_{\parallel}\sin\left[(n+\tfrac{1}{2})\tfrac{\pi z}{L}\right]\right\} \tag{5}$$

The contributions for the spontaneous emission rate associated with $\boldsymbol{d}_{12}^{\perp}$ and $\boldsymbol{d}_{12}^{||}$ are given respectively by:

$$A_{21}^{\perp}(z) = \frac{3\pi}{k_0 L} A_{21}^{0\perp} \sum_{n=0}^{N} \left[1 - \frac{\left(n+\frac{1}{2}\right)^2}{k_0^2 L^2} \pi^2 \right] \cos^2 \left[(n+\frac{1}{2}) \frac{\pi z}{L} \right] \tag{6}$$

and

$$A_{21}^{||}(z) = \frac{3\pi}{2k_0 L} A_{21}^{0||} \sum_{n=0}^{N} \left[1 + \frac{\left(n+\frac{1}{2}\right)^2}{k_0^2 L^2} \pi^2 \right] \sin^2 \left[(n+\frac{1}{2}) \frac{\pi z}{L} \right] \tag{7}$$

where $A_{21}^{0\perp}$ and $A_{21}^{0||}$are the corresponding contributions for the spontaneous emission rate in unbounded (free) space, namely:

$$A_{21}^{0||} = \frac{4|\boldsymbol{d}_{12}^{||}|^2 \omega_0^3}{3\hbar c^3} \quad \text{and} \quad A_{21}^{0\perp} = \frac{4|\boldsymbol{d}_{12}^{\perp}|^2 \omega_0^3}{3\hbar c^3} \tag{8}$$

and N is the greatest integer part of $k_0 L/\pi - 1/2$. The total emission coefficient is given by $A_{21} = A_{21}^{\perp} + A_{21}^{||}$ Recall that Einstein's coefficient for spontaneous emission is simply given by

$$A_{21}^{0} = A_{21}^{0||} + A_{21}^{0\perp} = \frac{4|\boldsymbol{d}_{12}|^2 \omega_0^3}{3\hbar c^3} \tag{9}$$

The graph displayed in figure (1) shows the ratio between A_{21} and A_{21}^0 as a function of the dimensionless variable $s := z/\lambda_0$ for the case of two conducting plates (dashed line) and the case of a conducting plate and a permeable plate (solid line). Although the two curves are analogous, in the sense that both present oscillations with s, they are different curves since the mode functions of the vacuum field in each case are not the same. It is worth emphasizing the lack of symmetry of the latter curve around the point that is equidistant from the plates. This was expected because in this case the two plates correspond to distinct electromagnetic media, with different properties.

The second example we shall be concerned with consists of two perfectly permeable plates. The boundary conditions for this case can be cast into the form:

$$\frac{\partial A_x}{\partial z}(x, y, 0) = \frac{\partial A_y}{\partial z}(x, y, 0) = A_z(x, y, 0) = 0 \tag{10}$$

$$\frac{\partial A_x}{\partial z}(x,y,L) = \frac{\partial A_y}{\partial z}(x,y,L) = A_z(x,y,L) = 0 \tag{11}$$

The corresponding mode functions are:

$$\boldsymbol{A}_{\boldsymbol{k}_1}(\boldsymbol{r}) = \left(\frac{2}{V}\right)^{1/2} (\boldsymbol{k}_{\|} \times \hat{\boldsymbol{z}}) \cos{(n\frac{\pi z}{L})} e^{i\boldsymbol{k}_{\|}\cdot\boldsymbol{r}_{\|}} \tag{12}$$

and

$$\boldsymbol{A}_{\boldsymbol{k}_2}(\boldsymbol{r}) = \left(\tfrac{1}{k}\right)\left(\tfrac{2}{V}\right)^{1/2} e^{i\boldsymbol{k}_{\|}\cdot\boldsymbol{r}_{\|}} \times$$

$$\times \left\{ k_{\|}\hat{\boldsymbol{z}} \sin{(n\tfrac{\pi z}{L})} + i(\tfrac{n\pi}{L})\boldsymbol{k}_{\|} \cos{(n\tfrac{\pi z}{L})} \right\} \tag{13}$$

The contributions for the spontaneous emission rate associated with $\boldsymbol{d}^{\perp}_{12}$ and $\boldsymbol{d}^{\|}_{12}$ are given respectively by:

$$A^{\perp}_{21}(z) = \frac{3\pi}{k_0 L} A^{0\perp}_{21} \sum_{n=1}^{N} \left(1 - \frac{n^2\pi^2}{k_0^2 L^2}\right) \sin^2{(\frac{n\pi z}{L})} \tag{14}$$

and

$$A^{\|}_{21}(z) = \frac{3\pi}{2k_0 L} A^{0\|}_{21} \left\{ 1 + \sum_{n=1}^{N} \left(1 + \frac{n^2\pi^2}{k_0^2 L^2}\right) \cos^2{(\frac{n\pi z}{L})} \right\} \tag{15}$$

where N is the greatest integer part of $k_0 L/\pi$.

Figure (2) shows the ratio between A_{21} and A^0_{21} as a function of $s = z/\lambda_0$ for the case of two conducting plates (dashed line) and the case of two permeable plates (solid line). The curve for this latter case also presents oscillations in the spontaneous emission rate as the distance from the atom to each plate varies and is also symmetric with respect to the equidistant point to the plates. However, there is a remarkable difference between these two curves: whenever there is an enhancement in the spontaneous emission rate of the former, there will be a depletion for the latter and vice versa. Particularly, their behaviour near the plates are quite different.

The strong suppression that occurs in the case of two conducting plates for $A^{\|}_{21}$ has its counterpart in the two cases discussed previously, as we shall see. However, we should emphasize that in the case of two permeable plates as well as in the case of a conductting plate and a permeable one, the suppression occurs for $A_{21}^{\perp}$, in contrast with the case of two conducting plates. For simplicity, let us just fix the atom at a point equidistant from the parallel infinite plates in both setups and vary the distance L between the plates. Also, for convenience, in the remaining figures of this letter we shall plot the graphs of the ratios $A_{21}^{||}/A_{21}^{0||}$ and $A_{21}^{\perp}/A_{21}^{0\perp}$ as functions of the dimensionless parameter $l := L/\lambda_0$. Figure (3) shows jointly the suppression of $A_{21}^{\perp}$ for the case of two permeable plates (solid line) and the suppression of $A_{21}^{||}$ for the case of two conducting plates (dashed line). Observe that both occur for the same value of the distance between the plates. Though not obvious, this result is quite reasonable, since for the case of two permeable plates the mode functions of the vacuum field are also symmetric with respect to $s = l/2$. In this sense, for the case of a conducting plate and a permeable one, for which mixed boundary conditions are used, it is natural to expect that the suppression will occur for a different value of l. This is indeed what happens and as it is shown separatly in figure (4), the suppression occurs for a value of l which is smaller than the value found for the other cases (shown in figure (3)). Concerning figures (3) and (4), a last comment is in order: observe that in figure (3) the distances between two successive peaks for $A_{21}^{||}/A_{21}^{0||}$ (dashed curve) or discontinuities in the derivative of $A_{21}^{\perp}/A_{21}^{0\perp}$ (continuous curve) are $\Delta l = \lambda_0$, in contrast with the value $\Delta l = \lambda_0/2$ observed for the discontinuities in the derivative of $A_{21}^{\perp}/A_{21}^{0\perp}$ in figure (4), since for the situation described by figure (4) there are no nodes for the vacuum modes at the midpoint between the two plates.

To conclude two final remarks. Firstly, it is very interesting to notice that though the Casimir energy density for the case of two perfectly parallel conducting plates is exactly the same as that for two infinitely permeable parallel plates, the influence of these two different surroundings in radiative properties of an atomic system (like the spontaneous emission rate of an atom) can be quite different. In other words, though the Casimir effect is "blind" to the change of the two conducting plates by the two infinitely permeable ones, the atom is not. The reason for that is simply because only the possible field frequencies enter in the calculation of the Casimir energy density, while the atom interacts directly with each vaccum field mode, it probes locally the vacuum field. Finally, we think it could be interesting to do experiments about the influence of the proximity of material walls in the spontaneous emission rate of atomic systems analogous to those mentioned before where conducting plates could be interchanged at will with permeable ones. Comparing the results thus obtained may add some new information to such an interesting problem as the atom-cavity interaction.

The authors acknowledge P.A.M. Neto and C. Lenz for valuable suggestions. D.T.A. would like to thank CAPES and UFPA for financial support and C.F. would like to thank CNPq for partial financial support.

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[1] A. Einstein, Z. Physik **18**, 121 (1917).

[2] Serge Haroche and Daniel Kleppner, Physics Today **january**, 24 (1989).

[3] S. Haroche, in *Fundamental Systems in Quantum Optics*,Les Houches Summer School, Session LIII, edited by J. Dalibard , J.-M. Raymond, and J. Zinn-Justin (North-Holland, Amsterdam,1992).

[4] *Cavity Quantum Electrodynamics*, ed. Paul R. Berman (Academic Press, 1994).

[5] E. Purcell, Phys. Rev. **69**, 681 (1946).

[6] H. B. G. Casimir and D. Polder, Phys. Rev. 73, 360 (1948).

[7] H.B.G. Casimir, Proc. K. Ned. Akad. Wet. **51** (1948) 793.

[8] H. Morawitz, Phys. Rev. **187** (1969), 1792.

[9] K.H. Drexhage, Sci. Am. **222** (March 1970) 108.

[10] G. Barton, Proc. Roy. Soc. Lond. **A320** (1970), 251.

[11] G. Barton, Proc. Roy. Soc. Lond. **A410** (1987), 141.

[12] M.R. Philpott, Chem. Phys. Lett. **19** (1973), 435.

[13] P.W. Milonni and P.L. Knight, Opt. Commun. **9** (1973), 119.

[14] P.W. Milonni, The Quantum Vacuum, Academic Press, San Diego 1994.

[15] *QED vacuum between an unusual pair of plates*, M.V. Cougo-Pinto, C. Farina, F. C. Santos e A. C. Tort, hep-th/9811062. To appear in J. Phys. **A**.

[16] Hulet, Hilfer and D. Kleppner, Phys. Rev. Lett. **55** (1985), 2137.

[17] E. A. Hinds, in *Advances in Atomic and Molecular Physics*, **Vol 20**, eds. D. Bates and B. Bederson (Academic Press, New York, 1990).

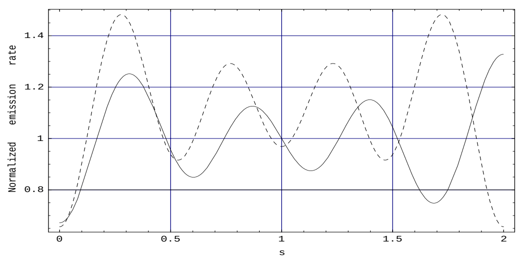

FIG. 1. The ratio A_{21}/A_{21}^0 as a function of the dimensionless variable $s = z/\lambda_0$ for the case of two perfectly conducting plates (dashed curve) and the case of a perfectly conducting plate and an infinitely permeable plate (solid curve). The range of the variable s is a typical one.

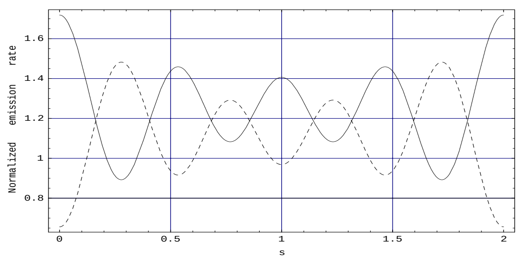

FIG. 2. The ratio A_{21}/A_{21}^0 as a function of the dimensionless variable $s = z/\lambda_0$ for the case of two perfectly conducting plates (dashed curve) and the case of two infinitely permeable plates (solid curve). The range of the variable s is a typical one.

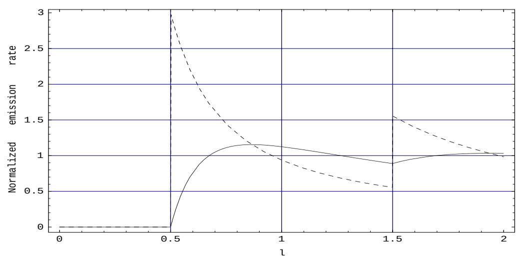

FIG. 3. The ratio $A_{21}^{||}/A_{21}^{0||}$ for the case of two perfectly conducting plates (dashed curve) and the ratio $A_{21}^{\perp}/A_{21}^{0\perp}$ for the case of two infinitely permeable plates (solid curve) as functions of the dimensionless variable $l = L/\lambda_0$.

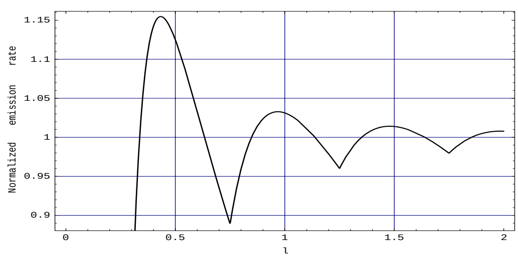

FIG. 4. The ratio $A_{21}^{\perp}/A_{21}^{0\perp}$ for the case of one perfectly conducting and one infinitely permeable plate. Suppression occurs at $l = L/\lambda_0 = 1/4$.